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JMMC

NOISE MODEL FOR INTERFEROMETRIC COMBINERS

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	First version, partially from copy of VSI report by G. Duchêne		
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	corrected typos, added more explanations		
3.0	28 Nov. 2017	GDuv	3.1
	Better table and caption.		
3.1	11 Jan. 2018	LBou	3.2 & 3.3
	Added thermal background noise in equations.		

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1 Scope

The scope of this memo is to provide estimates for the uncertainties on square visibilities and closure phases for interferometric combiners (aka "Noise Models").

2 Recombination Model

We want to model theoretical noise figures for "optical interferometry". We introduce **simple models** for **the interferometer** itself, understood as a mean to feed photons from a **science object** into an **interferometric recombinator** that produces after some **integration time** and **data reduction** several **interferometric observables**.

2.1 Common-Path

To be general, the telescopes beams are possibly corrected with an adaptive optics system and/or a tip-tilt system, as is currently the case on the VLTI. The value of Strehl can be estimated quite accurately on the basis of the object brightness in the optical and various typical atmospheric parameters using a similar estimator as was used for AMBER (memo amb-igr-011).

$transm$: transmission of flux. Can depend on each beam, however supposed constant in the following.

Mag_{lim}^{FT} : the limiting magnitude for use of the FT. This is a yes-no model : below Mag_{lim}^{FT} the FT works with 100% efficiency, above it is not used.

V_{inst}^{FT} : instrumental visibility loss induced by the Fringe Tracker.

N_{tel} : the number of incoming beams.

D : telescope diameter (supposed identical for simplicity).

θ : seeing (in arc sec)

N_{act} : number of actuators in the adaptive optics system. Minimum value is 1 for tip-tilt.

2.2 Recombiners

Recombiners are photon-counting detectors, even if they are more than often CCDs or IRCCDs which do not exactly "count" photons. Fringes are detected either temporally or spatially by reading several pixels of these detectors after some **integration time**. Thus the parameters to take into account are :

R : spectral resolution

λ : central wavelength of observation (thus the band used)

$\Delta\lambda$: spectral witdth of one element of spectral resolution

t_{int} : the individual integration time for a single fringe measurement **without FT**

t_{int}^{min} : the minimum individual integration time (can depend on the number of pixels read).

t_{int}^{max} : the maximum individual integration time of the detector (to saturation for example, or by design).

t_{int}^{tot} : the total integration time for a full measurement (i.e., the desired noise figure).

N_{obs} : the number of individual integrations of t_{int} producing t_{int}^{tot}.

σ_{det} : the readout noise of the detector.

Q_{det} : the detector Quantum Efficiency.

N_{th} : the number of thermal photons per channel in t_{int}.

V_{inst} : instrumental visibility loss. Can depend on each beam, however supposed constant in the following.

TABLE 1 – Reference values for a zeroth-magnitude star photometry. Note that F_0 is in $photons \cdot s^{-1} \cdot m^{-2} \cdot m^{-1}$.

Band	U	B	V	R	I	J
F_0	$6.694 \cdot 10^{16}$	$1.467 \cdot 10^{17}$	$1.016 \cdot 10^{17}$	$7.466 \cdot 10^{16}$	$5.026 \cdot 10^{16}$	$1.968 \cdot 10^{16}$
S_{Max}	0.3	0.48	0.5	0.65	0.75	0.77
λ_0	0.334	0.461875	0.556	0.6625	0.869625	1.2365
Δ_{λ_0}	0.066	0.08175	0.1105	0.10651	0.31176	0.426
Band	H	K	L	M	N	Q
F_0	$9.708 \cdot 10^{15}$	$4.741 \cdot 10^{15}$	$1.233 \cdot 10^{15}$	$6.582 \cdot 10^{14}$	$7.203 \cdot 10^{13}$	$5.641 \cdot 10^{12}$
S_{Max}	0.84	0.93	0.972	0.985	0.996	0.999
λ_0	1.679625	2.365625	3.45875	6.4035	11.63	16.575
Δ_{λ_0}	0.46425	0.912	1.2785	4.615	5.842	4.05

2.2.1 Multi-Axial Recombinators

In this section we assume that the recombiner is designed following a multi-axial recombination scheme (AMBER, MATISSE...). The output on the detector(s) typically to have all interference fringes on 1 "interferometric" row of pixels (hereafter referred to as "interferometric channel" or IC" and separate measurements of photometries (can be simultaneous or not) on $Ntel$ "photometric channels or PC". Fringes are generally stabilized with a Fringe Tracker (FT). Important factors in this design are the following :

f : the fraction of an entry beam that feeds into the interferometric channel.

f_p : the fraction of an entry beam that feeds into the photometric channel. Normally equals $1 - f$ except for non-simultaneaous photometric measurements.

N_{pix} : the number of pixels needed to read entirely the interferometric channel

N^p_{pix} : the number of pixels needed to read a single photometric channel.

F_{peak} : maximum flux per pixel in the linear regime of the detector.

F_{frac} : the fraction of the flux in the interferometric channel that falls in the peak pixel. Note that for the brightest objects, the total flux in the central pixel may become higher than F_{peak} in t_{int}. It is possible to "adapt" to this by reducing t_{int} towards t^{min}_{int}

2.2.2 MonoAxial Recombiners

Normally the above equations should hold with N_{pix} taken as the number of pixels read during a scan of the fringes.

2.3 Object

$V(u, v, \ldots)$: object visibility

mag_{obs} : object magnitude at fringes λ

mag_{FT} : object magnitude at FT wavelength.

mag_v : object magnitude at 0.5μm (needed for strehl estimate).

3 Equations

3.1 Photometric flux and errors

Let

$\bar{N}$: the number of photoevents in the interferometric channel ;

$\bar{N}_p$: the number of photoevents in a photometric channel ;

$\bar{N}$ and $\bar{N}_p$ are the number of detected photevents in the interferometric and individual photometric channels, respectively. They depend on the object brightness and on the throughput of the entire interferometer complex. Let us define N_{tot} as the total number of detectable photoevents arising from the observed object :

$$N_{tot} = F_0 \times 10^{0.4 \times mag_{obs}} \times t_{int} \times N_{tel} \times \pi D^2/4 \times \Delta\lambda \times trans \times S$$

where F_0 is the photon flux of a zeroth-magnitude star (in photons/m2/s/μm), $\Delta\lambda$ the spectral bandwith (per resolution element). F_0 is given in table 2.3 for usual bands. S is the Strehl ratio, see appendix A. From N_{tot} , we can calculate $\overline{N} = f_p N_{tot}$ and the photometric flux F_p^i of i^{th} photometric channel :

$$F_p^i = \overline{N_p} = f_p N_{tot}/N_{tel}$$

Then the variance is

$$\sigma^2(F_p^i) = \overline{N_p} + nb_{exp}(\overline{N_{th}^p} + N_{pix}^p \sigma_{det}^2)$$

where nb_{exp} gives the number of exposures per photometry measurement (2 for MATISSE chopping) and $\overline{N_{th}^p}$ the number of thermal background photons per telescope

3.2 Squared Coherent Flux

Here the estimator of the squared coherent flux is taken as the maximum of the spectral density peak at the fringe frequency $Q(f) = |F_c(f)|^2 = |\widehat{S}(f)|^2$, see VSI-SYS-001.

This estimate is biased including the thermal background photons N_{th} :

$$\overline{|\widehat{Biased\_F_c}(f)|^2} = \overline{N}^2 \frac{|V|^2}{N_{tel}^2} + \overbrace{\overline{N} + \overline{N_{th}} + N_{pix}\sigma_{det}^2}^{\text{bias}}$$

In general, a debiassing is done a posteriori and while the expected value of the squared coherent (debiassed) flux is $\overline{|\widehat{F_c(f)}|^2} = \overline{N}^2 \frac{|V|^2}{N_{tel}^2}$, its variance is still the variance of the biassed value :

$$\begin{aligned}
\sigma^2(\overline{|\widehat{F_c}(f)|^2}) = 2\overline{N}^3 \frac{|V|^2}{N_{tel}^2} + 4\overline{N}^2 \frac{|V|^2}{N_{tel}^2} + \overline{N}^2 + \overline{N}\ (photon\ noise) \\
+ N_{pix}^2 \sigma_{det}^4 + 3 N_{pix} \sigma_{det}^4\ (detector\ noise) \\
+ 2 N_{pix} \sigma_{det}^2 (\overline{N}^2 \frac{|V|^2}{N_{tel}^2} + \overline{N})\ (coupled\ terms) \\
+ \overline{N_{th}}^2 + \overline{N_{th}}\ (thermal\ background) \\
+ 2\overline{N_{th}}(\overline{N}^2 \frac{|V|^2}{N_{tel}^2} + \overline{N} + N_{pix}\sigma_{det}^2)\ (coupled\ terms)
\end{aligned}$$

Let's factorize $\overline{|\widehat{F_c(f)}|^2}$, the variance is :

$$\begin{aligned}
\sigma^2(\overline{|\widehat{F_c}(f)|^2}) = \overline{N}^2 \frac{|V|^2}{N_{tel}^2} (2\overline{N} + 2\overline{N_{th}} + 2N_{pix}\sigma_{det}^2 + 4) \\
+ (\overline{N} + \overline{N_{th}})^2 + (\overline{N} + \overline{N_{th}})(1 + 2N_{pix}\sigma_{det}^2) \\
+ (3 + N_{pix}) N_{pix} \sigma_{det}^4
\end{aligned}$$

Let $\overline{N_I} = \frac{\overline{N}}{N_{tel}}$ and $\overline{N_{th}^I} = \frac{\overline{N_{th}}}{N_{tel}}$, the interferometric and thermal background photons per telescope, then :

$$\begin{aligned}
\sigma^2(\overline{|\widehat{F_c}(f)|^2}) = \overline{N_I}^2 |V|^2 (2N_{tel}(\overline{N_I} + \overline{N_{th}^I}) + 2N_{pix}\sigma_{det}^2 + 4) \\
+ N_{tel}^2 (\overline{N_I} + \overline{N_{th}^I})^2 + N_{tel}(\overline{N_I} + \overline{N_{th}^I})(1 + 2N_{pix}\sigma_{det}^2) \\
+ (3 + N_{pix}) N_{pix} \sigma_{det}^4
\end{aligned}$$

3.3 Squared Visibility

The squared visibility being

$$\widetilde{|V^{ij}|}^2 \propto \frac{<|F^{ij}|^2>}{<F^iF^j>}$$

the relative error $\varepsilon(|V^{ij}|^2)$ on the square visibility is :

$$\varepsilon^2(|V^{ij}|^2) = \frac{1}{\mathrm{SNR}(|V^{ij}|^2)^2} = \frac{\sigma^2(|F_c^{ij}|^2)}{\overline{|F_c^{ij}|^2}^2} + \frac{\sigma^2(F^i)}{\widehat{F_p^i}^2} + \frac{\sigma^2(F^j)}{\widehat{F_p^j}^2}$$

As $\widehat{F_p^i}$ and $\widehat{F_p^j}$ are equivalent, then

$$\varepsilon^2(|V^{ij}|^2) = \frac{\sigma^2(|F_c^{ij}|^2)}{\overline{|F_c^{ij}|^2}^2} + 2\frac{\sigma^2(F^i)}{\widehat{F_p^i}^2}$$

It gives the detailled formula :

$$\varepsilon^2(|V^{ij}|^2) = \frac{2N_{tel}(\overline{N_I} + \overline{N_{th}^I}) + 2N_{pix}\sigma_{det}^2 + 4}{\overline{N_I}^2|V|^2}$$

$$+\frac{N_{tel}^2(\overline{N_I} + \overline{N_{th}^I})^2 + N_{tel}(\overline{N_I} + \overline{N_{th}^I})(1 + 2N_{pix}\sigma_{det}^2) + (3 + N_{pix})N_{pix}\sigma_{det}^4}{\overline{N_I}^4|V|^4}$$

$$+2\frac{\overline{N_p} + nb_{exp}(\overline{N_{th}^p} + N_{pix}^p\sigma_{det}^2)}{\overline{N_p}^2}$$

3.3.1 other estimates

VEGA estimates the variance of the energy of the fringe peak of the averaged spectral density, E_{fp} as $\sigma E_{fp} = \sqrt{\mathrm{N_{pix,fp}}\sigma_{dsp}}$ where $\mathrm{N_{pix,fp}}$ is the number of pixels support of the fringe peak and σ_{dsp} is the standard deviation of the spectral density noise (average outside peaks). They apply $F_c = 2\frac{E_{fp}}{E_{bf}}$ where E_{bf} is the flux of the continuum peak, i.e., $< F^i > + < F^j >$ in the above notation. This is not an estimate but a measure from the data, unavailable for simulators such as ASPRO. However they also estimate theoretically that

$$\mathrm{SNR_c} \simeq \frac{E_{fp}}{\sigma_{E_{fp}}}$$

which we can estimate as $\frac{\overline{|\widehat{F_c(f)}|^2}}{\sigma^2(|\widehat{F_c(f)}|^2}$ as above with no detector noise and $f = 1$.

PIONIER ?

4 The MATISSE instrument

LBO : To be updated & reviewed later

The SNR of the coherent flux is

$$\mathrm{SNR_c} = \frac{NV}{\sqrt{2(N + N_{th}) + 2N_{pix}\sigma_{det}^2}}$$

for the photometry

$$\mathrm{SNR_p} = \sqrt{\frac{6}{5}} \frac{N}{\sqrt{N + 2N_{th} + 2N_{pix}^{p}\sigma_{det}^2}}$$

for the squared visibility :

$$\mathrm{SNR_{v^2}} = \frac{1}{\sqrt{\frac{4}{\mathrm{SNR_c}^2} + \frac{2}{\mathrm{SNR_p}^2}}}$$

and for the variance of the phase closure

$$\sigma_{\Delta\phi}^2 \approx \frac{3}{2\mathrm{SNR_c}^2}$$

5 Calibration Errors

The previous equations give an estimate of the errors on single interferometric observations. However these are still the (true) object's observable (e.g., visibility) filtered by the instrument+atmosphere response. To retrieve the "true" value, il is necessary to divide by, (or substract, depending on the observable) the transfer function. The latter is estimated at regular time intervals on calibrators, i.e., objects whose interferometric observables are known.

The uncertainties on the transfer function due to the non-simultaneity with the observations limit the reachable accuracy and are at the origin of an additive and multiplicative "noise". One can mimic this behaviour as :

$$V_{cal} = visibilityCalibrationBias \times V + visibilityMinimumValue$$

$$V_{cal}^2 = (visibilityCalibrationBias)^2 \times V^2 + (visibilityMinimumValue)^2$$

$$\phi_{cal} = \phi + typicalCalibrationErrorOnPhaseClosure$$

A Strehl ratio

According to le Louarn et al (1998, mnras 295, 756), as reported in AMB-REP-001, the strehl ratio S can be simulated to a simple degree by a parametric function of the seeing θ.

θ is the FWHM of the Airy function due to the Fried coherence length of the atmosphere, $r0$:

$$\theta = 1.22\frac{\lambda}{r0}$$

It is important to note that the seeing θ usually refers to the V band ($\lambda = 0.5\mu m$). S is given by :

$$S = \exp(-\sigma_\phi^2) + \frac{1 - \exp(-\sigma_\phi^2)}{1 + (\frac{D}{r_0})^2}$$

with :

$$\sigma_\phi^2 = \sigma_{\mathrm{alias+fit}}^2 + \sigma_{\mathrm{photons}}^2 + \sigma_{\mathrm{fixed}}^2$$

$$\sigma_{\mathrm{alias+fit}}^2 = 0.87 N_{\mathrm{act}}^{5/3}$$

the 0.87 comes from COME-ON experience in AMB-REP-001. It is probably more realistic than the theoretical value of 0.54 (Le Louarn 1998). It depends on the type of AO used and is thus an AO-related parameter.

$$\sigma_{\mathrm{photons}}^2 = 1.59 \times 10^{-8} (\frac{D}{r_0})^2 (\frac{\lambda}{0.5\mu m})^{-2} N_{\mathrm{act}} 10^{0.4 mag_v}$$

$$\sigma_{\mathrm{fixed}}^2 = -\log S_{\mathrm{fixed}}$$

$$\frac{D}{r_0} = \frac{D}{1.22 * 0.5 \cdot 10^{-6}\mathrm{m}} \frac{\theta}{1\ \mathrm{arcsec}} (\frac{\lambda}{0.5\mu m})^{-6/5}$$

where r_0 is the Fried parameter, and S_{fixed} is a strehl ratio due to fixed abberrations (TBD typical for each interferometer).

B Thermal Emission

This is only useful for Mid-infrared instruments. P. Berio gives :

$$N_{th} = 0.025\ \pi\ D^2/4\ \Delta\Omega\ \Delta\lambda\ t_{int}\ B(\lambda)\ \frac{\lambda}{hc}$$

$$\Delta\Omega \simeq \frac{\pi\lambda^2}{2D^2}$$

$$B(\lambda) = \frac{2hc^2}{\lambda^5}\frac{1}{\exp\left[\frac{hc}{k\lambda T}\right] - 1}$$