Interferometric Image Reconstruction
4th work-package of Opticon FP7

Éric Thiébaut (CRAL), John Young (Cambridge), Gilles Duvert (IPAG/JMMC), Paulo Garcia (Univ. Porto), Martin Vannier (OCA), Jean-Baptiste le Bouquin (IPAG/JMMC), Guillaume Mella (IPAG/JMMC), Jörg-Uwe Pott (MPIA Heidelberg), Thibaut Paumard (LESIA)

JMMC, 17th of November 2015, Nice

Image reconstruction project for OPTICON FP7

Rationale and context

- with 4 to 6 telescopes recombiners, 2nd generation VLTI instruments and recent optical interferometers (*e.g.*, CHARA) are targeted at multi-spectral imaging ($R \sim 10^4$, $\Delta\theta \sim 10^{-3}$ arcsec)
- for the scientific returns of these instruments, tools for image reconstruction usable by non-expert astronomers are required
- image reconstruction algorithms for interferometry (BSMEM, WISARD, MiRA, *etc.*) are mature but require substantial expertise
- mostly provide monochromatic image reconstruction
- lack of documentation
- not necessarily freely available

Summary of the project

Make the R&D and tutorial/software development to provide image reconstruction algorithms from optical interferometry data to general astronomers.

Sub-workpackages

- WP1 – Project management (CRAL)
- WP2 – Data samples and astrophysical model images
  - model images of astrophysical objects (FEUP)
  - synthetic data for LINC-Nirvana, Gravity and Matisse (MPIA, LESIA, OCA)
  - real data from Amber, LINC-Nirvana, Vega/Chara and Pionier (IPAG, MPIA, OCA)
- WP3 – Image reconstruction algorithms
  - unified image reconstruction description (UC)
  - algorithms derived from BSMEM, MiRA and Wisard (UC, CRAL, OCA)
- WP4 – User interface and user guides
  - algorithm interface specification (CRAL)
  - graphical user interface (JMMC)
  - tests and benchmarks (JMMC)
  - documentation and cookbooks (FEUP)

Algorithm Zoo

Name	Authors	Optimization	Regularization
BSMEM	Buscher, Baron, Young	trust region gradient	MEM-prior
WISARD	Meimon, Mugnier, Le Besnerais	quasi-Newton$^{(\star)}$ plus self-calibration	many
MiRA	Thiébaut	quasi-Newton$^{(\star)}$	many
MACIM	Ireland, Monnier	simulated annealing	MEM
SQUEEZE	Baron, Monnier, Kloppenborg	parallel tempering	
BBM	Hofmann, Weigelt	matching pursuit	sparsity
IRBis	Hofmann, Weigelt	conjugate gradients	many
Sparco	Kluska	(based on MiRA)	
Self-Cal	Millour	(based on MiRA + self-calibration)	
Painter	Schutz *et al.*	ADMM	many
MiRA-3D	Soulez	ADMM	many

$^{(\star)}$ OptimPack

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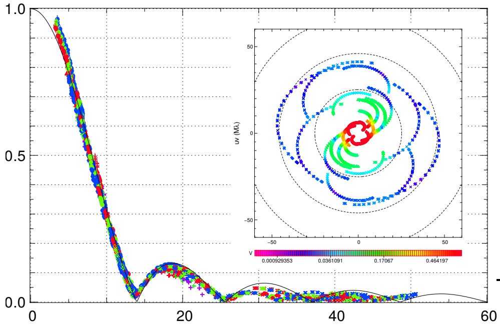
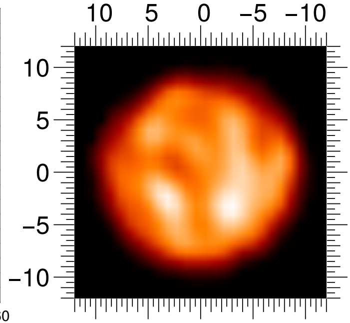
Reconstructiong a reliable image require expertise





- object: pi Gru
- instrument: Pionier
- credits: Claudia Paladini, Jean-Baptiste le Bouquin
- algorithm: MiRA with *carefully chosen*
  - priors
  - initial image

Objectives and Contents

- 1st deliverable of JRA
- formal description of image reconstruction in optical interferometry;
- **general:** all considered algorithms (BSMEM, WISARD, MiRA, etc.) can be expressed in this framework;
- required to homogenize and unify the inputs and outputs of these algorithms;
- **didactic:** give background information needed for the end users to understand the principle of image reconstruction (this knowledge is needed to properly use the software);
- document available at:
  `https://github.com/emmt/OI-Imaging-JRA`

Inverse approach provides a very general framework to describe most (if not all) image reconstruction algorithms.

The recipes involve:

1. a **direct model**: model of the brightness distribution and its Fourier transform;
2. a **criterion** to determine a unique and stable solution;
3. an **optimization strategy** to find the solution.

General image model

Object brightness distribution in angular direction $\boldsymbol{\theta}$:

$$I_\lambda(\boldsymbol{\theta}) = \sum\nolimits_n b_n(\boldsymbol{\theta})\,x_n \quad \text{with} \quad \begin{cases} b_n(\boldsymbol{\theta}) & \text{basis of functions} \\ \boldsymbol{x} \in \mathbb{R}^N & \text{image parameters} \end{cases}$$

$$\overset{\text{F.T.}}{\longmapsto} \quad \hat{I}_\lambda(\boldsymbol{\nu}) = \sum\nolimits_n \hat{b}_n(\boldsymbol{\nu})\,x_n$$

Complex visibility model

$$y_k = \hat{I}_\lambda(\boldsymbol{\nu}_k) = \sum\nolimits_n H_{k,n}\,x_n \quad \text{with} \quad \begin{cases} \boldsymbol{\nu}_k = \boldsymbol{B}_k/\lambda & \text{(sampled frequency)} \\ H_{k,n} = \hat{b}_n(\boldsymbol{\nu}_k) \end{cases}$$

in matrix notation:

$$\boldsymbol{y} = \mathbf{H}\cdot\boldsymbol{x}$$

Inverse problem approach for image reconstruction

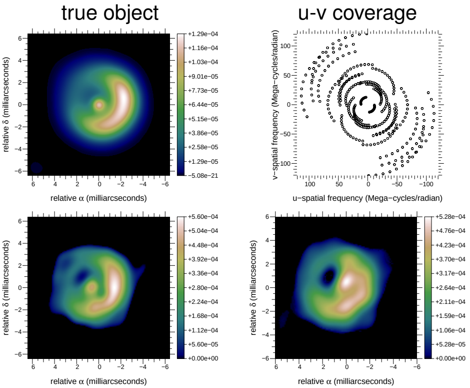
Image reconstruction amounts to solving an optimization problem

$$x^{+} = \underset{x \in \mathbb{X}}{\arg\min} \{ f_{\mathsf{data}}(\mathbf{H} \cdot x) + \mu \, f_{\mathsf{prior}}(x) \}$$

- f_{data} enforces **agreement with the data**;
- $\mathbf{H}$ implements the **direct model** (*e.g.*, nonequispaced Fourier transform)
- f_{prior} enforces **priors** ($\mu \geq 0$ is a tuning parameter);
- $\mathbb{X}$ enforces **strict constraints** (normalization, positivity):

$$\mathbb{X} = \left\{ x \in \mathbb{R}^N \;\middle|\; \sum_n x_n = 1; x_n \geq 0, \forall n = 1, \ldots, N \right\}$$

- **Wisard** (Meimon et al. 2005): phase closure + powerspectrum;
- **BSMEM** (Buscher 1994; Baron and Young 2008), **MiRA** (Thiébaut 2008): any available data;
- **BBM** (Hofmann and Weigelt 1993), **IRBis** (Hofmann et al. 2014): bispectrum;
- *etc.*
- consensus: data in **OI-FITS** format (Pauls et al. 2005)
- *no consensus: definition of $f_{\mathrm{data}}(x)$;*



reconstruction with powerspectrum and phase closures

reconstruction with powerspectrum only

Various regularizations

1. **Quadratic smoothness**:

$$f_{\text{prior}}(\boldsymbol{x}) = \|\boldsymbol{x} - \mathbf{S}\cdot\boldsymbol{x}\|^2$$

where $\mathbf{S}$ is a smoothing operator (by finite differences).

2-3. **Compactness** (le Besnerais et al. 2008):

$$f_{\text{prior}}(\boldsymbol{x}) = \sum_n w_n^{\text{prior}} \boldsymbol{x}_n^2$$

with $w_n^{\text{prior}} = \|\boldsymbol{\theta}_n\|^\beta$ and $\beta = 2$ or 3 yields ***spectral smoothness***.

4-5. **Non-linear smoothness:**

$$f_{\text{prior}}(\boldsymbol{x}) = \sum_n \sqrt{\|\nabla x_n\|^2 + \epsilon^2}$$

where $\|\nabla x_n\|^2$ is the squared magnitude of the spatial gradient in the image at nth pixel and $\epsilon \to 0$ yields **total variation** (Rudin et al. 1992) while $\epsilon > 0$ yields **edge-preserving smoothness** (Charbonnier et al. 1997).

Various regularizations (continued)

6-8. **Separable norms (ℓ_p):**

$$f_{\text{prior}}(\boldsymbol{x}) = \sum\nolimits_n \left(x_n^2 + \epsilon^2\right)^{p/2} \approx \sum\nolimits_n |x_n|^p$$

where $\epsilon > 0$ and $p = 1.5$, 2, and 3. Note that $p = 1$ is what is advocated in ***compress sensing*** (Donoho 2006) while $p = 2$ corresponds to regular ***Tikhonov regularization***.

9-11. **Maximum entropy methods** (Narayan and Nityananda 1986):

$$f_{\text{prior}}(\boldsymbol{x}) = -\sum\nolimits_n h(x_n; \bar{x}_n).$$

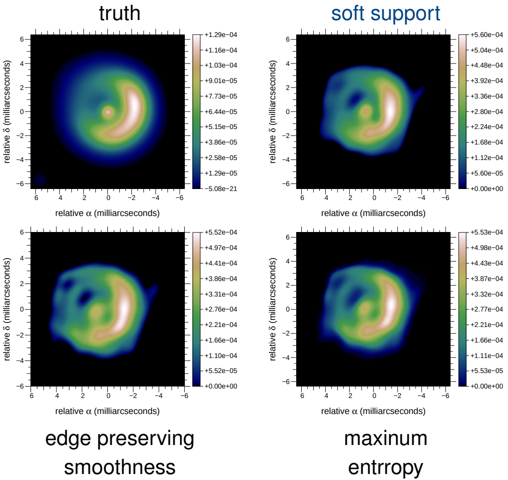
Here the prior is to assume that the image is drawn toward a prior model $\bar{\boldsymbol{x}}$ according to a non quadratic potential h, called the ***entropy***:

MEM-sqrt: $\quad h(x; \bar{x}) = \sqrt{x}\,;$

MEM-log: $\quad h(x; \bar{x}) = \log(x)\,;$

MEM-prior: $\quad h(x; \bar{x}) = x - \bar{x} - x\,\log\left(x/\bar{x}\right)\,.$

- there are many different possibilities
- a specific choice affects the result:
  - sparsity,
  - smoothness,
  - *etc.*
- users must be aware and choose wisely
- users must be encouraged to try different settings and compare



Tuning the regularization level

Observer has to choose the regularization level $\mu \geq 0$.

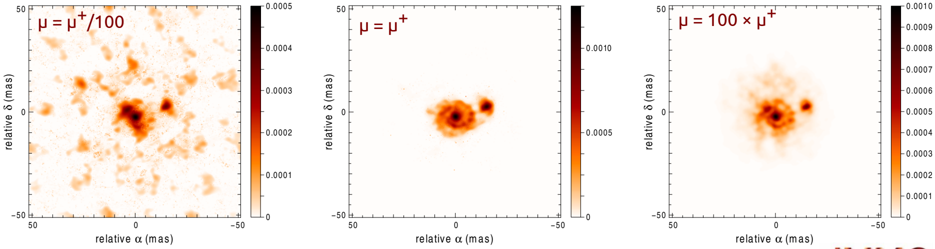
- by visual assesment of:

$$\boldsymbol{x}^{+} = \underset{\boldsymbol{x} \in \mathbb{X}}{\arg\min} \left\{ f_{\mathsf{data}}(\mathbf{H} \cdot \boldsymbol{x}) + \mu\, f_{\mathsf{prior}}(\boldsymbol{x}) \right\}$$

  a GUI will help

- by solving the equivalent problem:

$$\boldsymbol{x}^{+} = \underset{\boldsymbol{x} \in \mathbb{X}}{\arg\min}\, f_{\mathsf{prior}}(\boldsymbol{x}) \quad \text{s.t.} \quad f_{\mathsf{data}}(\mathbf{H} \cdot \boldsymbol{x}) \leq \eta$$



under-regularized | correct? | over-regularized

Algorithm Interface Specification$^{(*)}$

unified inputs and outputs of image reconstruction:

- input (in a FITS file):
  - OI-FITS for the data
  - image parameters (pixel size, *etc.*)
  - optional initial image
  - choice for the regularization and its parameters
- output (in a FITS file):
  - OI-FITS-like for the model of the data
  - current/final image

features:

- compatibility with OI-FITS (version 1 and 2)
- easy to resume a reconstruction or change parameters
- history maintained
- can be generalized to model fitting
- easy to display the results (image and actual fit to the data)

$^{(*)}$ draft available at: `https://github.com/emmt/OI-Interface-JRA`

Software Developments

Sharing data

- support for OIFITS $\rightarrow$ OIFITS-2 (in C/C++[1], in Julia[2], in Yorick[3])

Sharing Software

- make algorithms **freely available** (done)
- make software **portable** (at least easy to install)
  - current software: C/C++, IDL/GDL, FORTRAN, Yorick, MatLab
  - alternatives: Java, Julia, NumPy, ...
- make software **easy to use** (that's R&D in progress)
- provide **state of the art algorithms** (*e.g.*, massive rewrite of **OptimPack**[4] for numerical optimization with support in C, Yorick and Julia[5])
- **preserve future developments** of algorithms (multi-λ)

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[1] `https://github.com/jsy1001/oifitslib`
[2] `https://github.com/emmt/OIFITS.jl`
[3] `https://github.com/emmt/YOIFITS`
[4] `https://github.com/emmt/OptimPack`
[5] `https://github.com/emmt/OptimPack.jl`

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Achievements and Roadmap

(almost) done

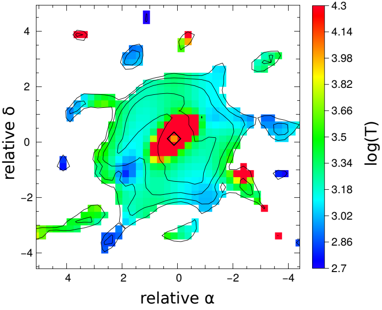
- unified description of image reconstruction
- interface specification
- portable code
  - OI-FITS-2
  - numerical libraries (optimization, *etc.*)

in progress

- modify algorithms (BSMEM, MiRA and WISARD) to account for input/output format
- design **graphical user interface** (GUI)
- test algorithms and interface on real and synthetic datasets
- write documentation and cookbooks

Perspectives: multi-wavelength reconstruction

- **a few years ago:**
  - reconstruction from **individual spectral slices** (*e.g.* le Bouquin et al. 2009)
- **now:**
  - **Sparco** (Kluska et al. 2014): semi-parametric approach
  - **MiRA-3D** (Soulez, *et al.* 2013): spatio-spectral regularization
  - **Painter** (Schutz et al. 2014)
  - **Self-Cal** (Millour)
  - *good side effects of sharing algorithms*
- **truly multi-λ image reconstruction is:**
  - much more powerful not only due to the improved u-v coverage
  - mandatory to fully exploit instruments (in particular GRAVITY and MATISSE)
  - more difficult to implement and more complex to use



Herbig Be HD98922, PIONIER data in H band, MiRA-3D algorithm (Soulez *et al.*, 2014)

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Partnership

- Lyon: CRAL (Éric Thiébaut)
  - image reconstruction (MiRA)
- Univ. Porto (Paulo Garcia)
  - sciences cases, cookbooks, tests
- Univ. Cambridge (John Young)
  - image reconstruction algorithm (BSMEM)
- MPIA Heidelberg (Jörg-Uwe Pott)
  - LINC-Nirvana (LBT) case
- IPAG/JMMC (Gilles Duvert, Guillaume Mella, Jean-Baptiste le Bouquin)
  - data from PIONIER and AMBER
  - graphical user interface
- OCA (Martin Vannier)
  - data from VEGA, simulations (MATISSE)
- LESIA (Thibaut Paumard)
  - simulation and test cases (GRAVITY)

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