Robust estimation of angular diameters of interferometric calibrators

Pierre CRUZALEBES

Fizeau Lab. UMR CNRS 6525

Nice Univ. – Côte d’Azur Obs.

Premières Rencontres JMMC – Grenoble 2-3 avril 2009

The need for calibrator

"true" visibility

$$V_{\text{sci,true}} = \frac{V_{\text{sci,raw}}}{R_V}$$

where

system response

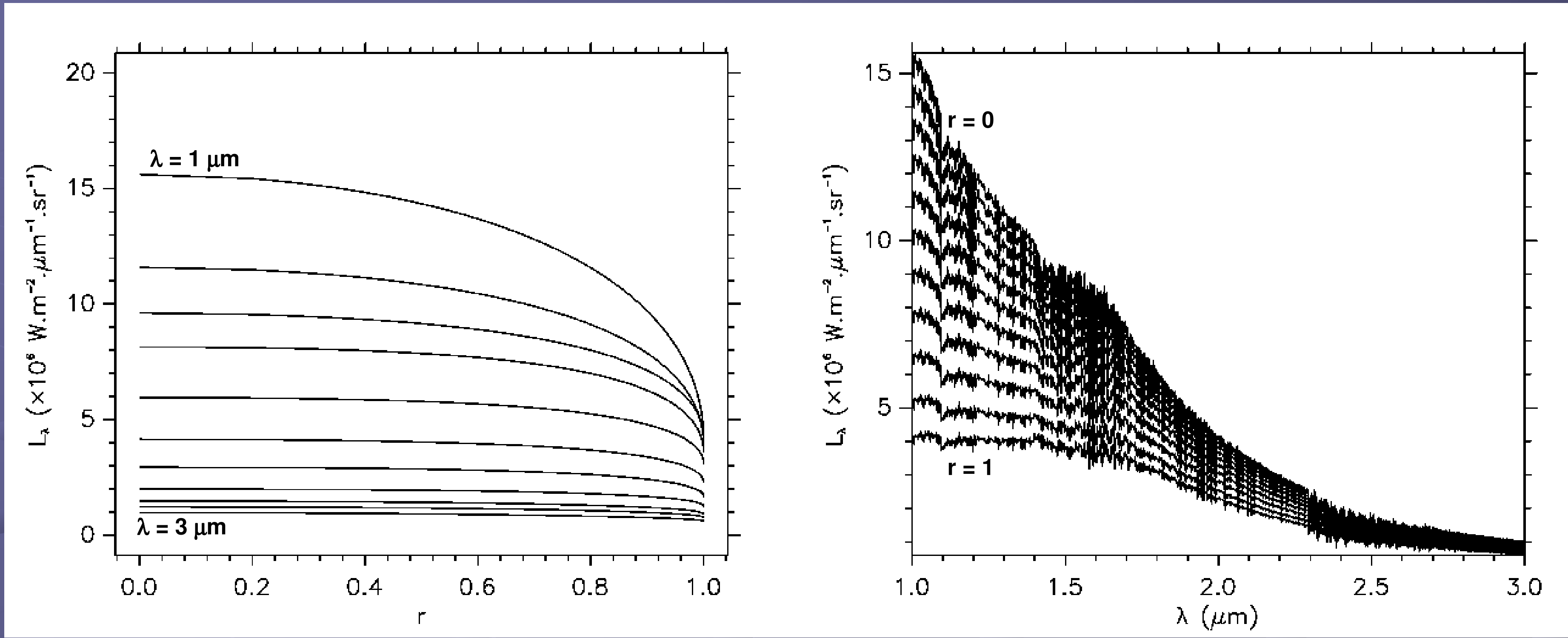
$$R_V = \frac{V_{\text{cal,raw}}}{V_{\text{cal,model}}}$$

circularly symmetric calibrator model :

$$V_{\text{model}}\left(f = B/\lambda\right) = \frac{\left|\int_0^1 L_\lambda(r) J_0(\pi r \phi f) r dr\right|}{\int_0^1 L_\lambda(r) r dr}$$

L_λ = model spectral radiance (emission intensity)

Example of model radiance



MARCS photospheric model :

T_{eff} = 4250 K

log(g) = 2.0

[Fe/H] = 0.0

ξ = 2 km/s

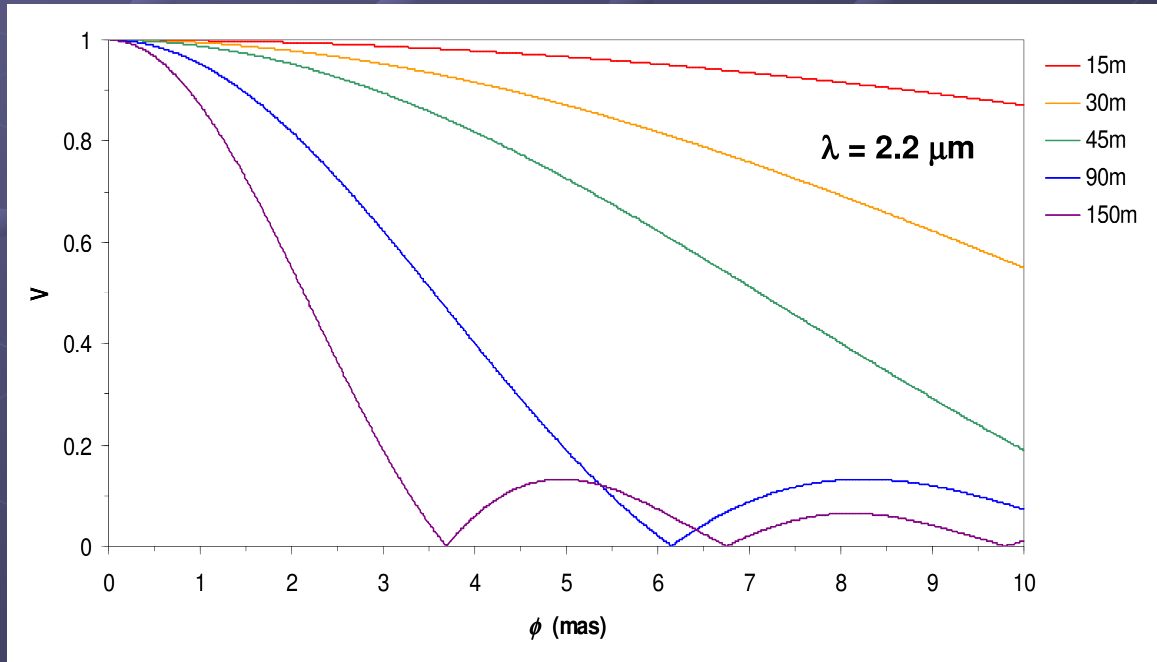
M = 1.0 M_{Sun}

Effect of calibrator diameter error

$$\Delta V_{\mathrm{model}} = \left| \frac{\partial V_{\mathrm{model}}}{\partial \phi} \right| \Delta \phi$$

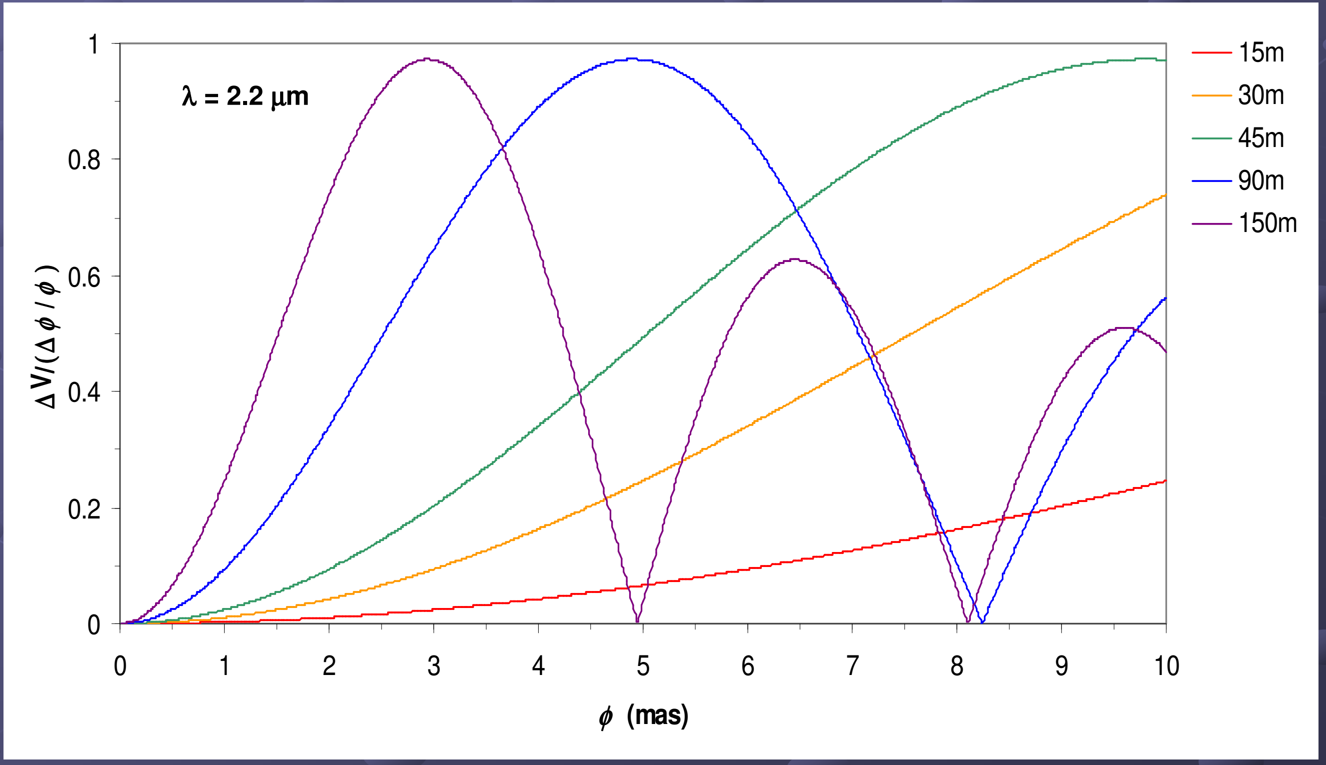
uniform disk model :

$$V_{UD} = \left| \frac{2J_1(q = \pi \phi_{UD} f)}{q} \right|$$



Error on visibility

$$\Delta V_{\text{model}} = 2\left|V_{UD} - \text{sgn}(J_1(q))\,J_0(q)\right| \frac{\Delta\phi_{UD}}{\phi_{UD}} \quad \leq \quad 0.973\,\frac{\Delta\phi_{UD}}{\phi_{UD}}$$

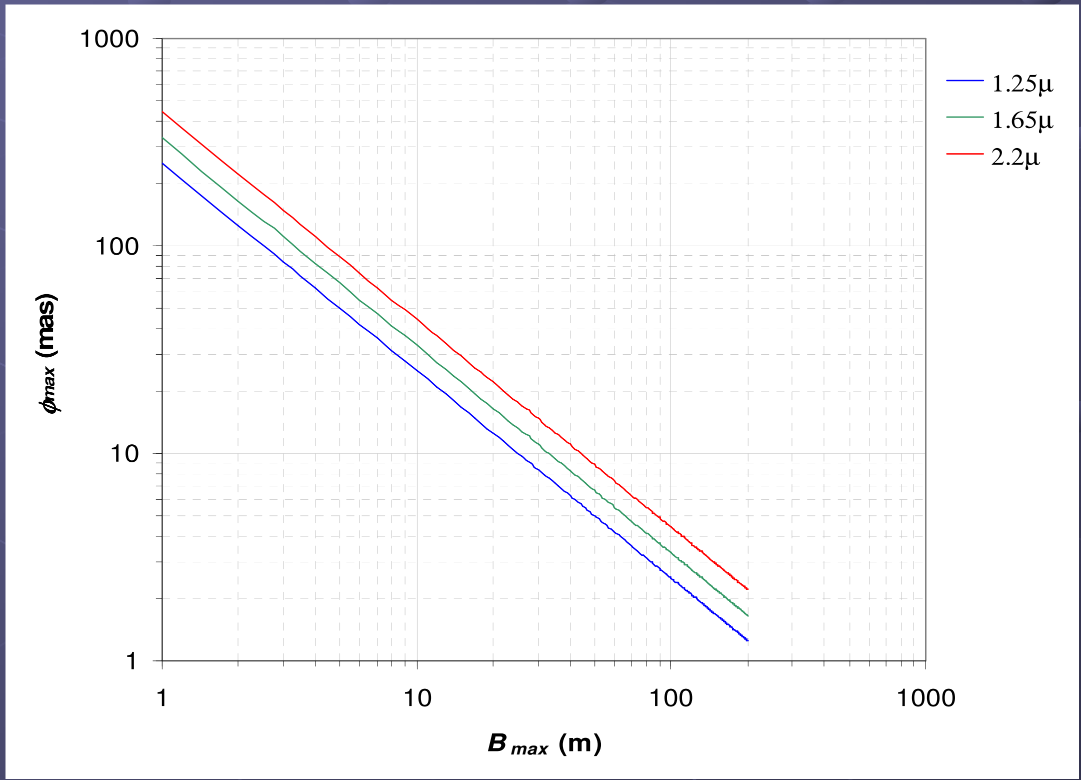


Maximum calibrator diameter allowable

if $\phi < \phi_{\max}(mas) \approx 200.527 \frac{\lambda(\mu m)}{\mathrm{B}_{\max}(\mathrm{m})}$ then

$$\Delta V_{\mathrm{model}} = J_2(q)\frac{\Delta\phi_{UD}}{\phi_{UD}} < 0.973\,\frac{\Delta\phi_{UD}}{\phi_{UD}}$$

for each spatial frequency



Diameter from color index

☞ *Van Belle et al. (1999, PTI)*

113 stars
spectral types G to M
luminosity class III
1.75 < V-K < 9.0

$$T_{eff}\left(\text{in Kelvins}\right) \approx 3030 + 4750 \times 10^{-0.187(V-K)}$$

$$\frac{\phi}{p} \approx 1.64 \times 10^{-2}\left(V-K\right)^{2.36}$$

where p = parallactic angle

☞ *Bonneau et al. (2006, JMMC)*

171 stars
spectral types O to M
luminosity classes I to V
-0.4 < B-V < 1.3
-0.25 < V-R < 2.8
-1.1 < V-K < 7.0

$$\phi(mas) \approx 9.306 \times 10^{-(m_V/5)} \sum_k a_k CI^k$$

Diameter from magnitudes

$$\frac{\phi^2}{4\Delta\lambda}\int_{\lambda_0-(\Delta\lambda/2)}^{\lambda_0+(\Delta\lambda/2)} M_\lambda d\lambda = F_0 \times 10^{-(m_0/2.5)}$$

m_0 = deredenned magnitude
λ_0 = effective wavelength
$\Delta\lambda$ = total spectral bandwidth
F_0 = zero-mag flux
M_λ = model spectral irradiance (emission flux)

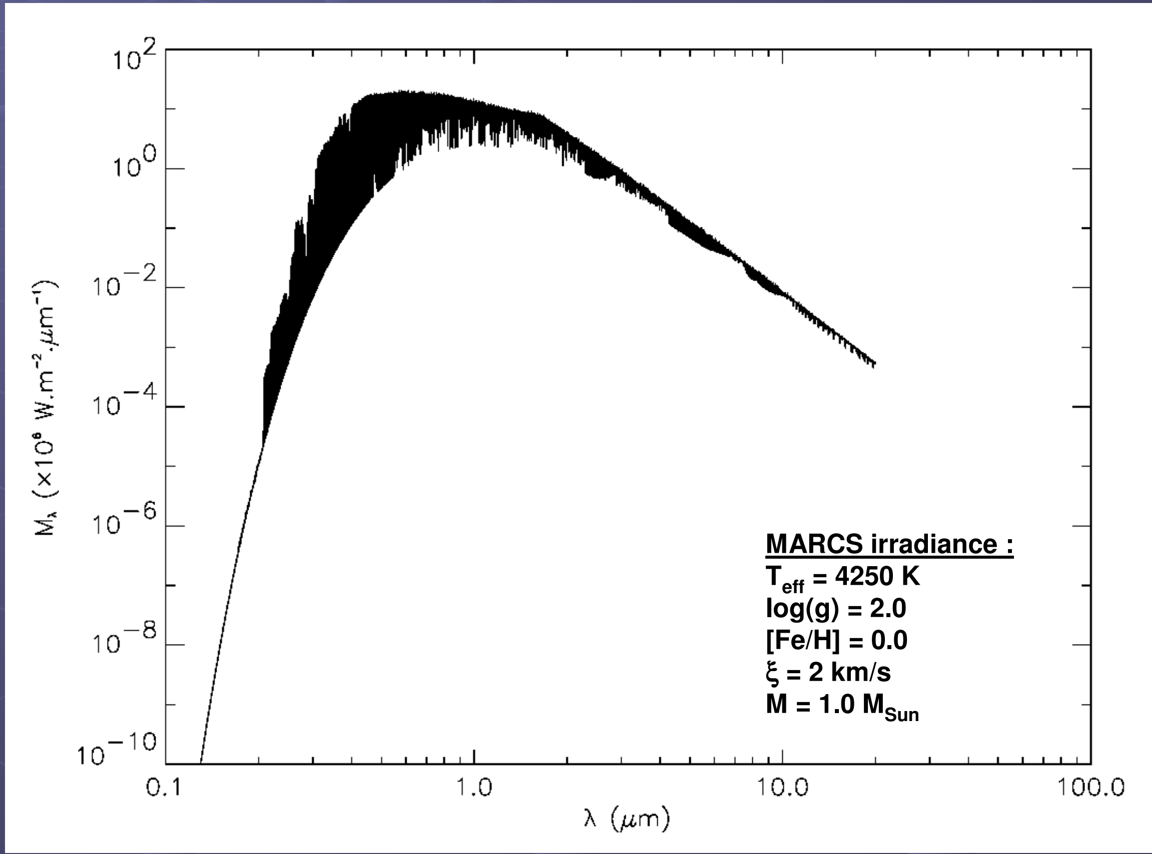
circularly symmetric model : $M_\lambda = 2\pi \int_0^1 L_\lambda(r)\ r dr$

Stellar photospheric model irradiance

→ High resolution sampled energy fluxes

☞ *MARCS library (Gustaffson et al., 2008) 2 500 to 8 000 K*

http://marcs.astro.uu.se/

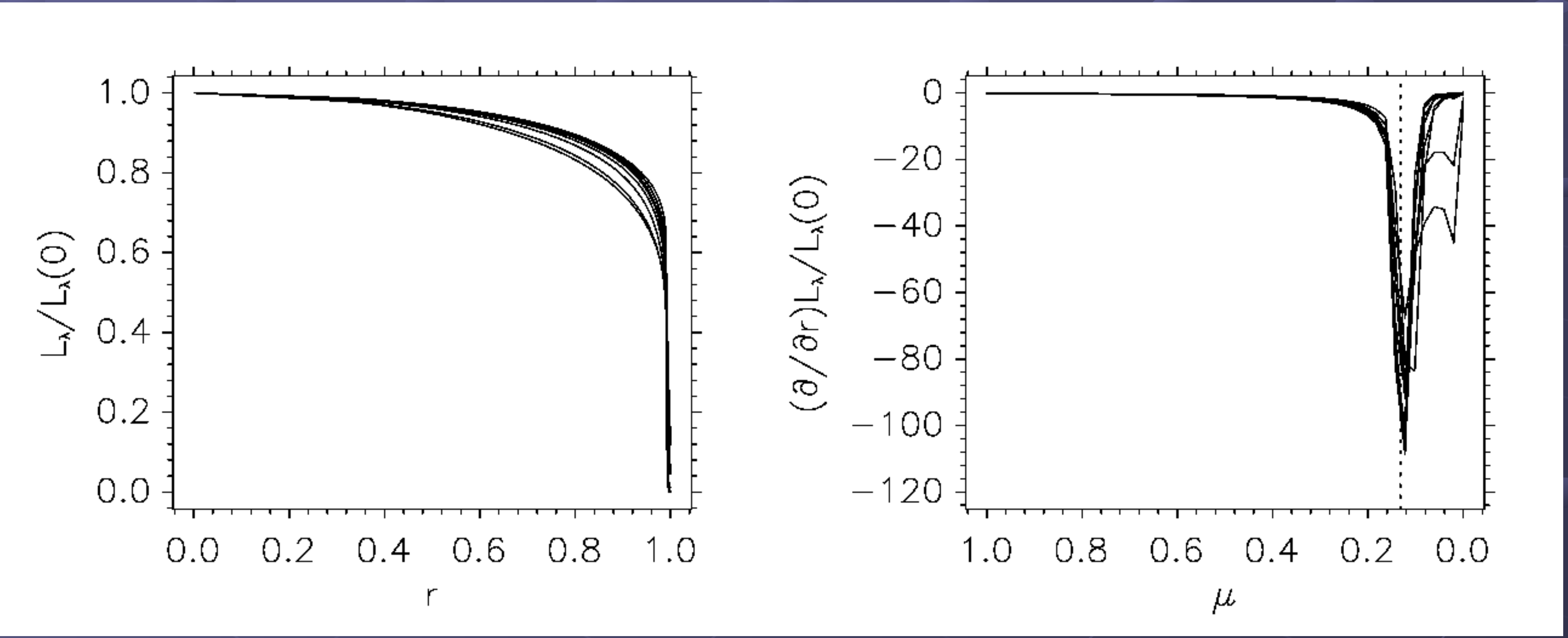


☞ *KURUCZ atlas (1993) 3 000 to 50 000 K*

http://www.stsci.edu/hst/observatory/cdbs/k93models.html

Rosseland diameter

Rosseland to true diameter conversion factor (wavelength-independent) given by shape of radial distribution of photospheric model spectral radiance



MARCS model : T_{eff} = 4250 K, log(g) = 2.0, [Fe/H] = 0.0, ξ = 2 km/s, M = 1.0 M_{Sun}

$$\Rightarrow \frac{\phi_{Ross}}{\phi_{true}} \approx 0.991$$

Diameter from SED fit

N values of measured fluxes $F_i \pm \sigma_i$
with spectral resolutions $R_i = \lambda_i / \delta_i$

ϕ_{best} given by minimization of merit function $\chi^2(\phi)$
(Levenberg-Marquardt)

$$\chi^2(\phi) = \sum_{i=0}^{N-1} \left[\frac{F_i - \hat{F}_i(\phi)}{\sigma_i} \right]^2 \quad \text{where } \hat{F}_i(\phi) = \frac{\phi^2}{4\delta_i} \int_{\lambda_i-(\delta_i/2)}^{\lambda_i+(\delta_i/2)} M_\lambda d\lambda$$

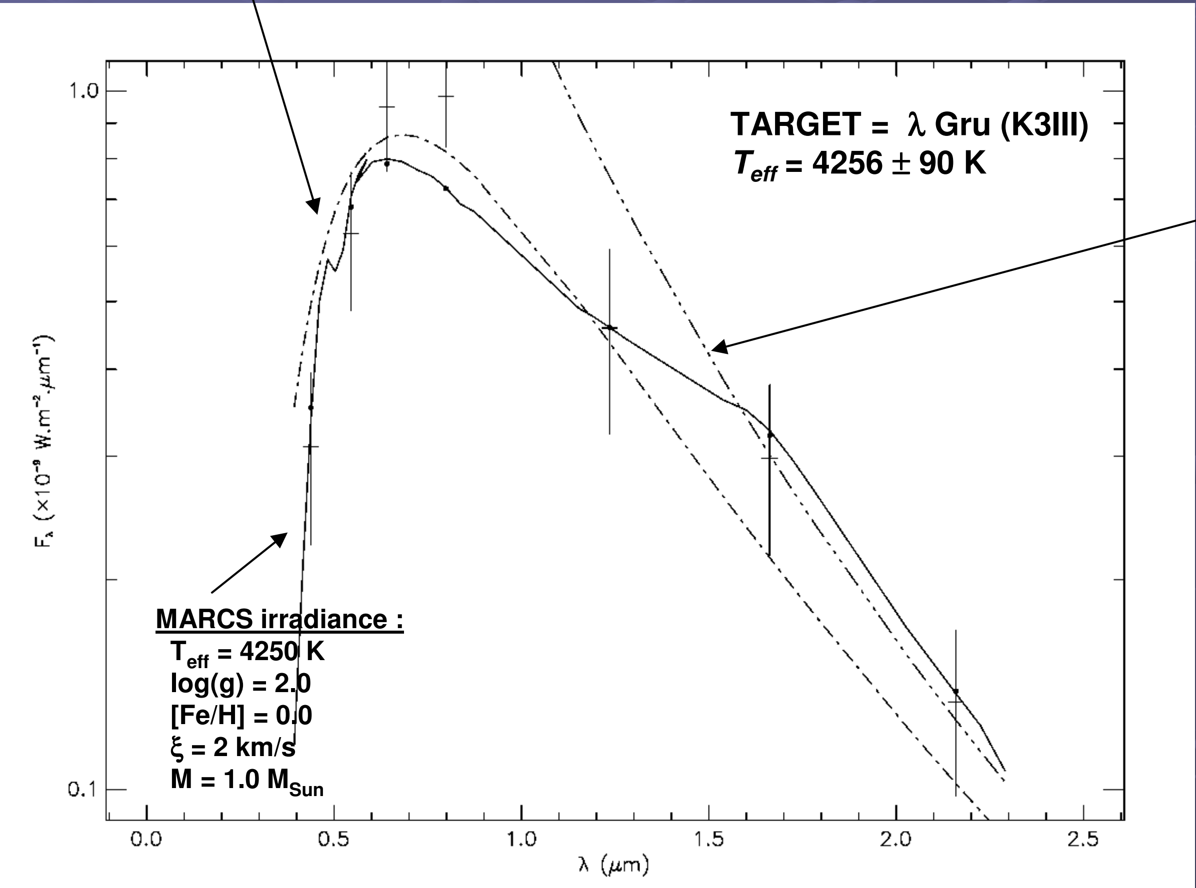
goodness-of-fit parameter : $F2 = \sqrt{\frac{9\nu}{2}} \left(\sqrt[3]{\left(\frac{\chi^2}{\nu} \right)} + \frac{2}{9\nu} - 1 \right)$

where ν = number of degrees of freedom

Fit on photometry

blackbody irradiance (Planck's law) :

$$M_\lambda = B_\lambda(T) = \frac{\pi C_1}{\lambda^5} \frac{1}{e^{(C_2/\lambda T)} - 1}$$



Engelke irradiance (based on IR continua of G, K, M giants) :

$$M_\lambda = B_\lambda\left\{C_3 T_{eff}\left(1 + C_4/\lambda T_{eff}\right)^{C_5}\right\}$$

Fit outputs :

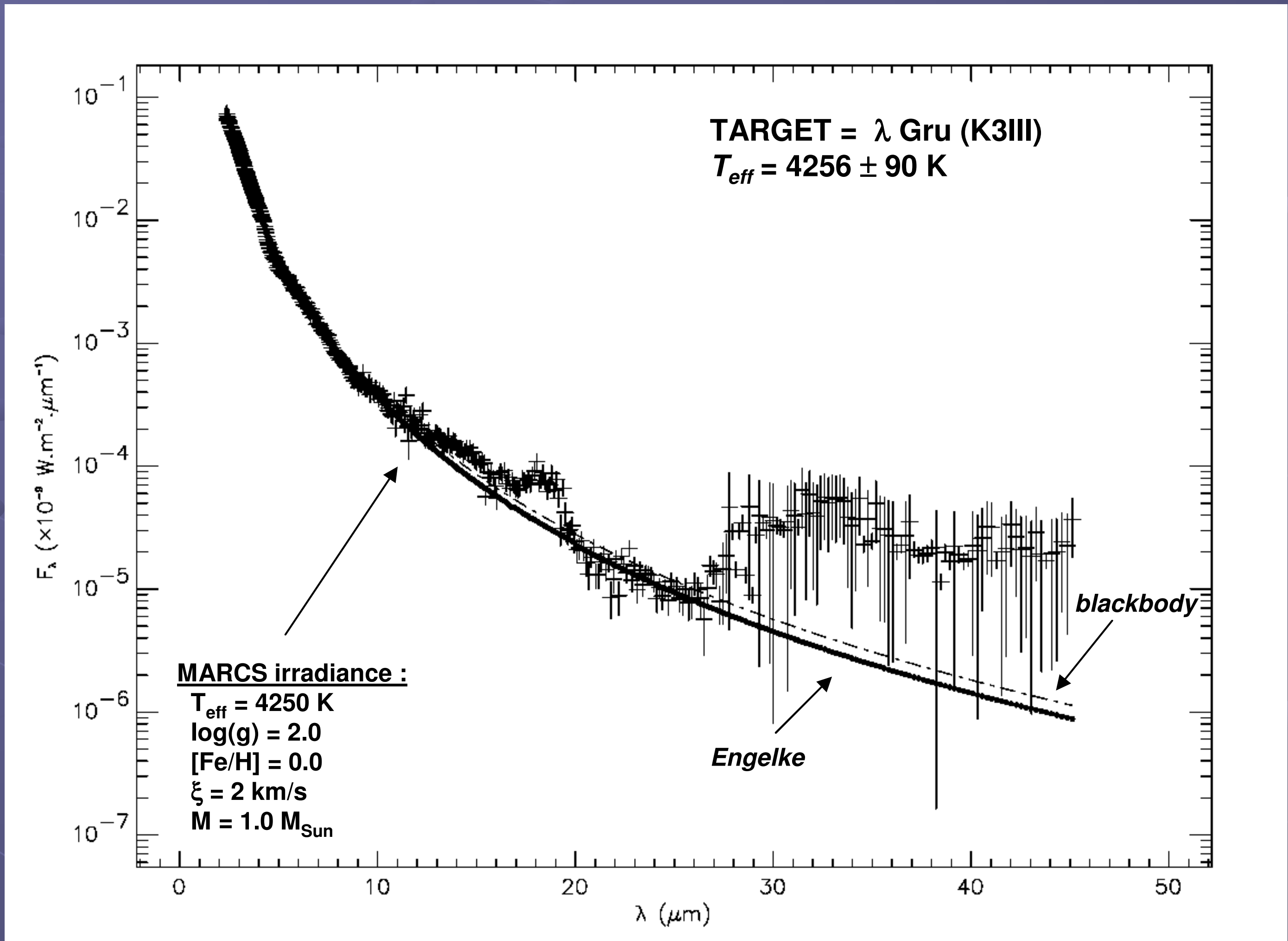
ϕ_{best} = 2.90 mas

fit formal error = 4.4%

χ_R^2 = 0.67

F2 = -0.47

Fit on ISO-SWS SED



Fit outputs :

ϕ_{best} = 2.717 mas

fit formal error = 0.02%

χ_R^2 = 12.9

$F2$ = 63.7

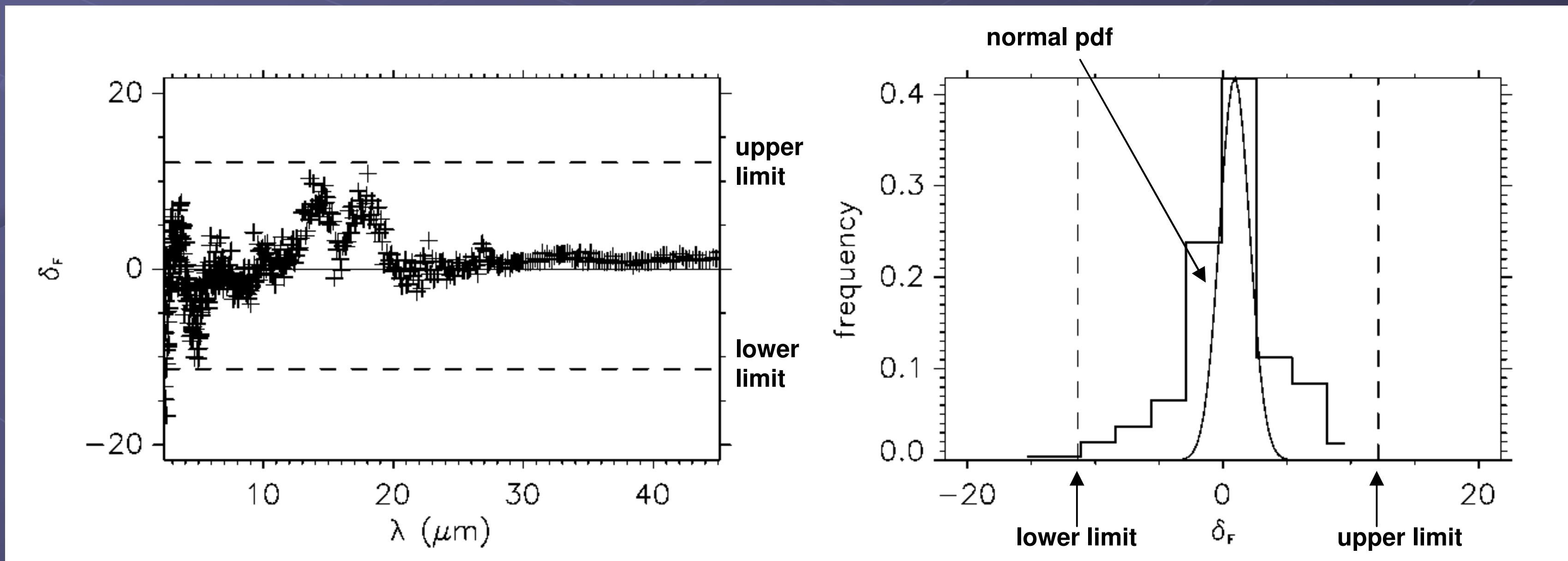
Outlier detection

Extreme outliers identified in tails of distribution of fit-residuals :

$$Q_1 - 3\times IQR < \delta_F < Q_3 + 3\times IQR$$

- Q_1 = lower quartile
- Q_3 = upper quartile
- IQR = interquartile range

centered residuals : $\delta_F = e_i - \bar{e}$ with $e_i = \dfrac{F_i - \hat{F}_i(\phi_{best})}{\sigma_i}$



Uncertainty of best-fit diameter

nonparametric residual bootstrap :
→ fabrication of many (M>1000) "new" data sets by random resampling of fit-residuals

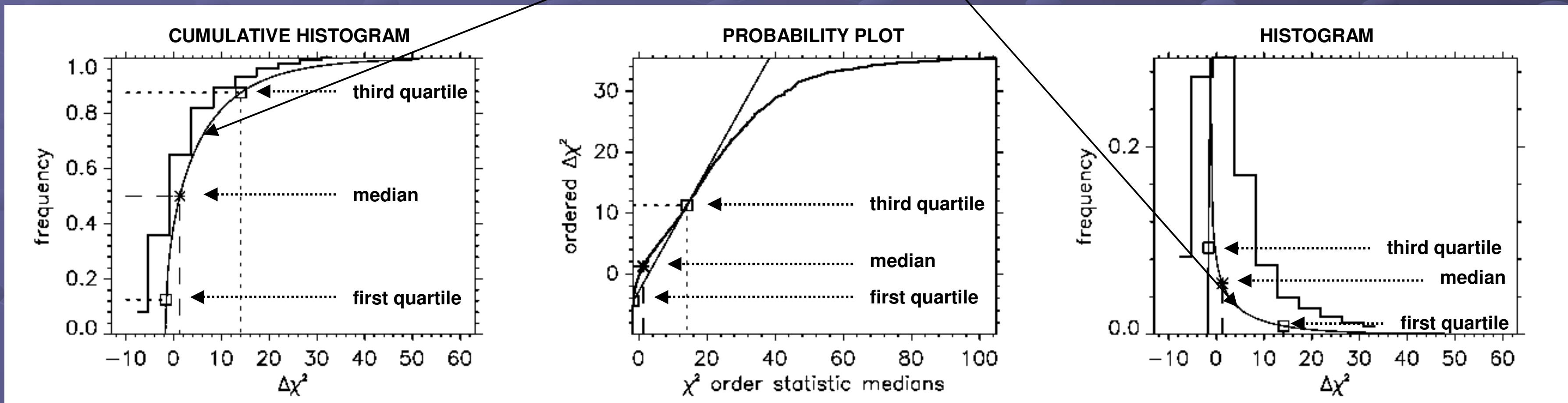
$$F_i \rightarrow \left(F_i^*\right)_k = \hat{F}_i + \left(\delta_F\right)_k \sigma_i \qquad k = 0 \ldots M-1$$

→ M bootstrapped estimates $[\phi_{best}]_k$ given by χ^2-minimizations
→ confidence interval given by χ^2-distribution with $\nu = 1$ degree of freedom (1 free parameter)

Bootstrap outputs

$$\left(\Delta\chi^2\right)_k = \chi^2\left\{\left(F^*\right)_k ; \left(\phi_{best}\right)_k\right\} - \chi^2\left\{F ; \phi_{best}\right\}$$

follows chi-square distribution with ν = 1 DOF



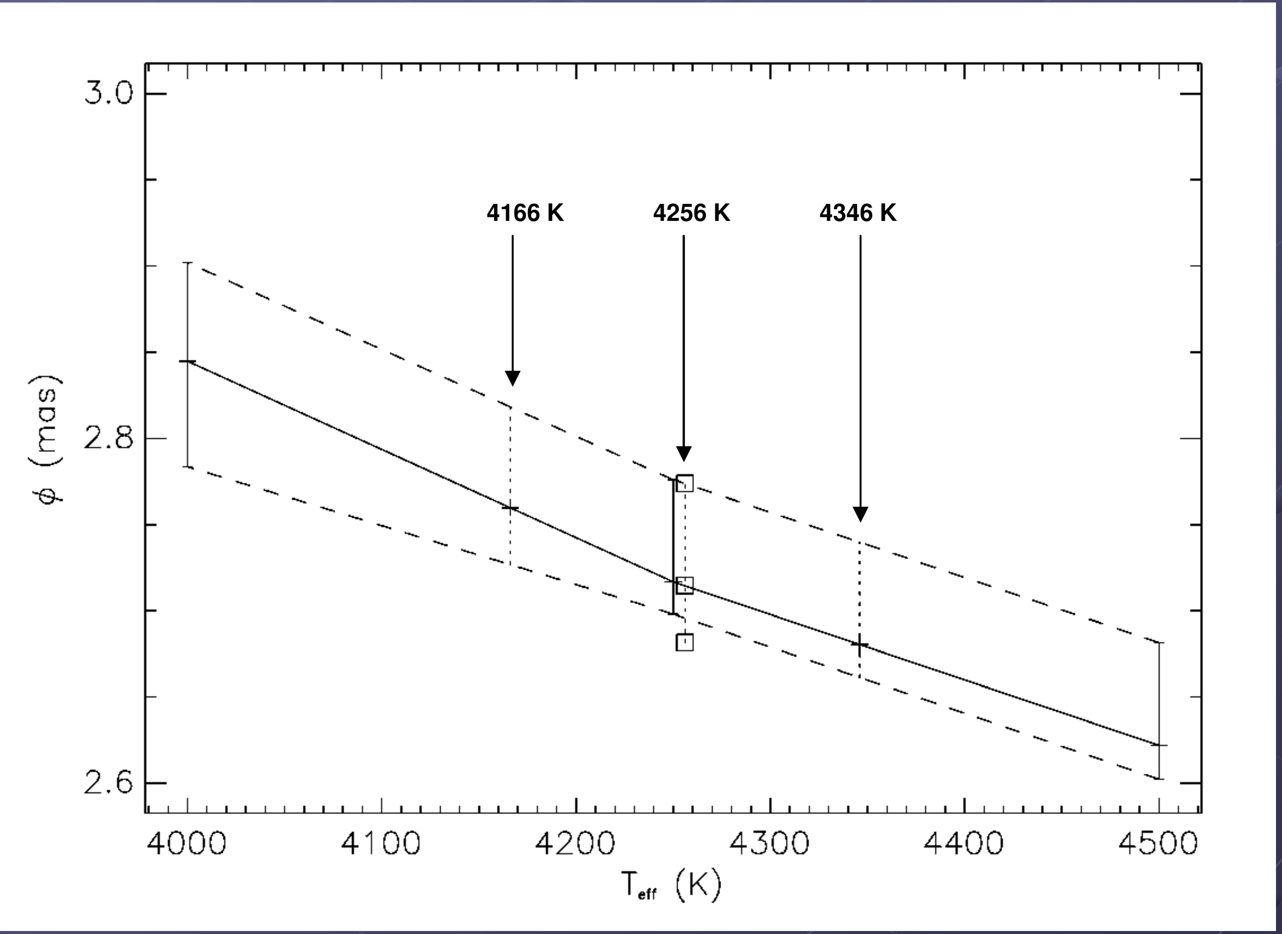
if α = confidence level (e.g. 95% ⇔ ±2σ for normal distribution) :

$$\left(\phi_{best}\right)_{\inf(\sup)} = \min(\max)\left\{\left(\phi_{best}\right)_k \text{ for } (1\text{-}\alpha)/2 < pdf\left(\Delta\chi^2\right) < (1+\alpha)/2\right\}$$

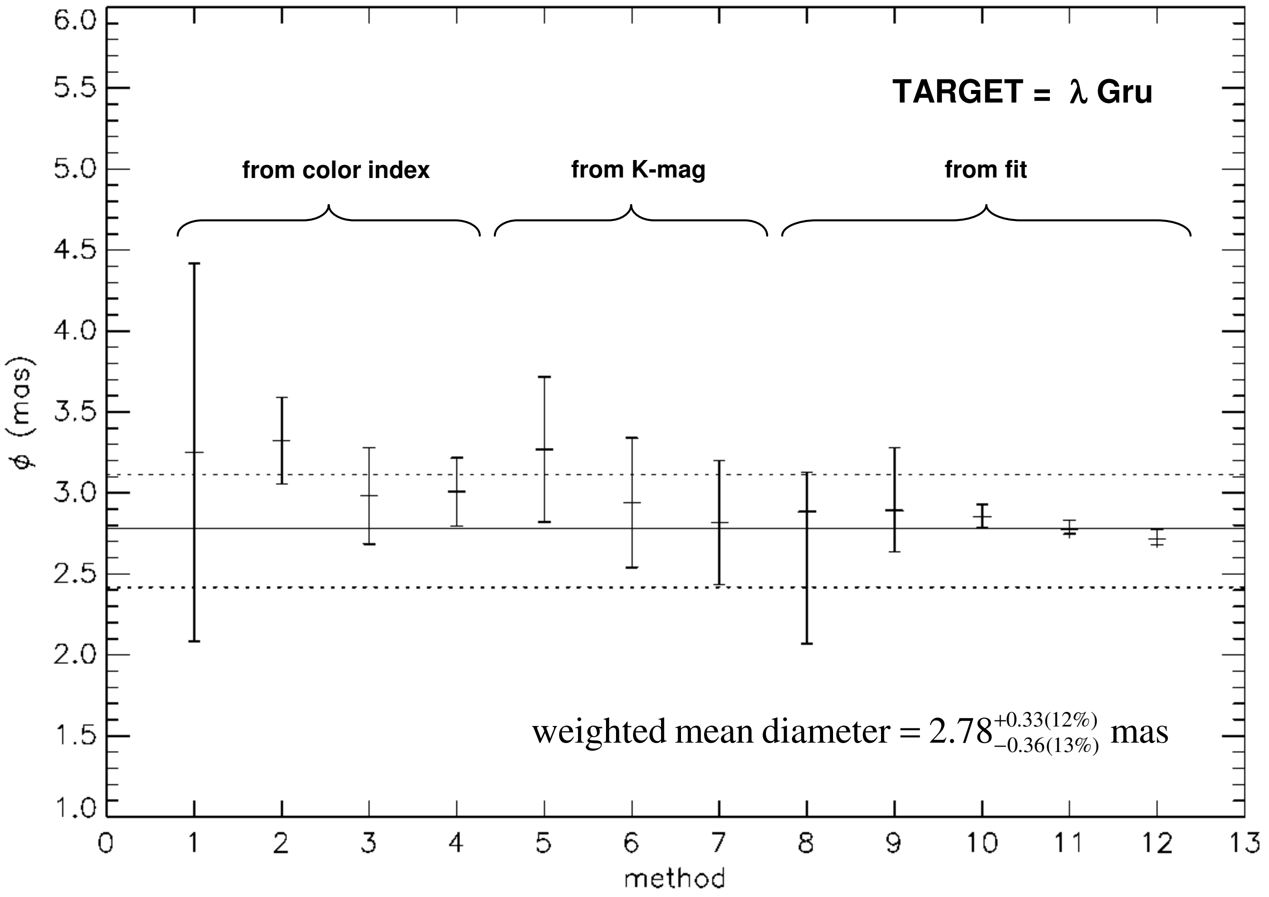
$$\Rightarrow \qquad \phi = \left[\phi_{best}\right]_{-\left\{\phi_{best} - \left(\phi_{best}\right)_{\inf}\right\}}^{+\left\{\left(\phi_{best}\right)_{\sup} - \phi_{best}\right\}}$$

True temperature

Linear interpolation of best-fit diameter estimates on "true" (= 4 256 ± 90 K) effective temperature



Diameter estimates



1. $\phi_{V\text{-}K}$ (Van Belle)
2. $\phi_{B\text{-}V}$ (Bonneau)
3. $\phi_{V\text{-}R}$ (Bonneau)
4. $\phi_{V\text{-}K}$ (Bonneau)
5. ϕ_{K} with Planck model
6. ϕ_{K} with Engelke model
7. ϕ_{K} with MARCS model
8. ϕ_{best} with Planck fitted on B,V,R,I,J,H,K
9. ϕ_{best} with MARCS fitted on B,V,R,I,J,H,K
10. ϕ_{best} with Planck fitted on ISO-SWS SED
11. ϕ_{best} with Engelke fitted on ISO-SWS SED
12. ϕ_{best} with MARCS fitted on ISO-SWS SED

Comparison with previous works

λ Gru = HD 209688

K3III Spec. Type, T_{eff} = 4256 ± 90 K

☞ This work : fit of MARCS spectral irradiance photospheric models on ISO-SWS flux measurements, nonparametric bootstrap of fit residuals, and angular diameter interpolation on true effective temperature

$$\phi(4256K) = 2.71^{+0.06(2\%)}_{-0.03(1\%)} \text{ mas}$$

after 10000 bootstrap loops with 95% confidence level

remark : with 68% confidence level ⇒ $\phi(4250K) = 2.72^{+0.05(1.7\%)}_{-0.02(0.7\%)} \text{ mas}$

☞ Cohen (99) & Bordé (2002) : fit of KURUCZ spectral irradiance photospheric model on calibrated "spectral templates" obtained from ground-taken spectral fragments, Kuiper Airborne Observatory, and IRAS-LRS

$$\phi = 2.71 \pm 0.03 \text{ mas}$$

diameter uncertainty directly given by fit error = 1%

Calibrator model visibility

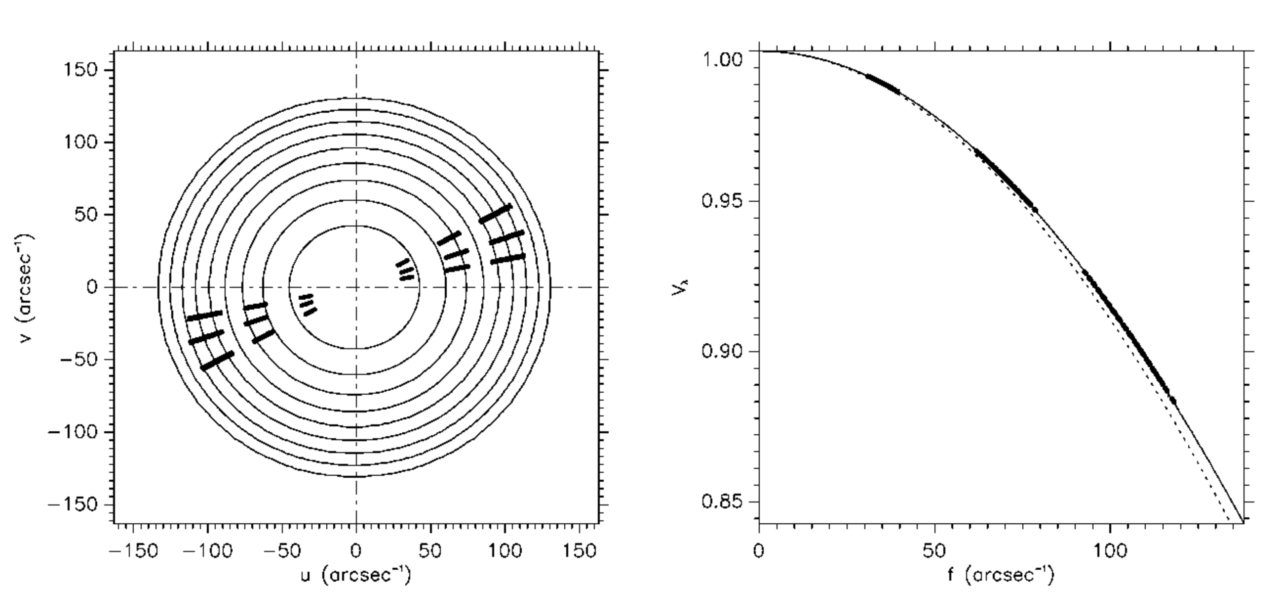
$$V_{\text{model}}(f) = \frac{2\pi}{M_\lambda} \left| \int_0^1 L_\lambda(r) J_0(\pi r \phi f) r dr \right|$$

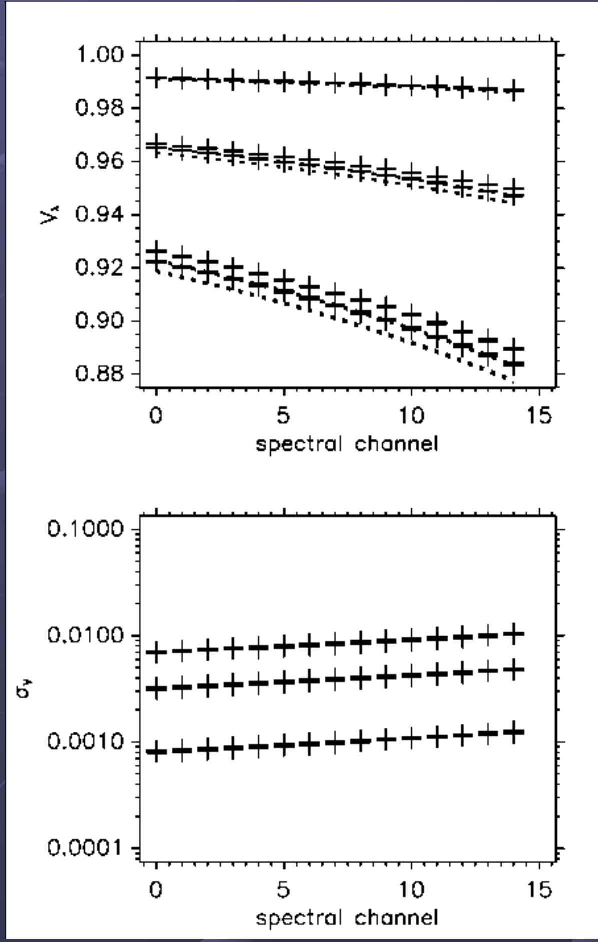
projection on each instrumental spectral channel defined by $\lambda_i \pm (\delta_i/2)$ ($\delta_i = \lambda_i/R$), and on each baseline $B_j \in [(B_{min})_j;(B_{max})_j]$ for each calibrator observing file

$$V_{\text{model}}(B_j/\lambda_i) = \int_{\lambda_i-(\delta_i/2)}^{\lambda_i+(\delta_i/2)} \left[\int_{(B_{\min})_j}^{(B_{\max})_j} V_{\text{model}}(B/\lambda)\, dB \right] d\lambda$$

Example : AMBER JHK-LR

K-band calibrator visibility for VLTI configuration E0-G0-H0 (16-32-48m)





Collaborators

- A. Jorissen, IAA, Bruxels Univ.
- S. Sacuto, Inst. of Astron., Vienna Univ.
- B. Plez, GRAAL, Montpellier Univ.
- D. Bonneau, Fizeau, UNS-OCA
- A. Spang, Fizeau
- O. Chesneau, Fizeau

The SPIDAST© software tool

interferometric data = visibility, spectrum, coherent spectrum, complex bispectrum (including closure phase)

photospheric model → calibrator model generation ← *measured SED*

↓

calibrator model data

↓

calibrator raw data → interferometric data calibration ← *science raw data*

↓

true data

↓

absolute data scaling

↓

scaled data

↓

fit of chromatic 2D models

↓

model parameters with their uncertainties