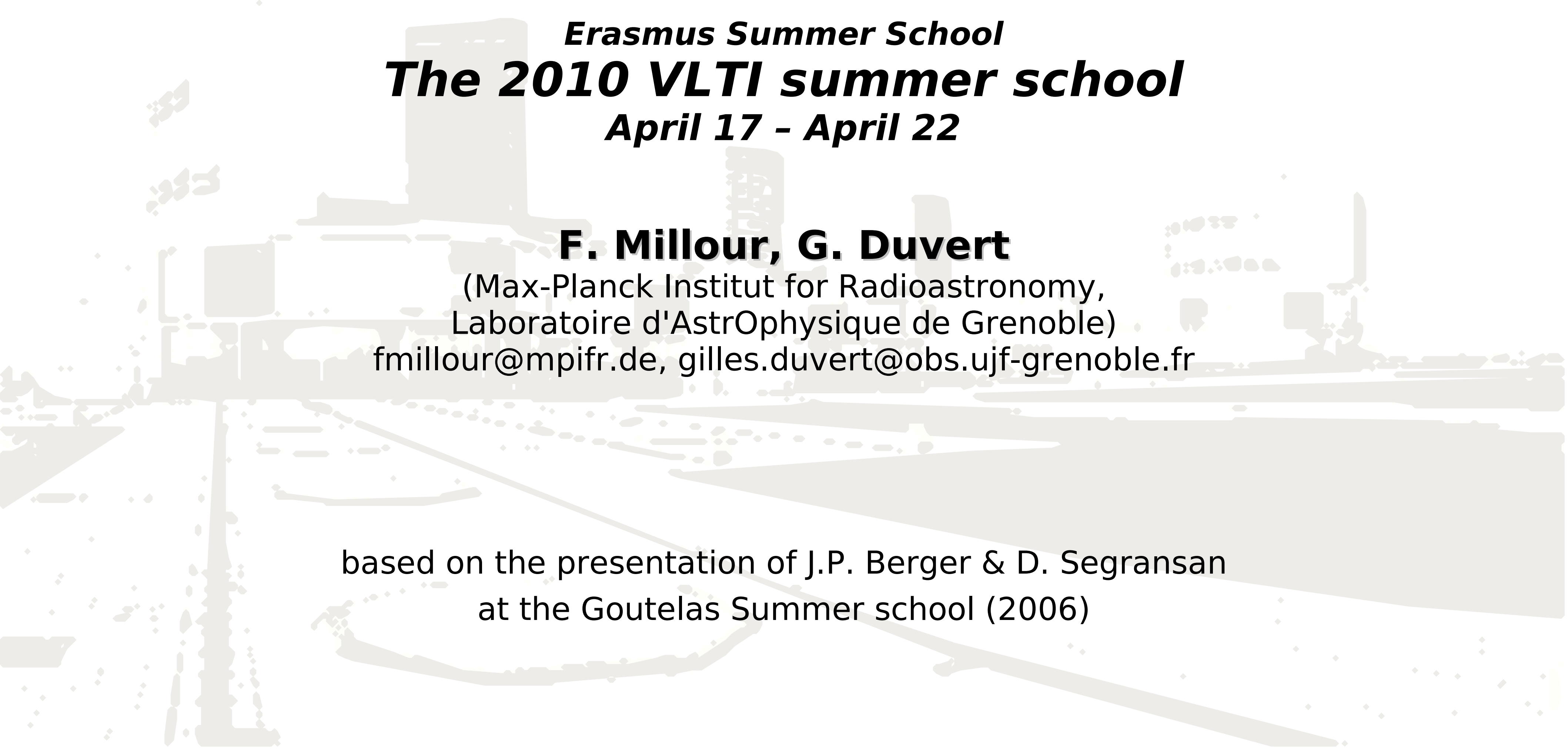
Introduction to visibility and model fitting tips



Erasmus Summer School

The 2010 VLTI summer school

April 17 – April 22

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Laboratoire d'AstrOphysique de Grenoble)
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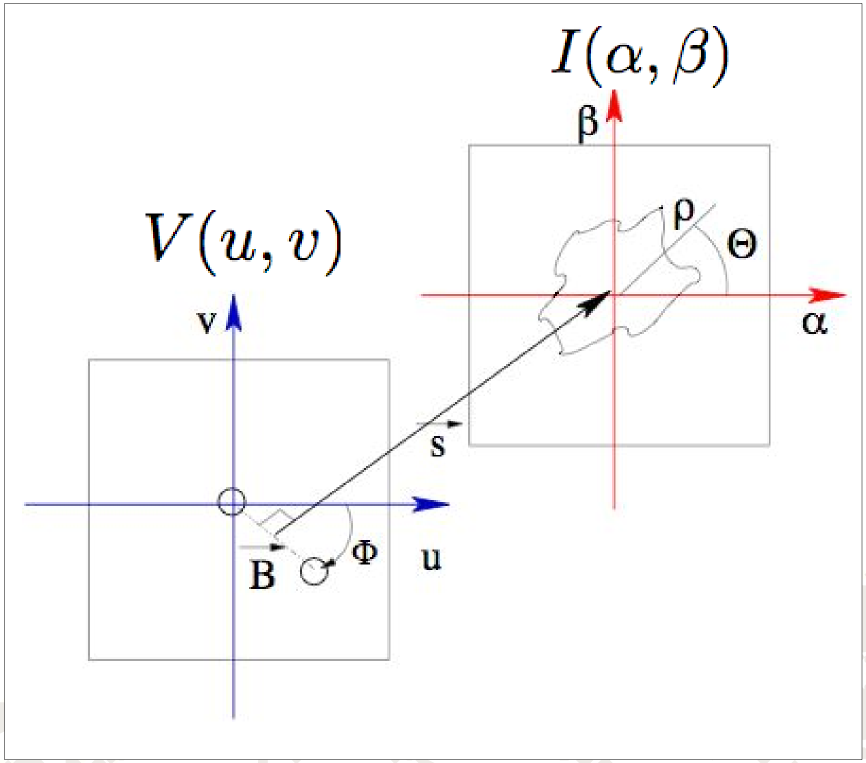
based on the presentation of J.P. Berger & D. Segransan
at the Goutelas Summer school (2006)

What is "visibility" ?

A practical application of the Van-Cittert / Zernike theorem



The VCZ theorem links:

the intensity distribution of an object in the sky plane (far field)

to the complex Fourier-transorm, measured in the array plane.

$$V(u,v) = \frac{\int\int I(\alpha,\beta)\exp^{-2i\pi(\alpha u+\beta v)}d\alpha d\beta}{\int\int I(\alpha,\beta)d\alpha d\beta}$$

This relation is a normalized **Fourier transform** (i.e. total flux does not matter).

Spatial frequency coordinates $\mathbf{u = B_x / \lambda}$, $\mathbf{v = B_y / \lambda}$
where B_x and B_y stand for projected baselines coordinates on the x and y axes of telescope

Visibility and single dish telescope

Example : **resolved binary star** (HIP 4647) observed at the Special Astronomical Observatory (Zelentchouk)

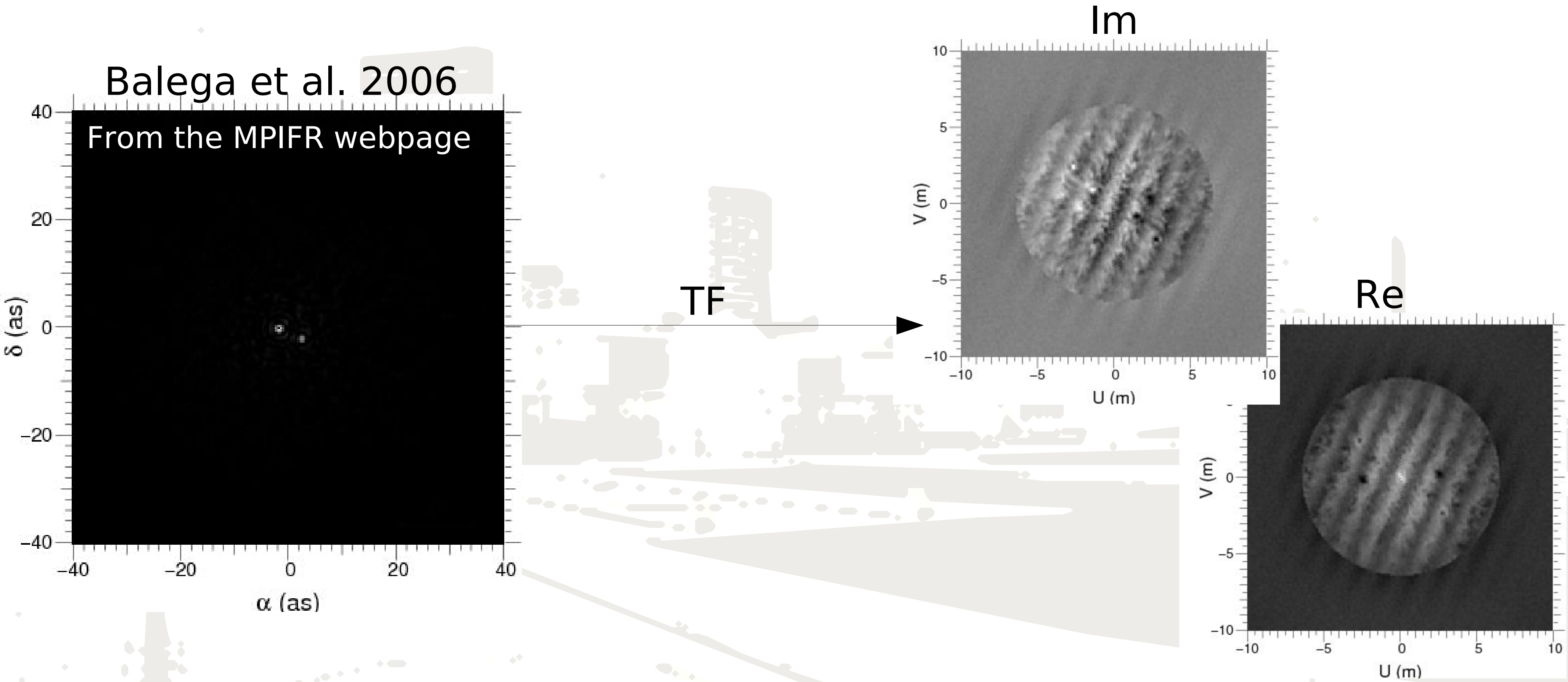


Image : $I(x,y) = O*PSF \rightarrow |V(u,v)|, \phi(u,v)$ & cut-off frequency at D/λ

Visibility and single dish telescope

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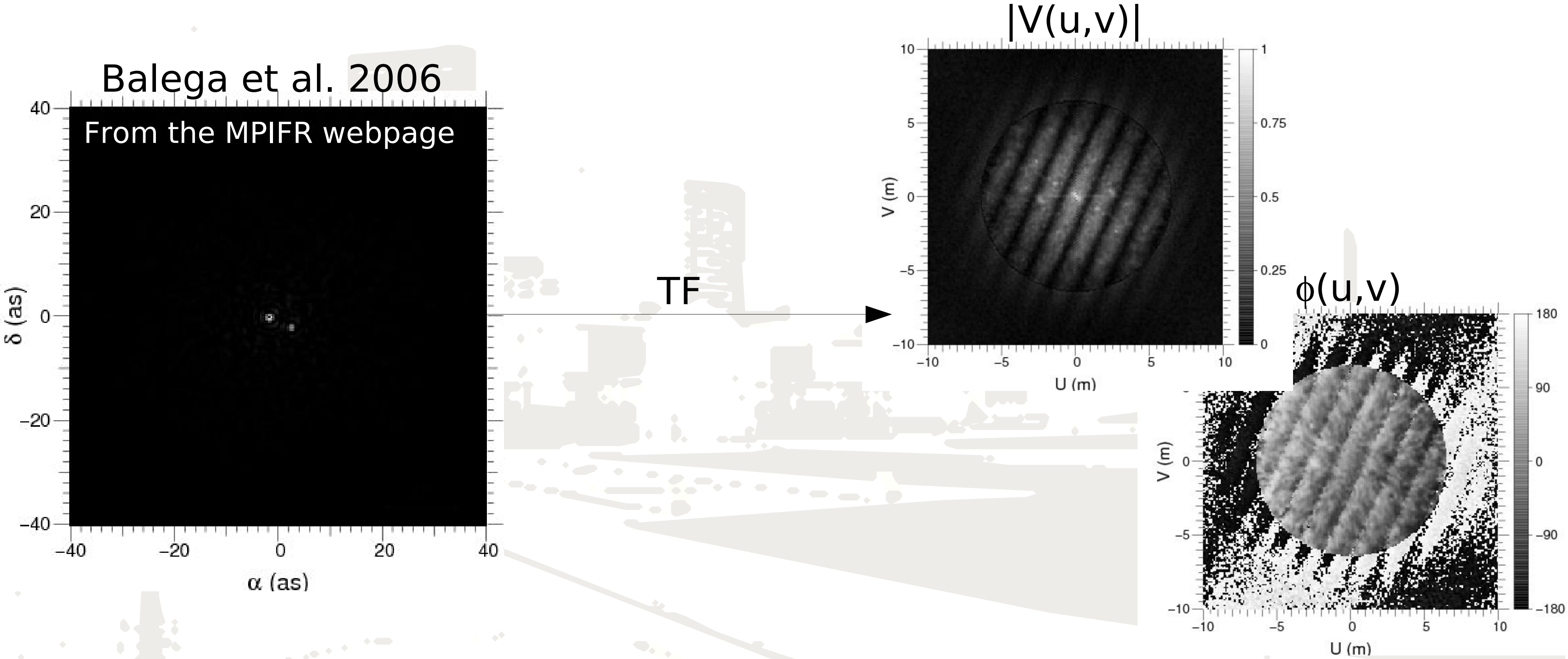

Balega et al. 2006

From the MPIFR webpage

Image : $I(x,y) = O*PSF$ → $|V(u,v)|$, $\phi(u,v)$ & cut-off frequency at D/λ

Visibility and single dish telescope

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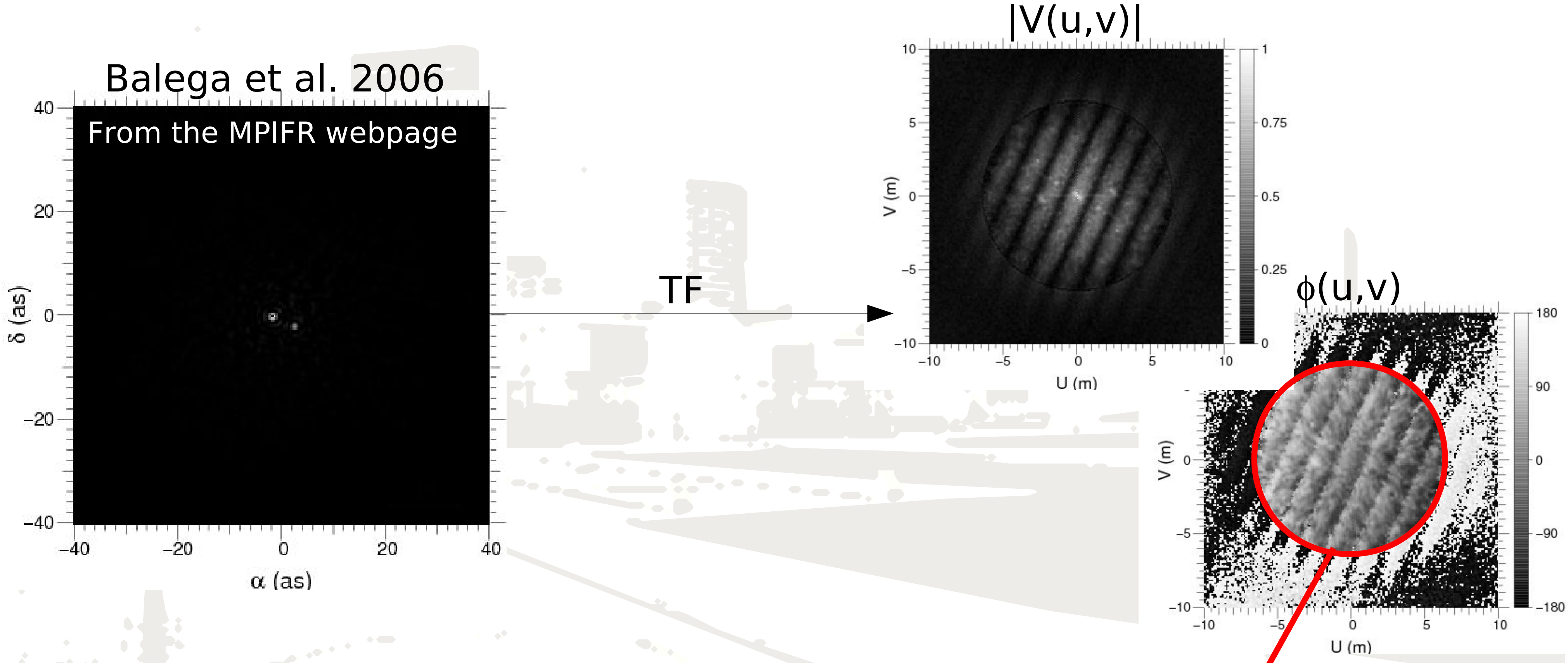


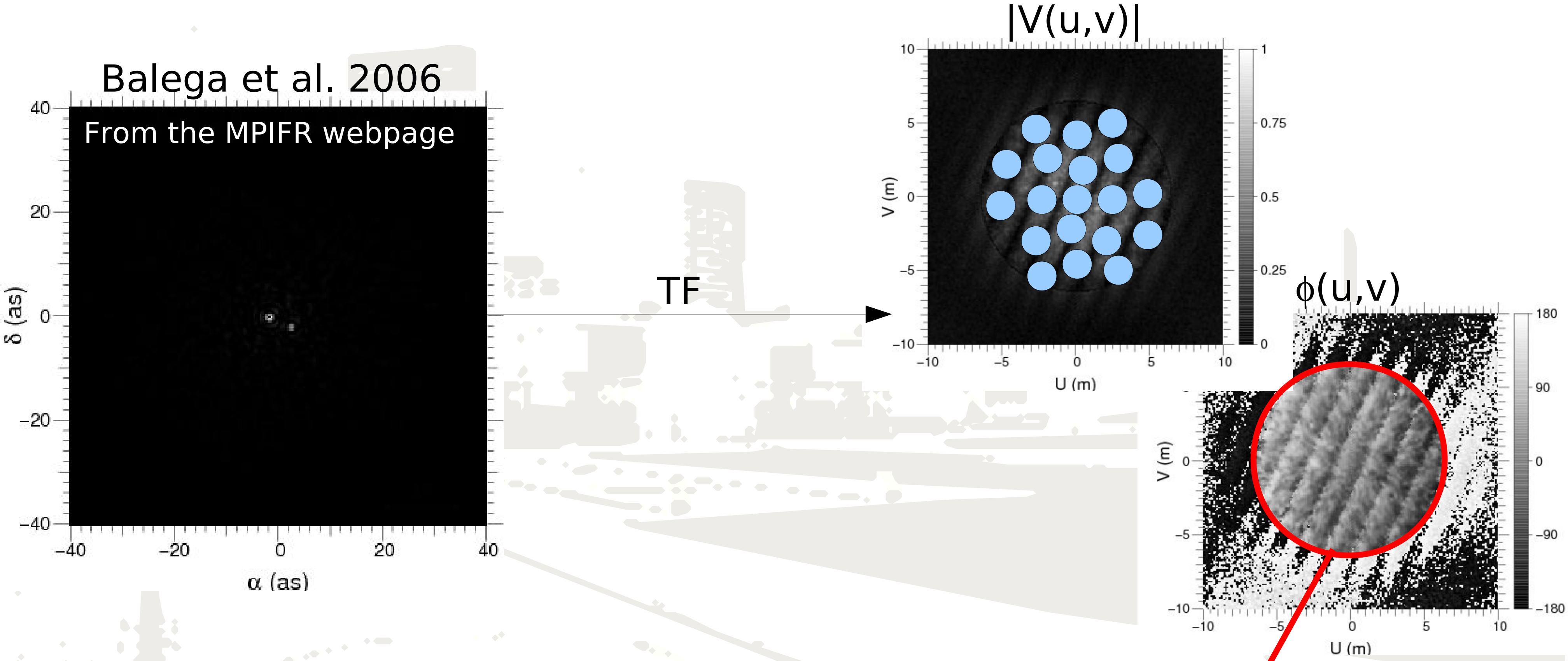
Balega et al. 2006

From the MPIFR webpage

Image : $I(x,y) = O*PSF \rightarrow |V(u,v)|, \phi(u,v)$ & cut-off frequency at D/λ

Visibility and single dish telescope (speckle masking)

Example : **resolved binary star** (HIP 4647) observed at the Special Astronomical Observatory (Zelentchouk)



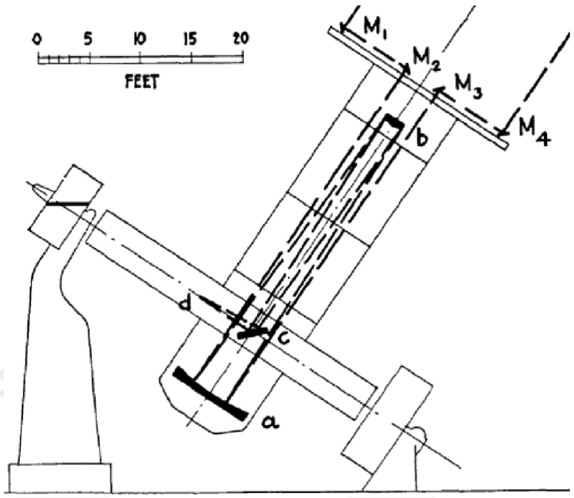
Balega et al. 2006

From the MPIFR webpage

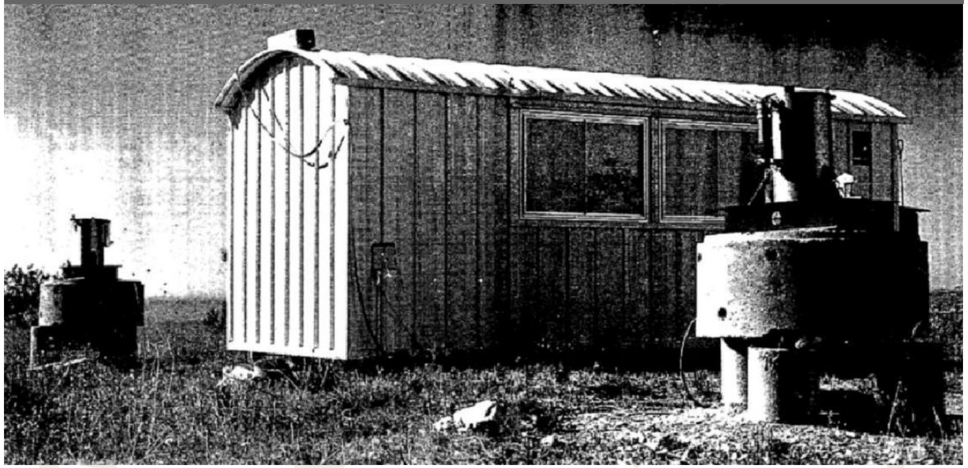
Image : $I(x,y) = O*PSF \rightarrow |V(u,v)|, \phi(u,v)$ & cut-off frequency at D/λ

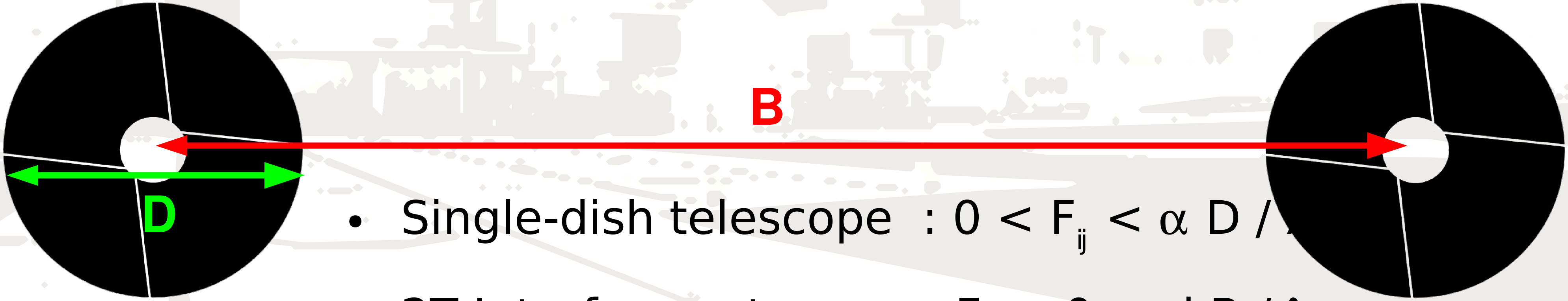
Long-baseline optical/IR interferometry

Michelson & Pease, 1921



Labeyrie, 1975

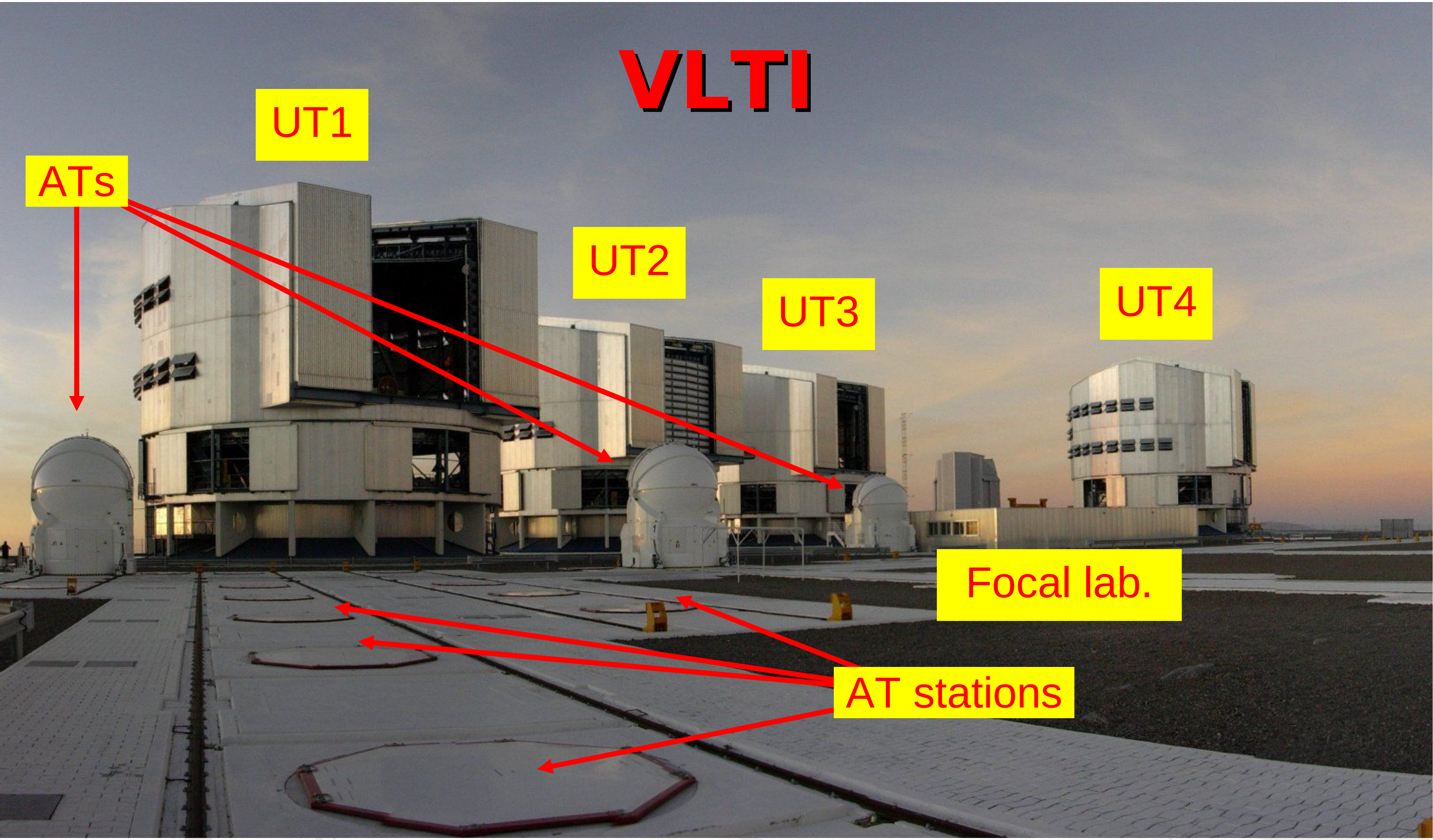


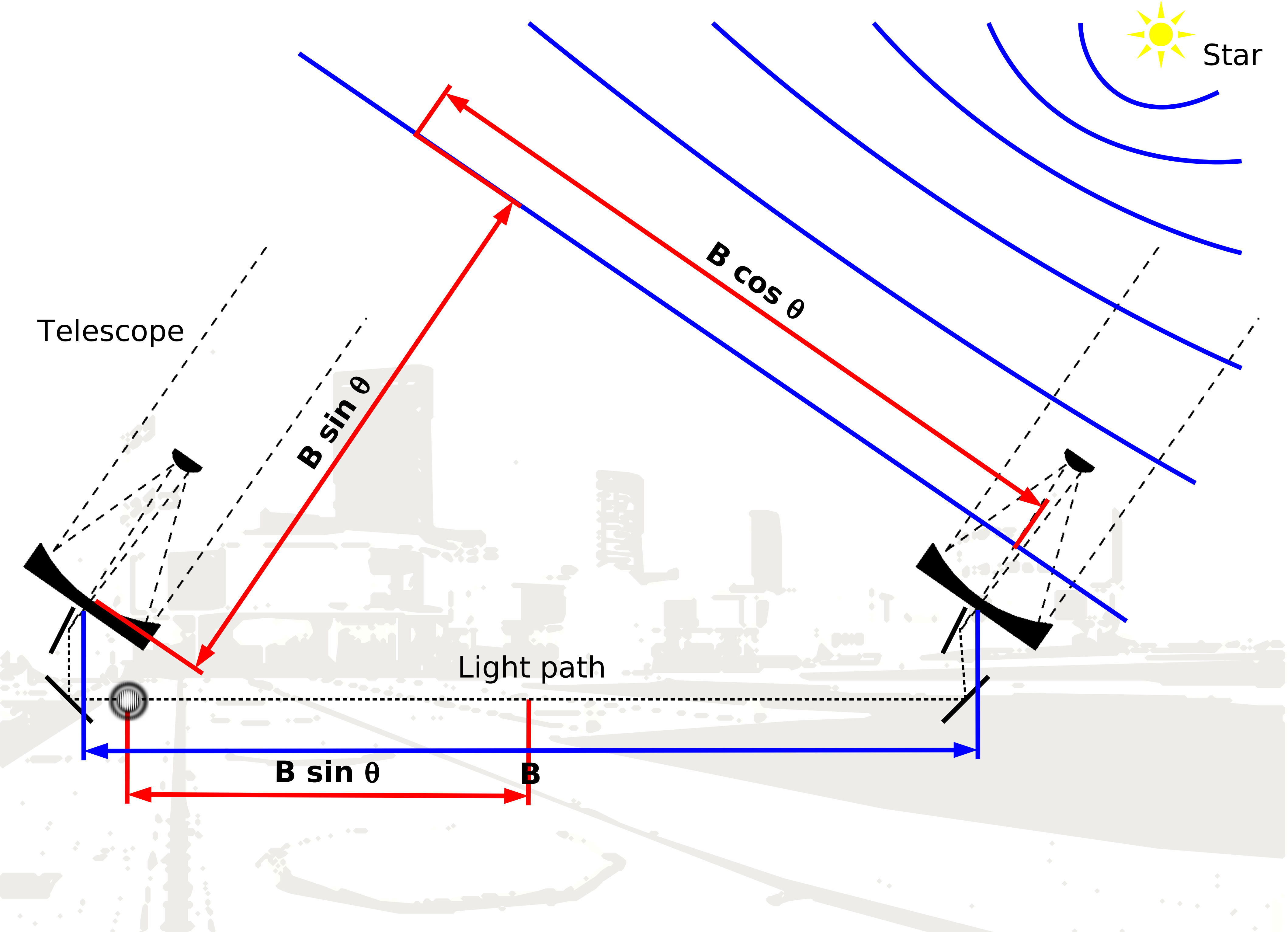


- Single-dish telescope : $0 < F_{ij} < \alpha D /$
- 2T Interferometer : $F_{ij} = 0$ and B / λ

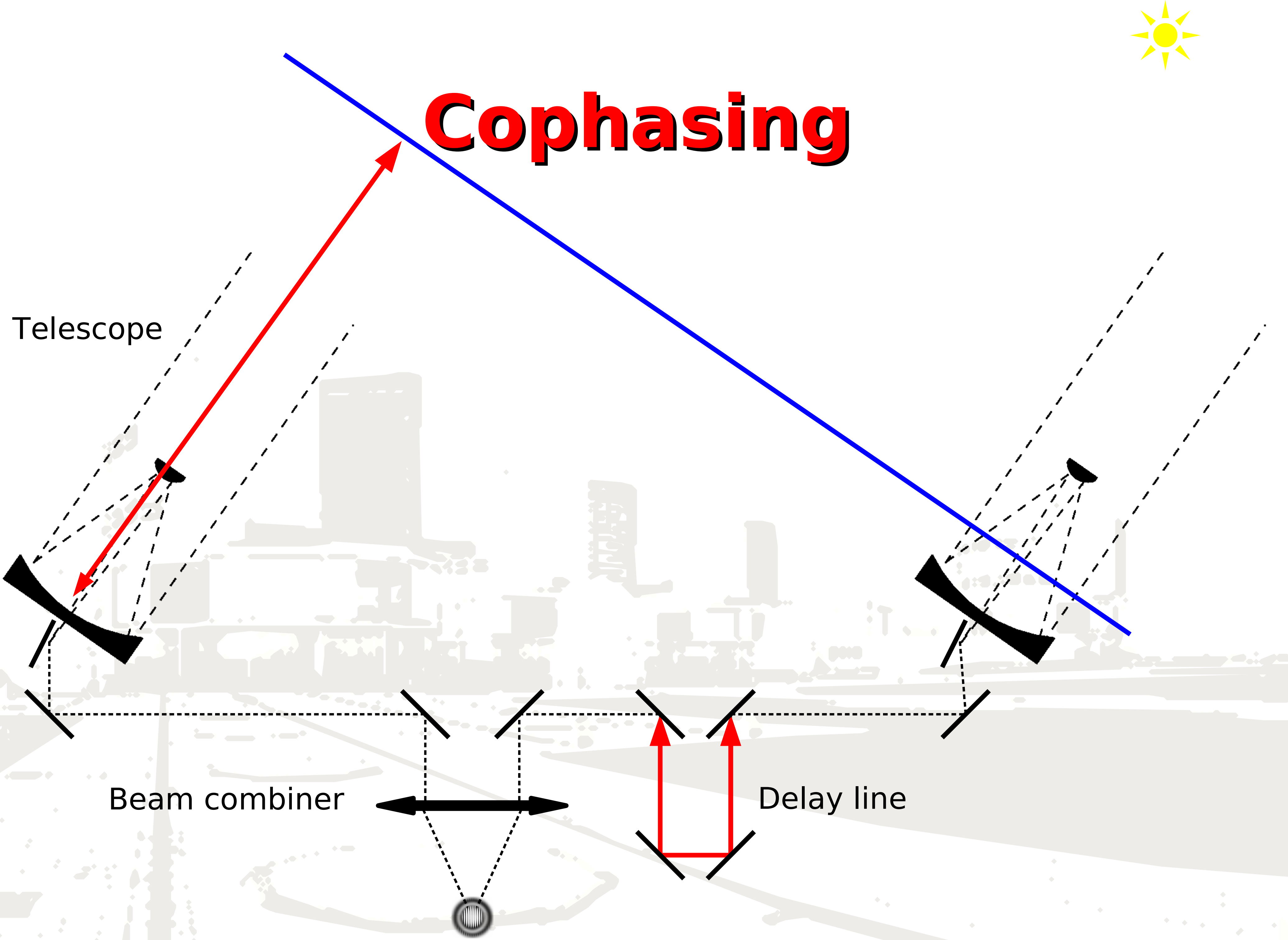
=> Only one (or very few) spatial frequency is scanned at once by an interferometer

VLTI

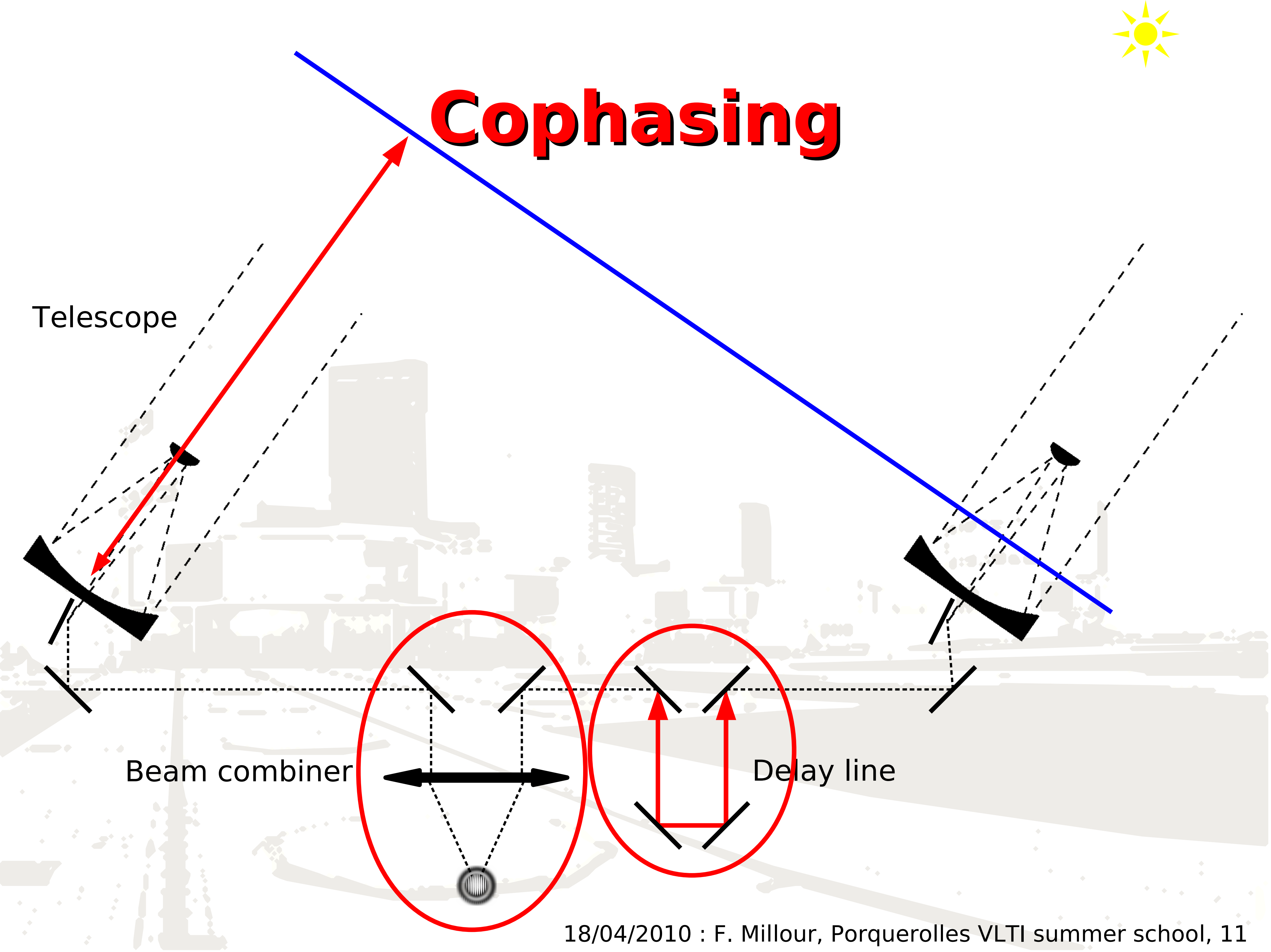



Star
Telescope
B sin θ
B cos θ
Light path
B sin θ
B

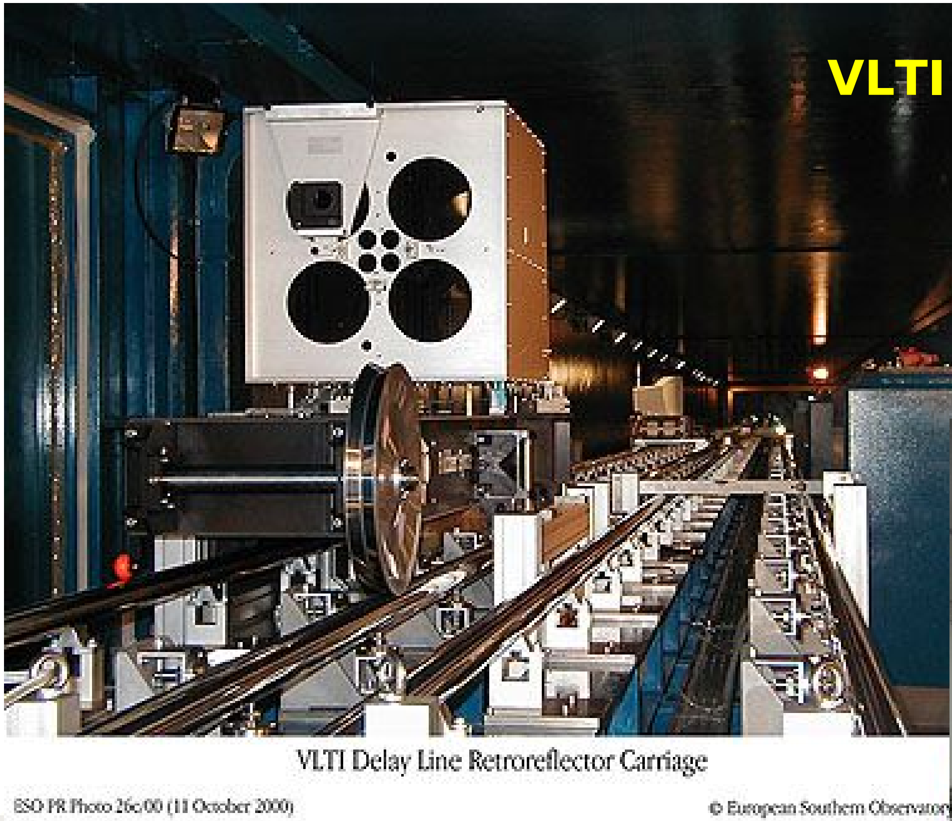
Cophasing



Cophasing



Delay Lines



VLTI Delay Line Retroreflector Carriage

ESO PR Photo 26c/00 (11 October 2000)

© European Southern Observatory

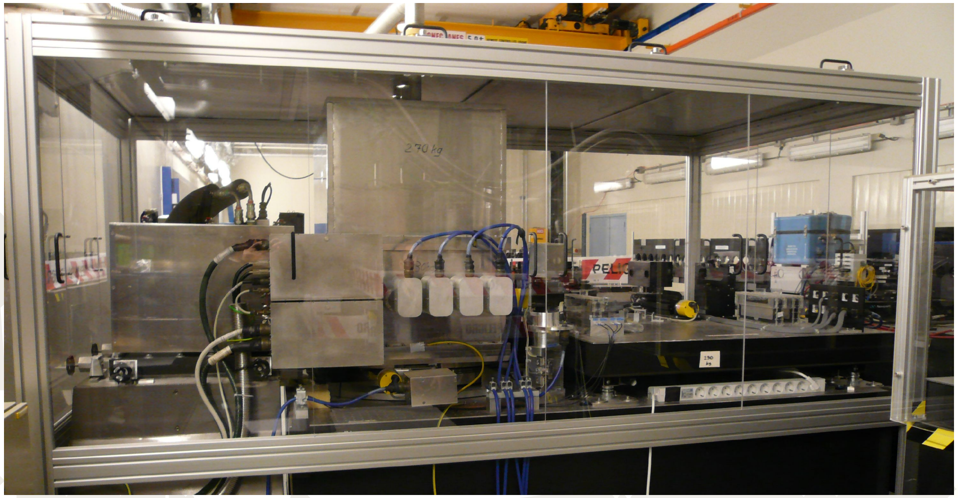
Instruments

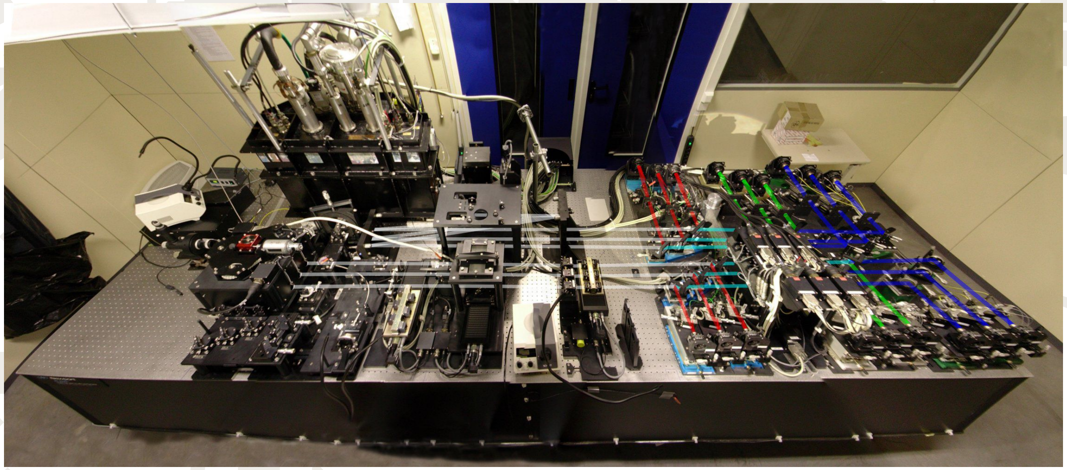
VINCI

- 2 telescopes
- K Band (2µm)
- Wideband

MIDI

- 2 telescopes
- N Band (8-13µm)
- Spectral resolutions R=30 & 300

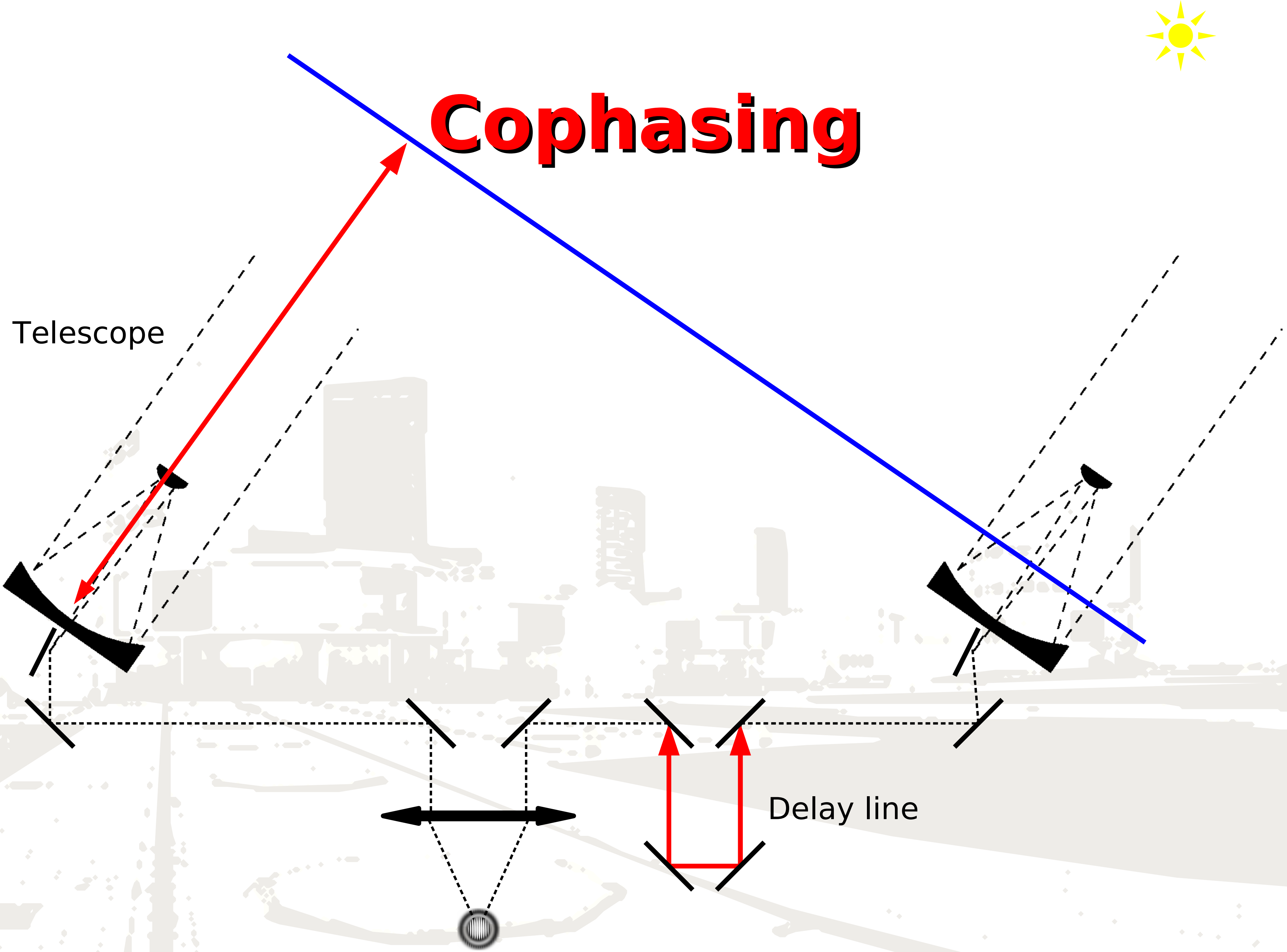




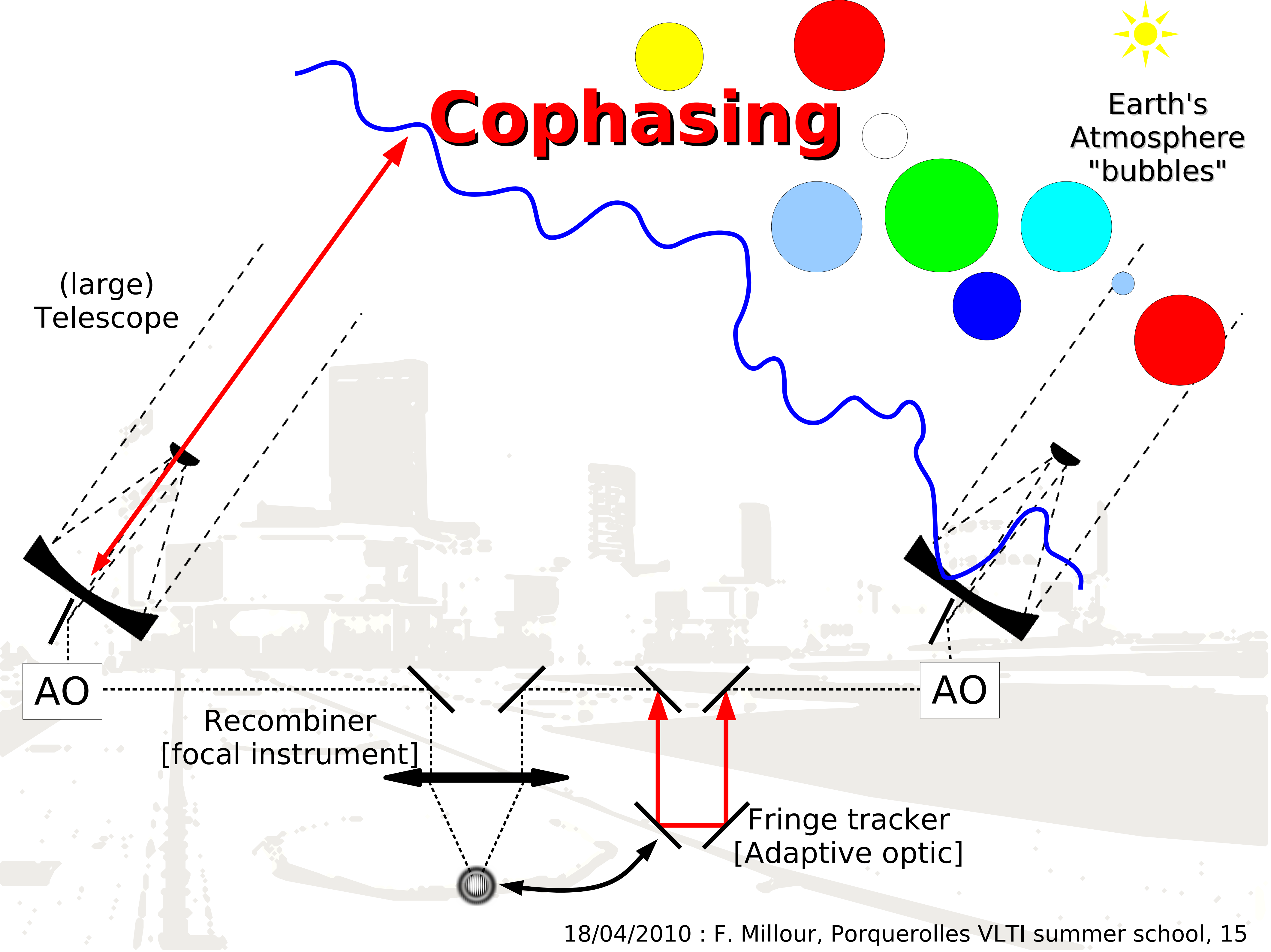
AMBER

- 3 telescopes (imaging capabilities)
- Simultaneous (J), H & K (1-2.5µm)
- Spectral resolutions
  R=35, 1500 & 12000

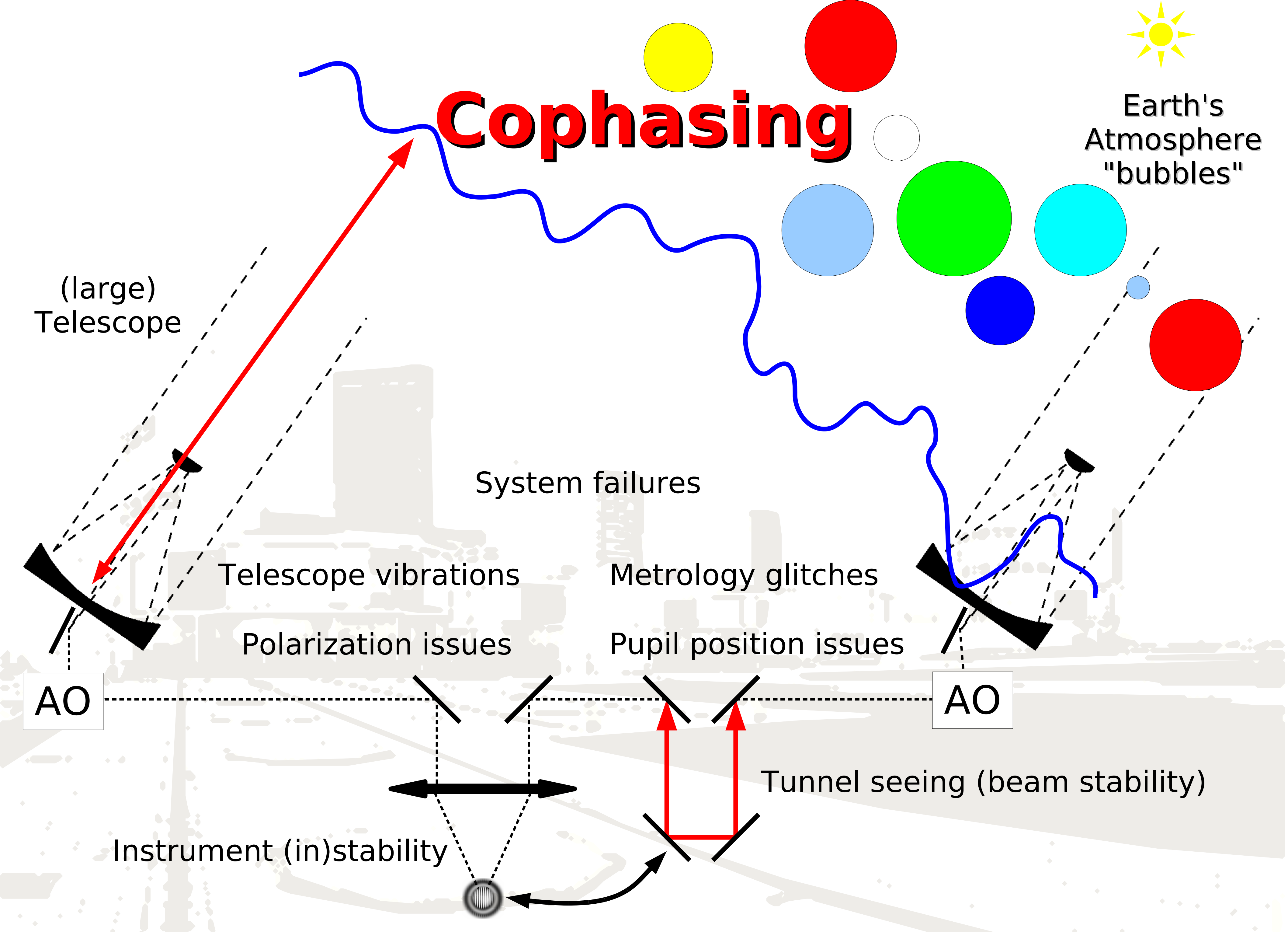
Cophasing



Cophasing


Earth's
Atmosphere
"bubbles"
(large)
Telescope
AO
AO
Recombiner
[focal instrument]
Fringe tracker
[Adaptive optic]

Cophasing



Earth's Atmosphere "bubbles"

(large) Telescope

System failures

Telescope vibrations

Metrology glitches

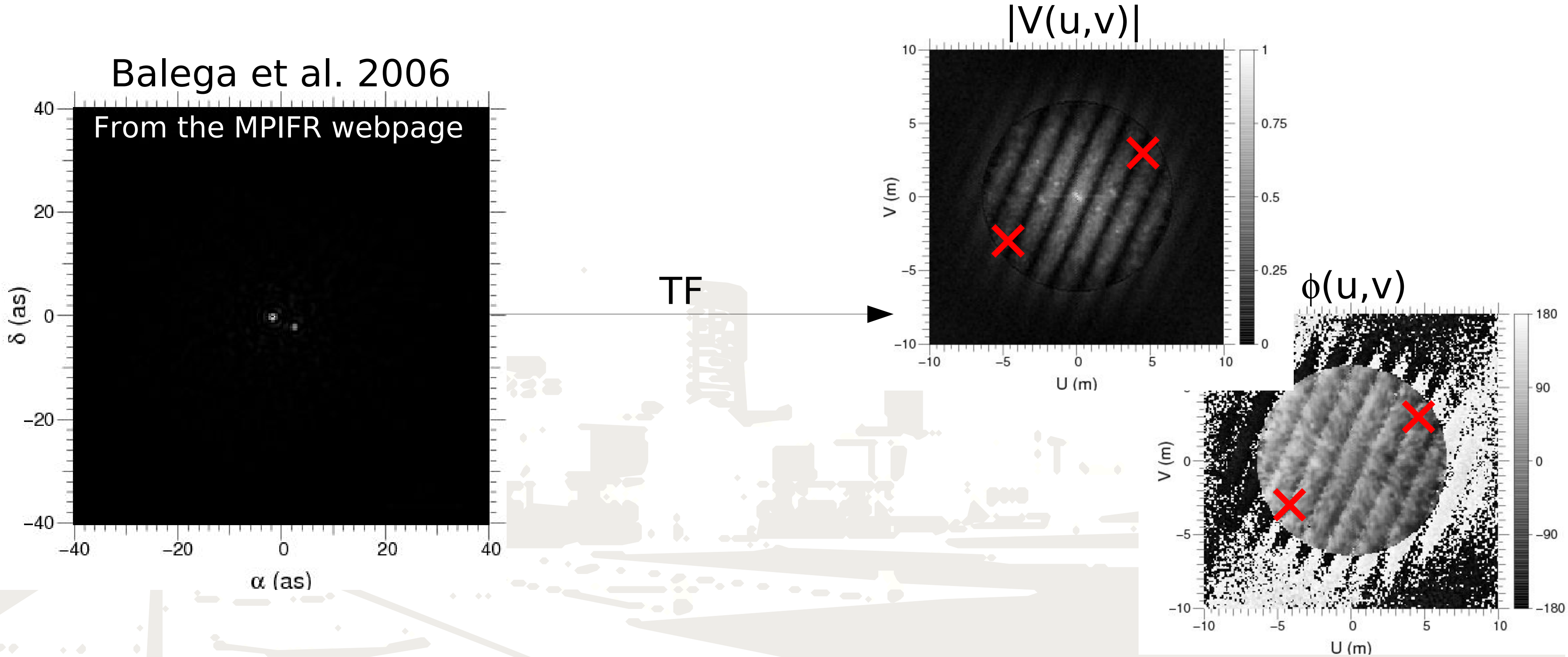
Polarization issues

Pupil position issues

Tunnel seeing (beam stability)

Instrument (in)stability

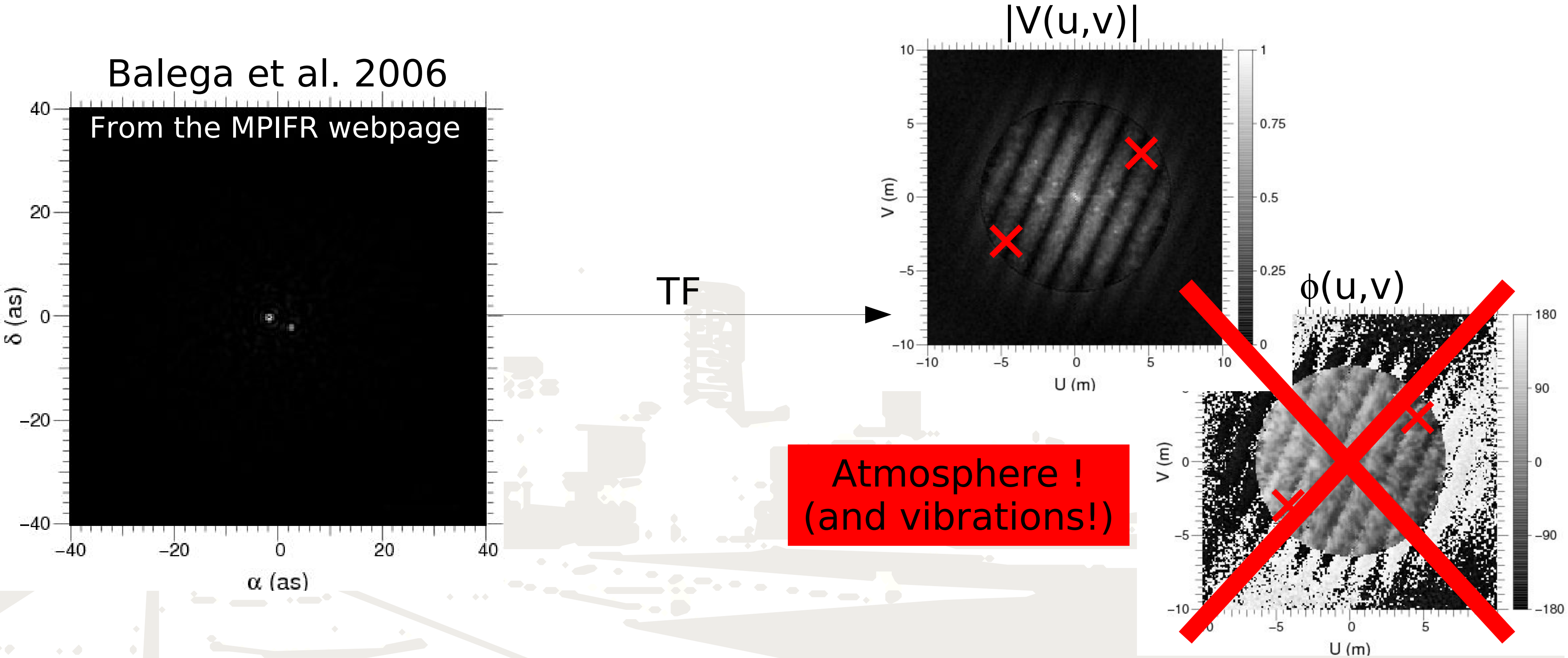
What visibility with interferometry ?


Balega et al. 2006

From the MPIFR webpage

Image : $I(x,y) = O*PSF \rightarrow |V(u,v)|, \phi(u,v)$ & cut-off frequency at D/λ

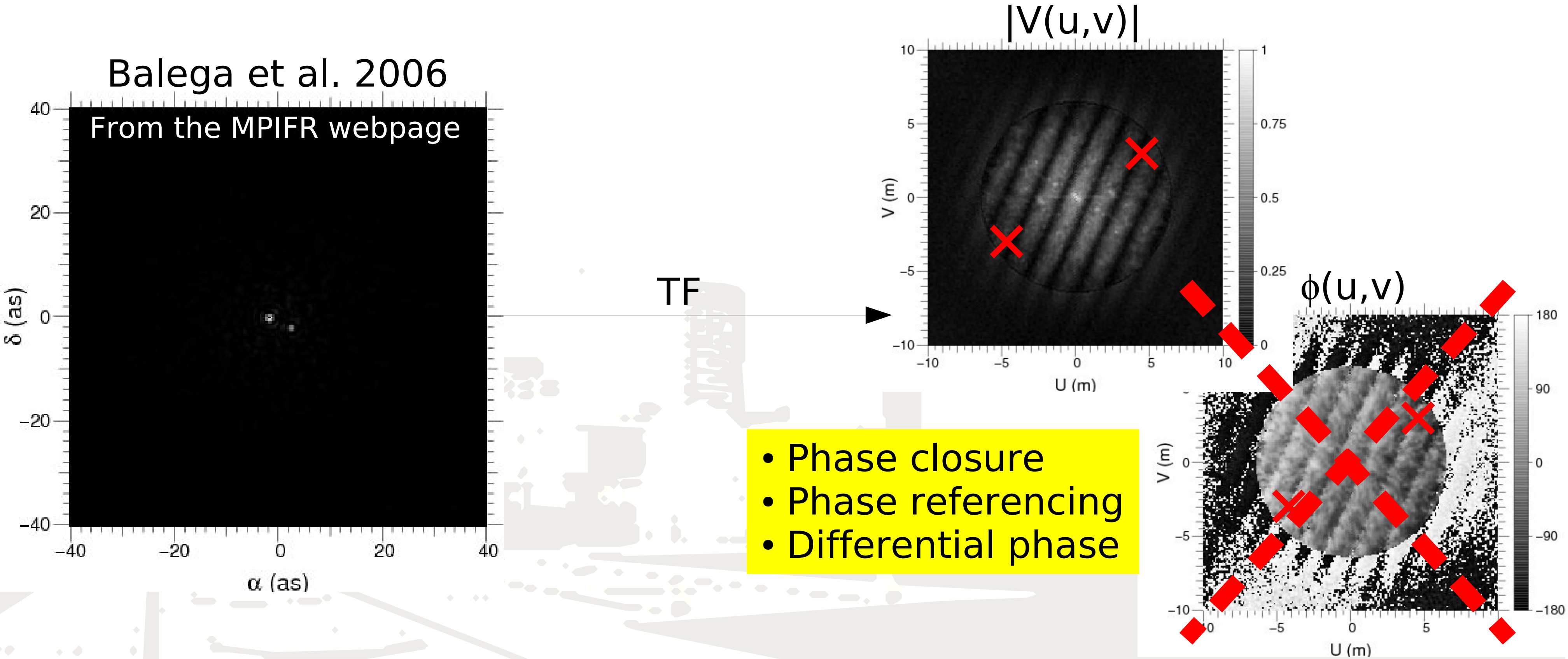
What visibility with interferometry ?



Atmosphere !
(and vibrations!)

Image : I(x,y) = O*PSF → |V(u,v)|, ~~ϕ(u,v)~~ & cut-off frequency at D/λ

What visibility with interferometry ?

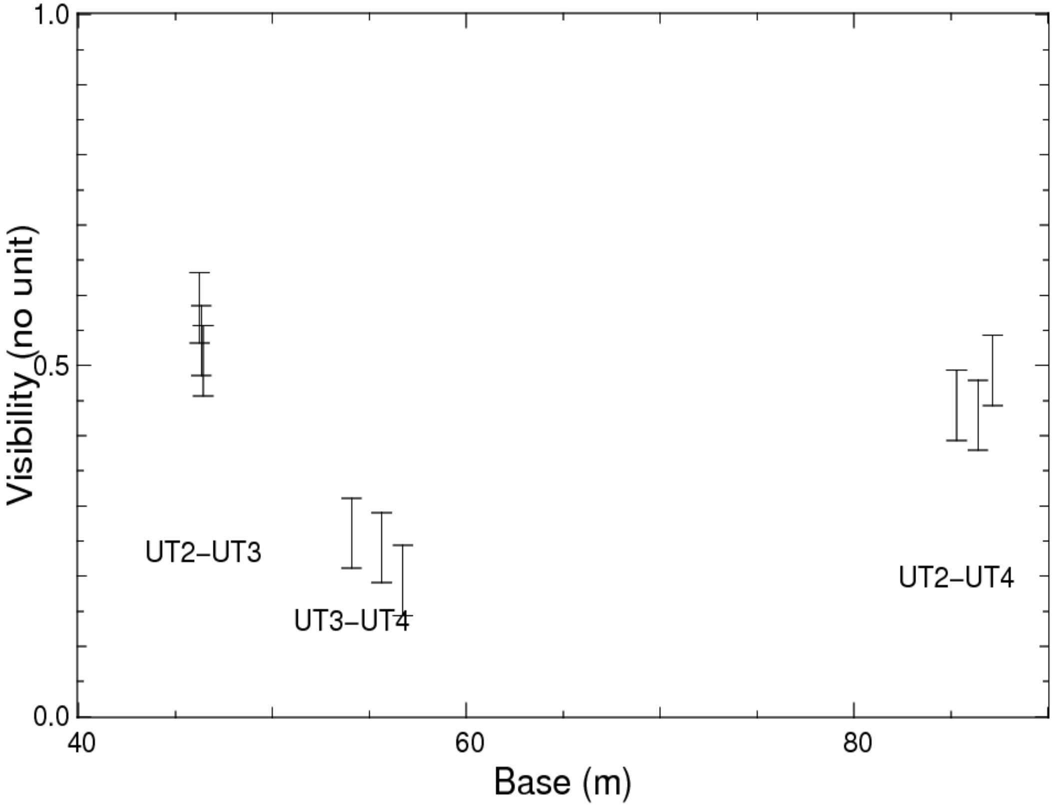


- Phase closure
- Phase referencing
- Differential phase

Image : $I(x,y) = O*PSF \rightarrow |V(u,v)|$, ~~$\phi(u,v)$~~ & cut-off frequency at D/λ

This session

is about what you can do with that ...



Model fitting in the Fourier plane domain is “attractive”:

Domain where interferometric measurements are made
=> errors easier to take into account (ex: Gaussian noise)

When (U,V) plane sampling is poor (almost always the case)

Better than nothing when no imaging is possible (ex: variable source)

Realistic in the VLTI AMBER and MIDI contexts

Simple first step : parametric analysis using basic visibility functions.

Ad-hoc modeling

Allows you to get a first idea of what you have observed!

- Use Fourier transform properties
- Use basic intensity distribution functions

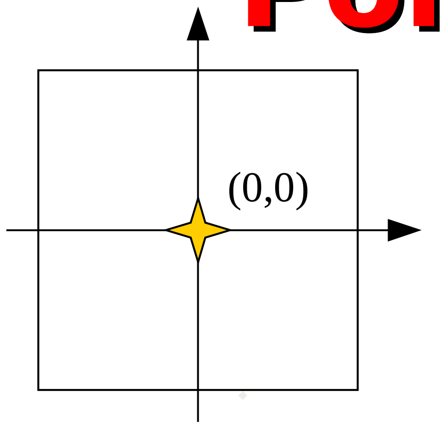
Important first step
towards modelling with
real physical models

Fourier transform properties:

- **Addition** $\mathrm{FT}\{f(x,y)+g(x,y)\} = F(u,v)+G(u,v)$
- **Convolution** $\mathrm{FT}\{f(x,y)\times g(x,y)\} = F(u,v).G(u,v)$
- **Shift** $\mathrm{FT}\{f(x-x_0,y-y_0) = F(u,v)\exp[2\pi i(ux_0+vy_0)]$
- **Similarity** $\mathrm{FT}\{f(ax,by)\} = \frac{1}{|ab|}F(u/a,v/a)$

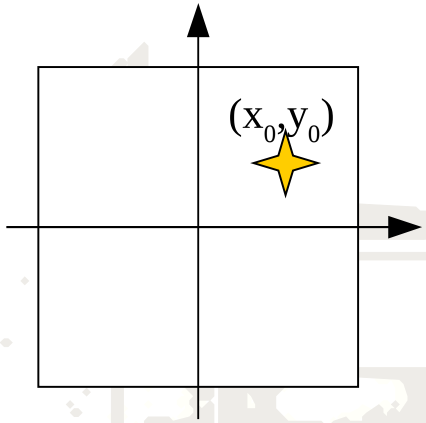
Point source function

Use: Multiple stars



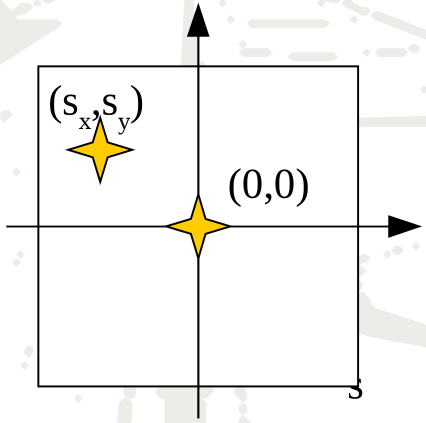
Centered source

$$I(x,y)=\delta(x,y) \Rightarrow V(u,v)=1$$



Off-axis source

$$I(x,y)=\delta(x-x_0)\delta(y-y_0) \Rightarrow V(u,v)=\exp[-2i\pi(x_0u+y_0v)]$$



Binary system

$$A\delta(x,y)+B\delta(x-sx,y-sy) \text{ with } s=\sqrt{sx^2+sy^2}$$

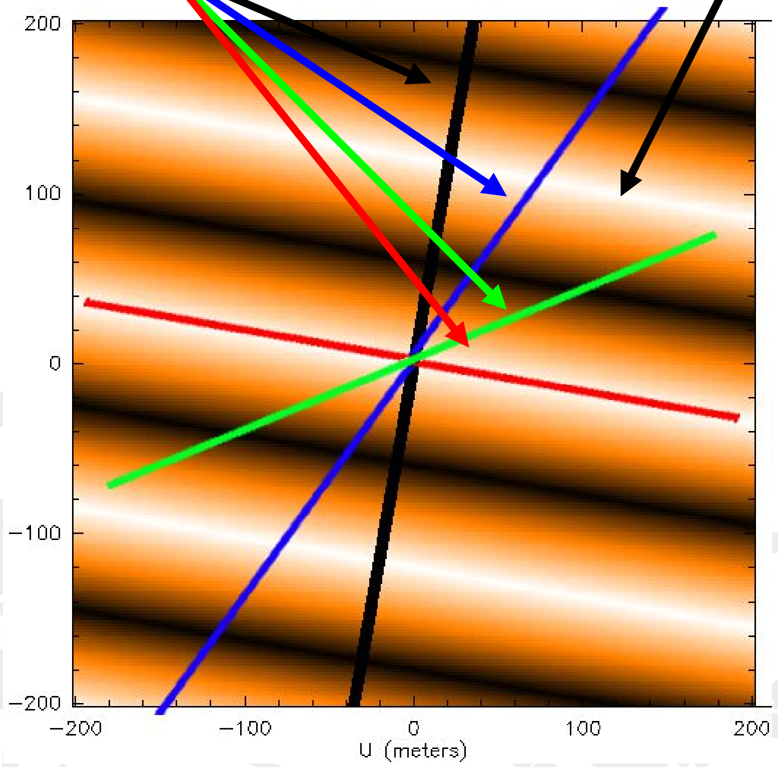
$$\Rightarrow \begin{array}{lcl} \mathrm{V(u,v)} & = & \sqrt{\frac{1+r_{ab}^2+2r_{ab}\cos 2\pi\vec{L_b}\vec{s}/\lambda}{1+r_{ab}^2}} \\ \text{with } \mathrm{r}_{ab} & = & \mathrm{A/B} \\ \text{with } \vec{L_b} & = & \text{Baseline vector} \end{array}$$

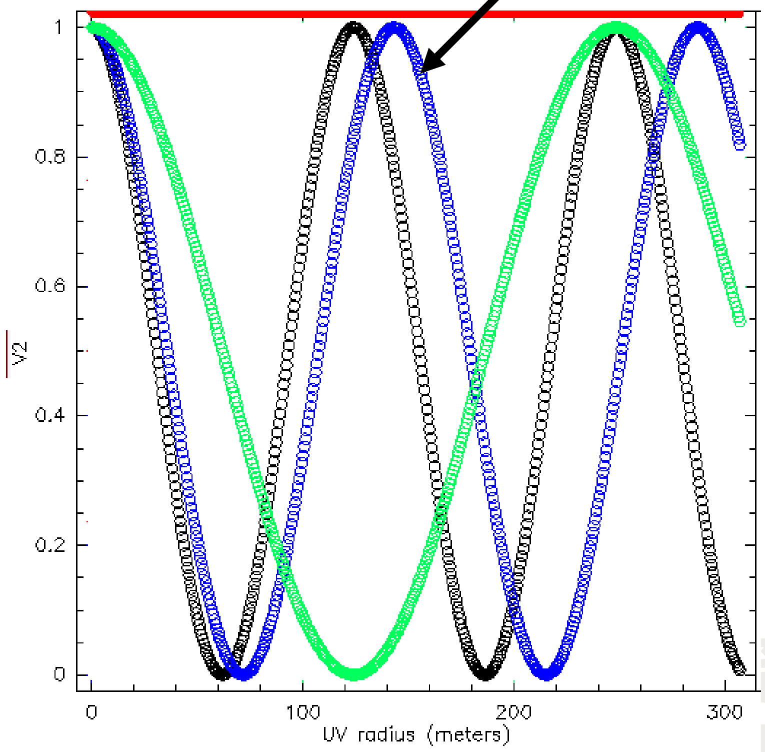
Binary star

Projection of baseline in the plane of sky

The visibility amplitude squared in (uv) plane

Squared visibility curves for three baselines as a function of baseline length

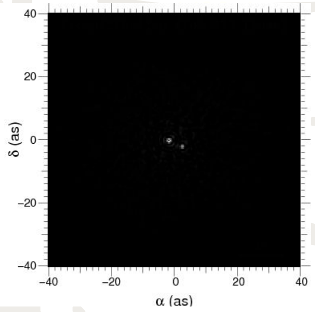




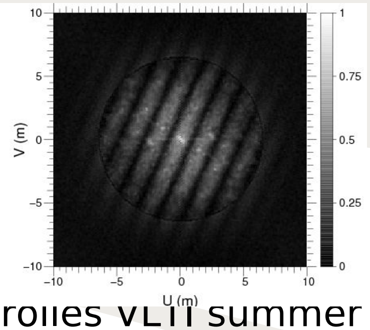
image

Remember:

Visibility

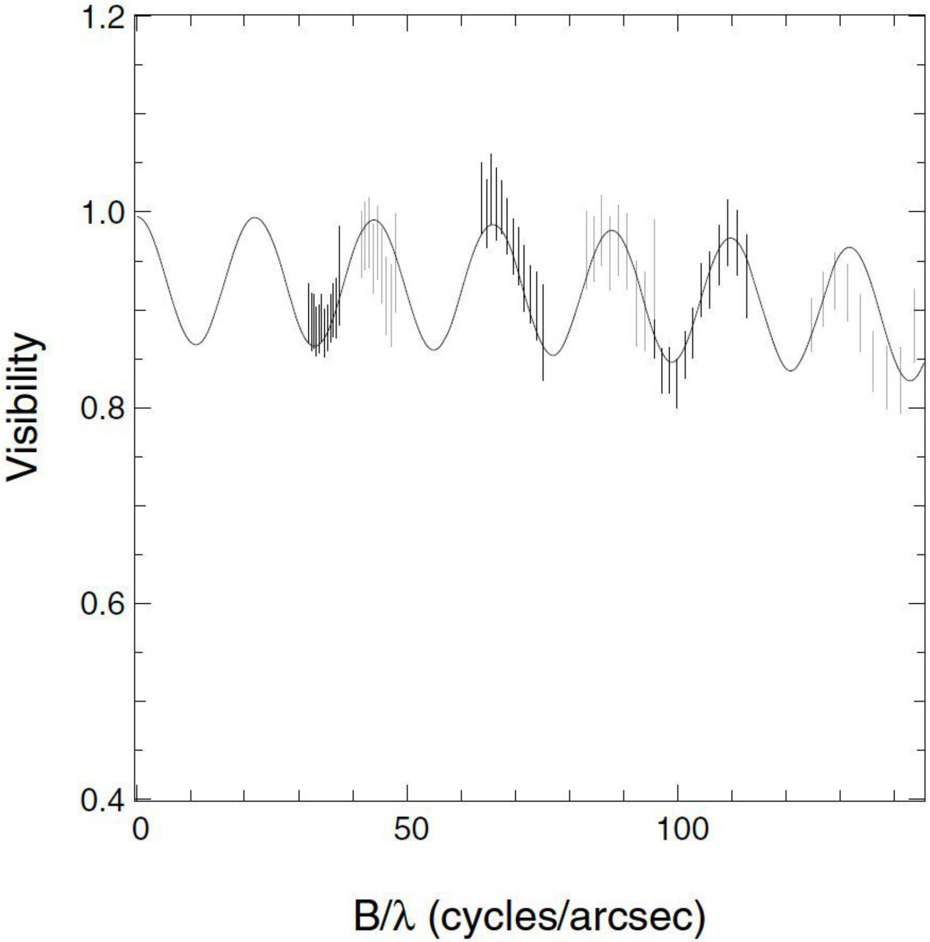


TF



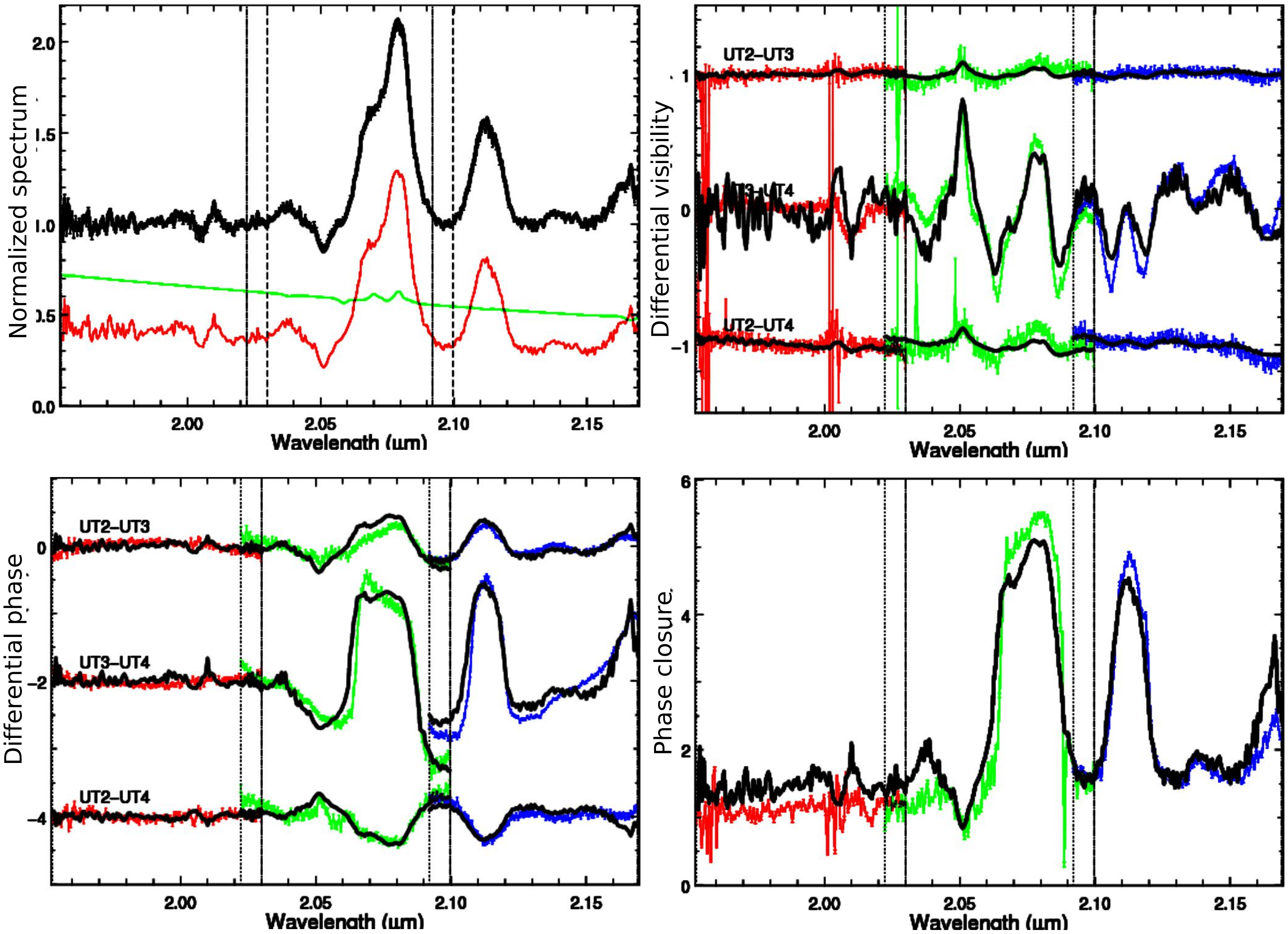
Binary star (exemple 1)

Binary star visibility curve as a function of spatial frequency



δ Cen, Meilland et al. 2008

Binary star (exemple 2)

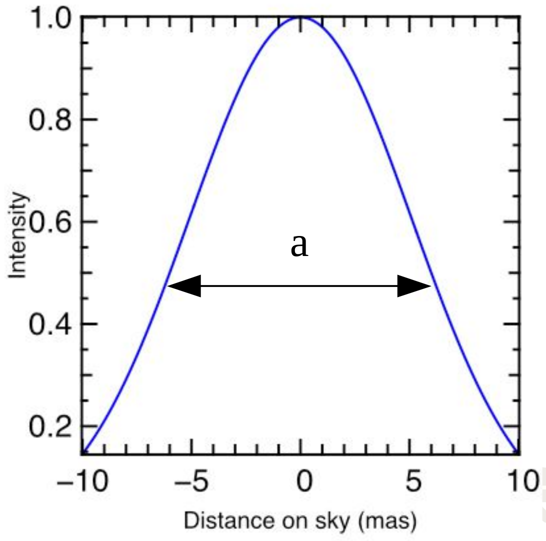


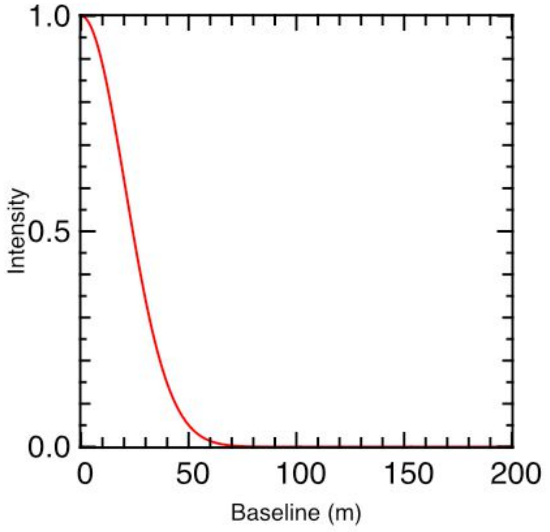
Spectrally varying flux ratio makes it working !

γ^2 Vel, Millour et al. 2007

Gaussian brightness distribution.

Use: Estimate for angular sizes of envelopes, disks, etc.





$$I(r) = \frac{I_0}{\sqrt{\pi / 4 \ln 2 a}} \exp(-4 \ln 2 r^2 / a^2)$$

Where a = FWHM intensity, I_0 = Peak intensity and

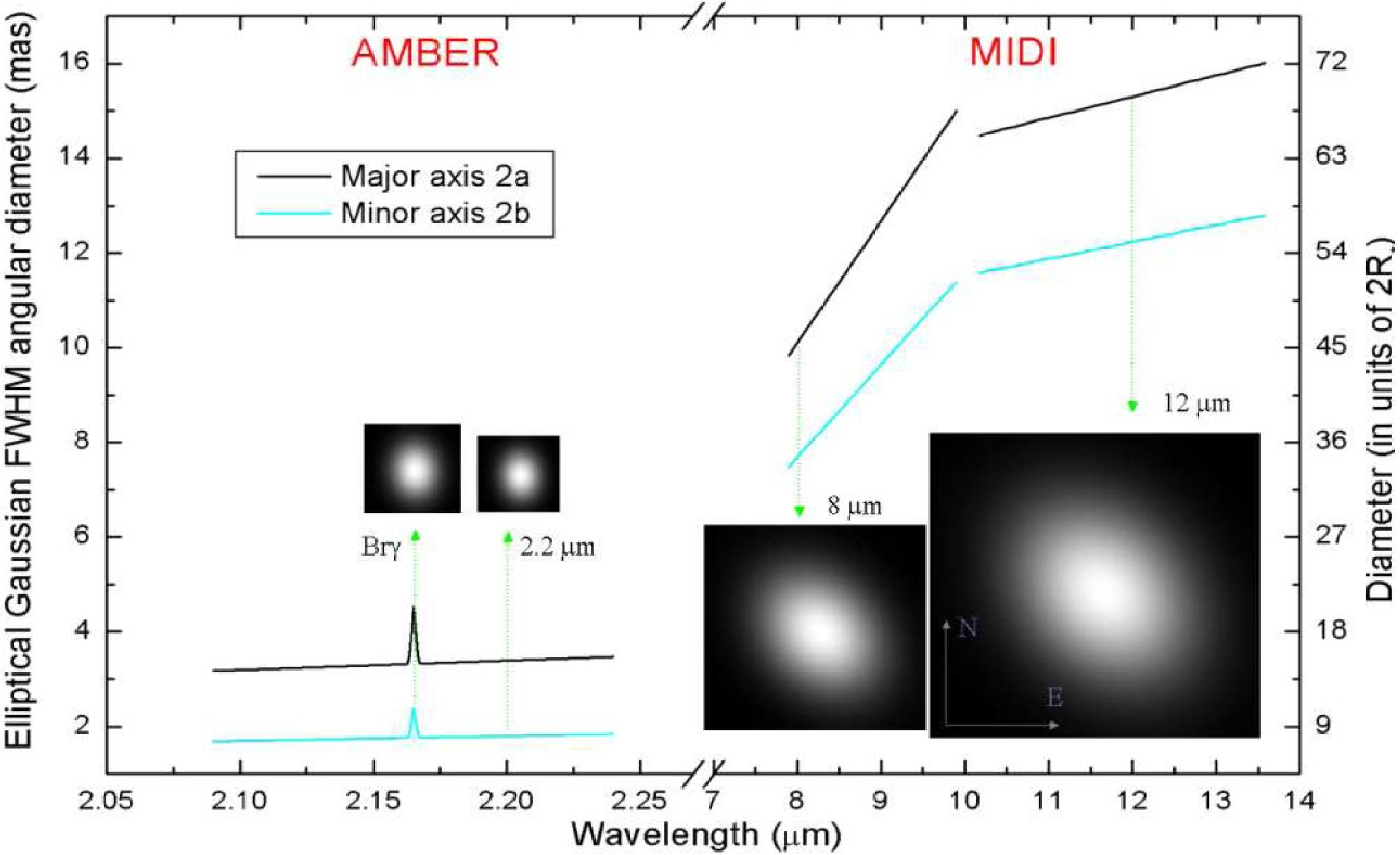
$$r = \sqrt{x^2 + y^2}$$

$$V(\rho) = \exp[-(\pi a \rho)^2 / (4 \ln 2)]$$

Where

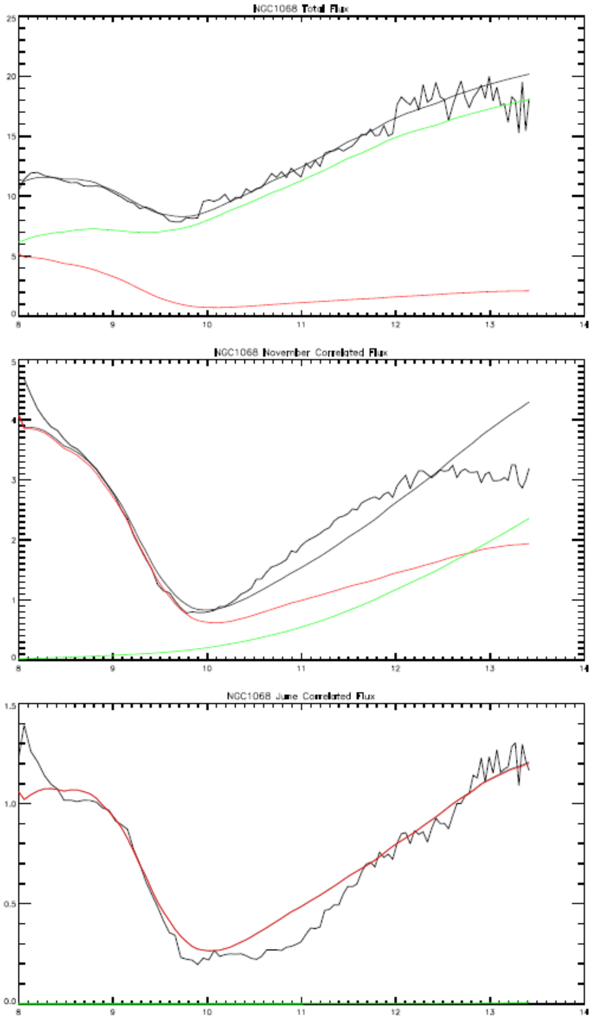
$$\rho = \sqrt{u^2 + v^2}$$

Gaussian (example 1)



Dominiciano da Souza et al A&A 2007

Gaussian (example 2)



Rottgering et al 2004

- MIDI observations of NGC 1068
- 1st-order interpretation with a series of Gaussian disks

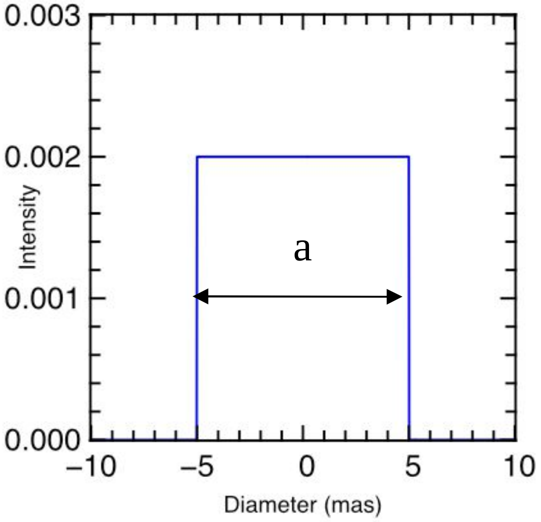
Uniform disk

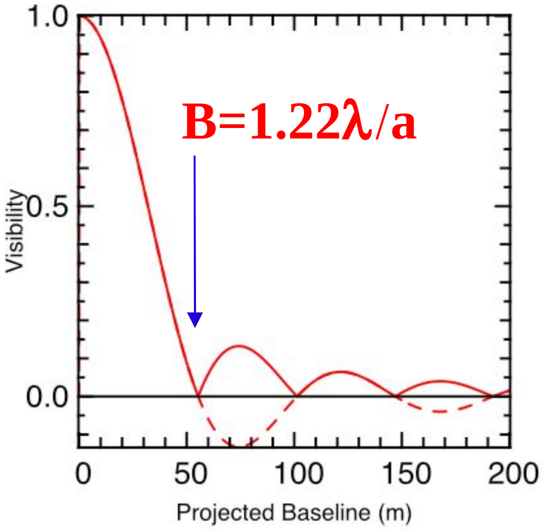
Use: aproximation for brightness distribution of photospheric disk.

$$\begin{aligned} \mathrm{I(r)} &= 4/(\pi a^2), \mathrm{if r} = \sqrt{\mathrm{x}^2+\mathrm{y}^2} \le \mathrm{a}/2 \\ \mathrm{I(r)} &= 0 \quad \mathrm{otherwise} \end{aligned}$$

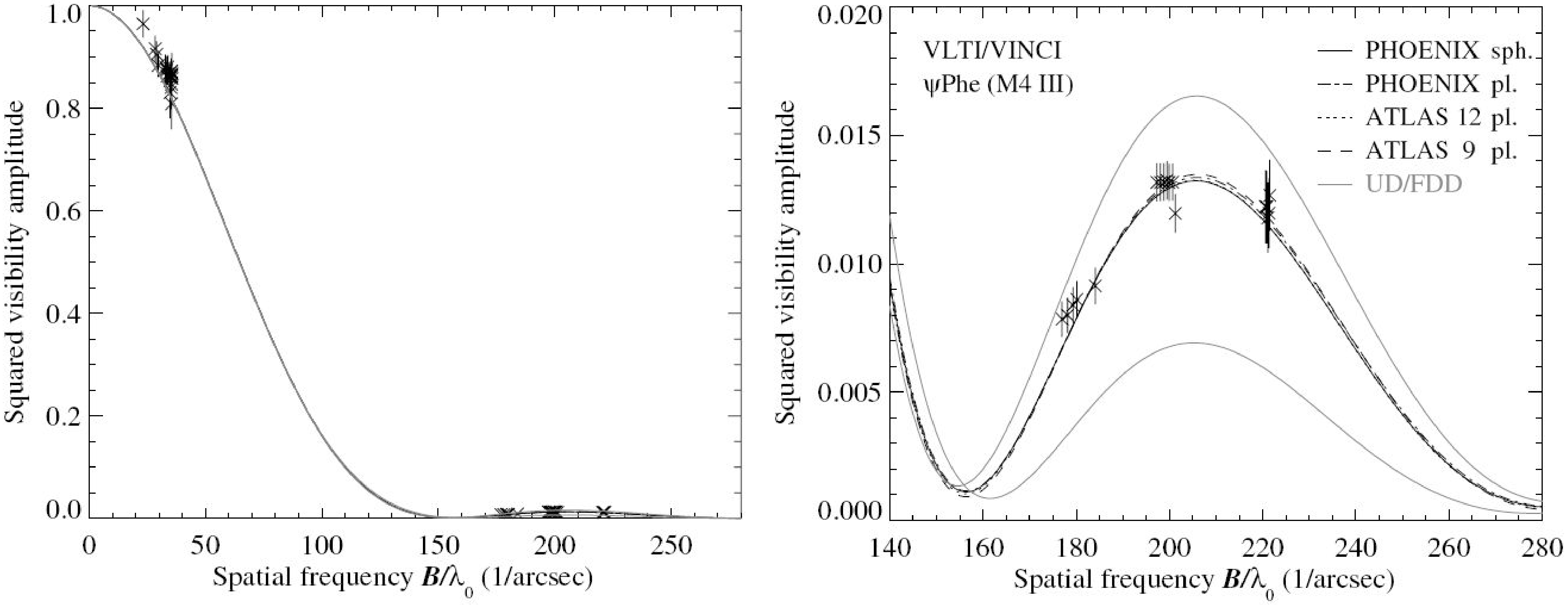
$$F(\rho) = \frac{J_1(\pi a \rho)}{\pi a \rho} \mathrm{with} \rho = \sqrt{\mathrm{u}^2+\mathrm{v}^2}$$

a = diameter
Sophistication of the model
I= f(r), limb darkening
Cf Hankel transformation
(afterwards)





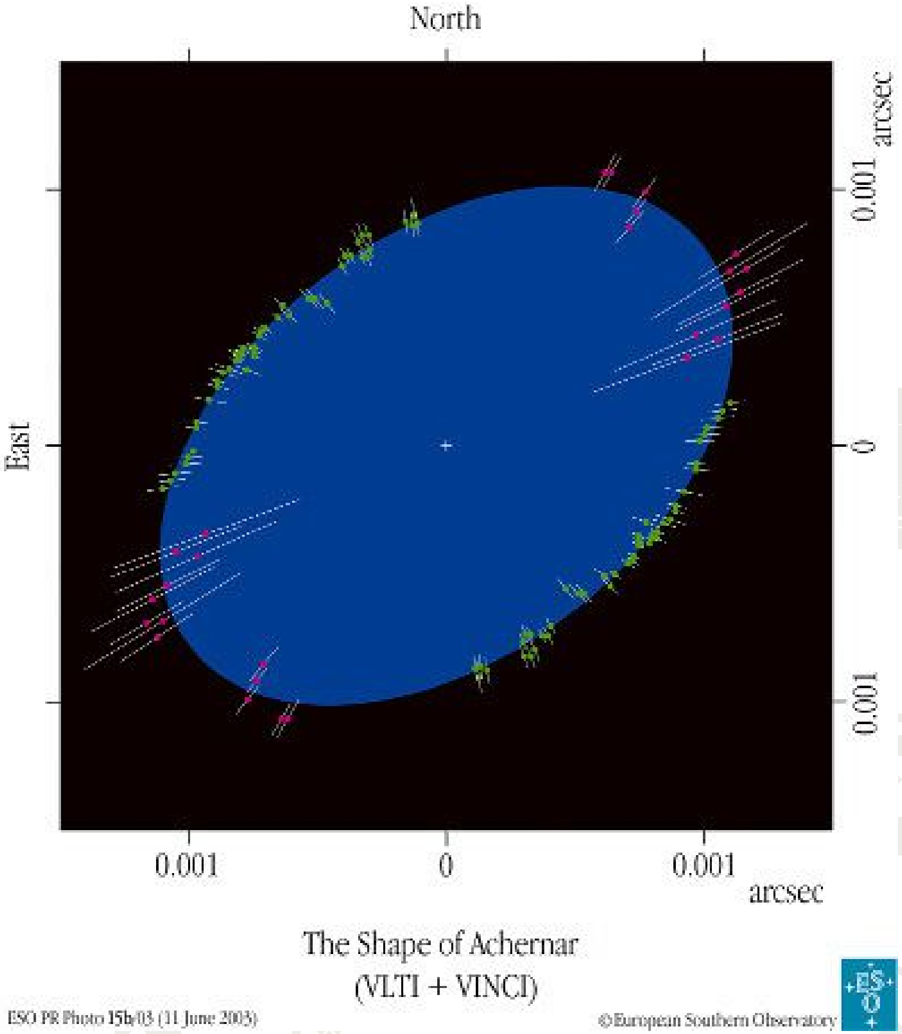
Uniform disk (example 1)



Wittkowski et al. 2003

- Comparison of ψ Phe VLTI/VINCI observations with uniform disk model (gray line)
- Second lobe points are the most constraining

Uniform disk (example 2)



The Shape of Achernar
(VLTI + VINCI)

ESO PR Photo 15b/03 (11 June 2003)

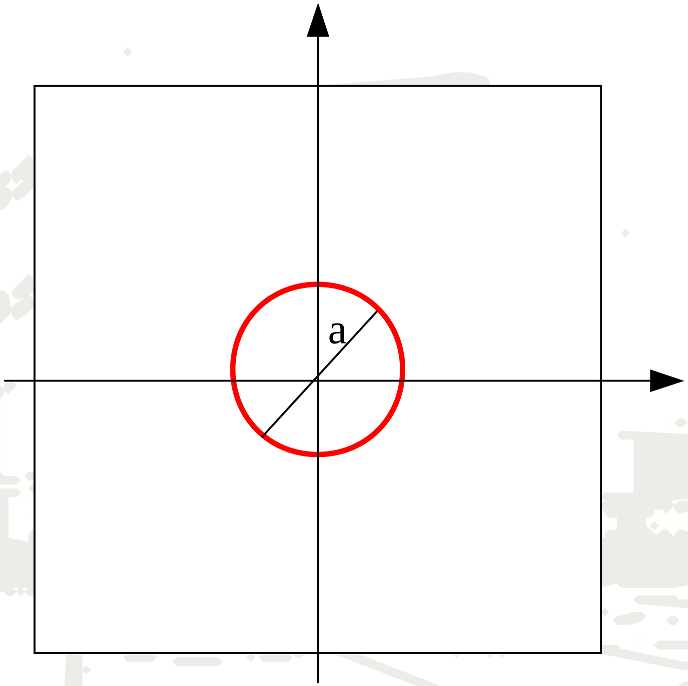
©European Southern Observatory

- Determination of uniform diameter of Archenar (VLTI/VINCI)
- Different positions angles shows evidence for flattening due to to fast rotation

Dominiciano da Souza et al A&A 2003

Ring

Use: complex centro-symmetric structure



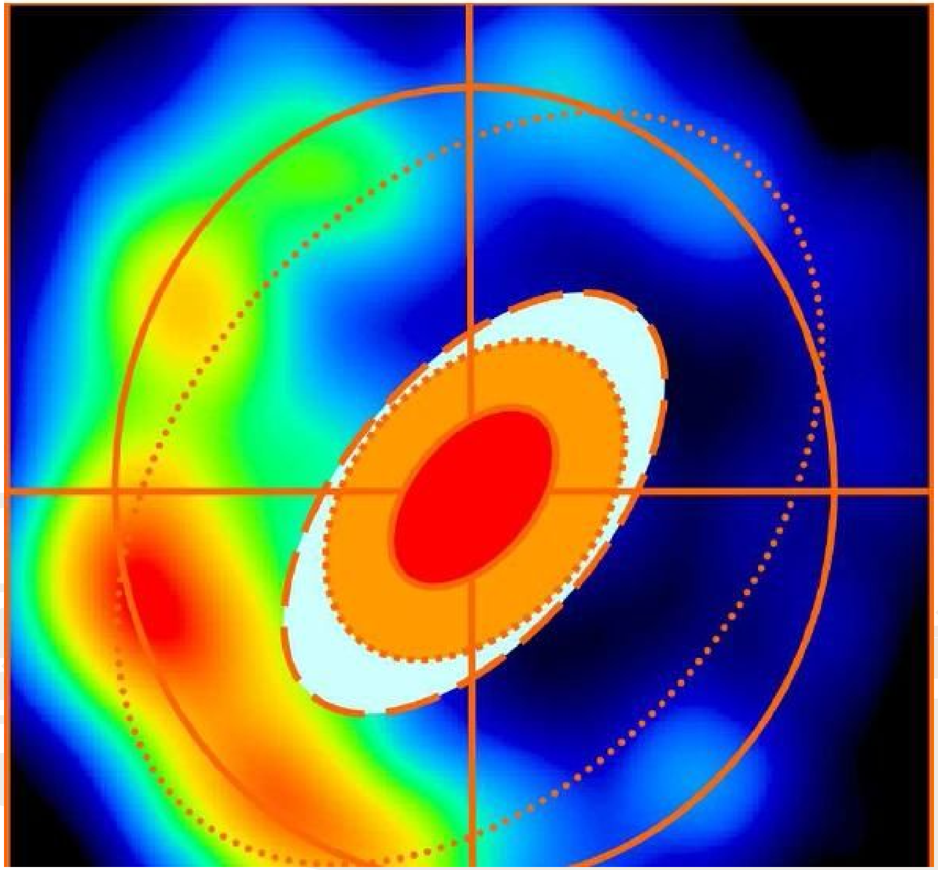
$$I(r) = 1/(\pi a)\delta(r - a/2)$$

$$V(\rho) = J_0(\pi a \rho)$$

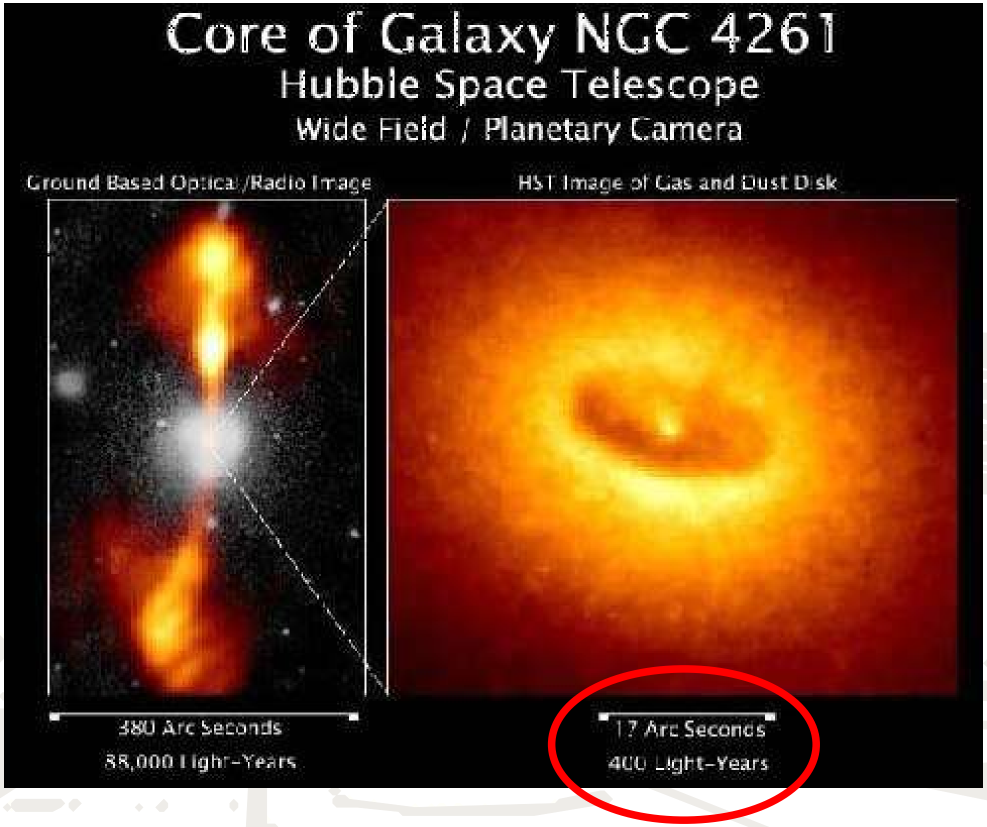
Ring (example 1)

- RS Oph aspherical Nova explosion

Chesneau et al., A&A 2007

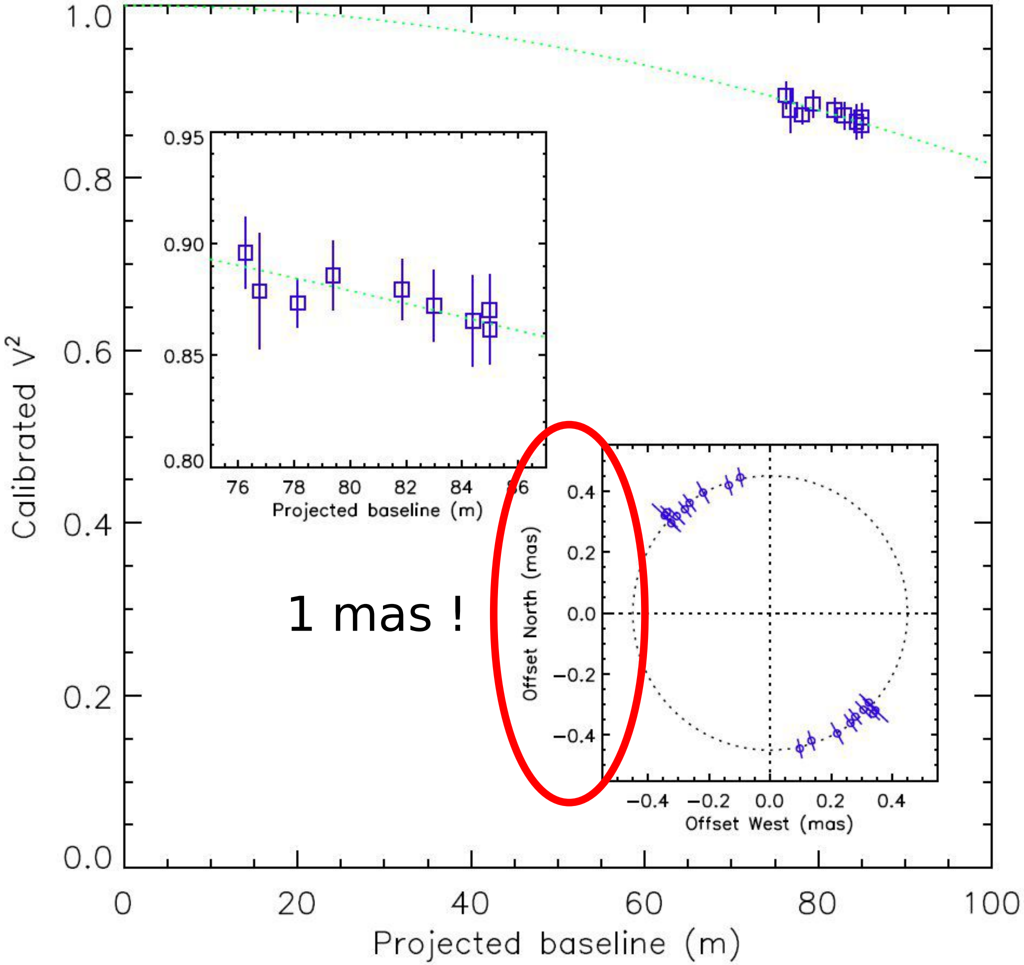


Ring (example 2)



17 arc-seconds

Core of galaxy NGC 4151
Keck-interferometer



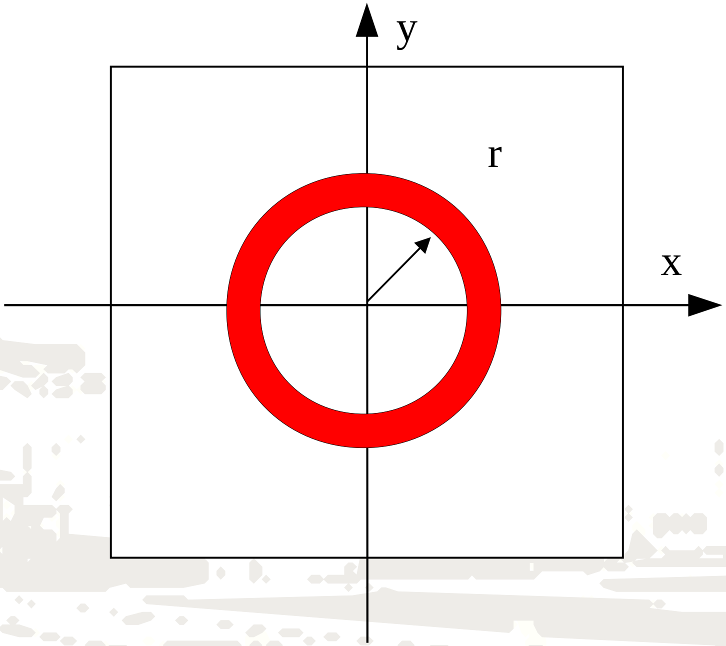
1 mas !

Kishimoto et al. 2009

Circularly symmetric object

e.g: an accretion disk made of a finite sum of annulii with different effective temperatures

Circularly symmetric component I (r)
centered at the origin of the (x,y) coordinate system.



The relationship between brightness distribution and visibility is a ***Hankel function***

$$V(\rho) = 2\pi \int_0^\infty I(r) J_0(2\pi r \rho) r dr$$

with

$$r = \sqrt{x^2 + y^2}$$

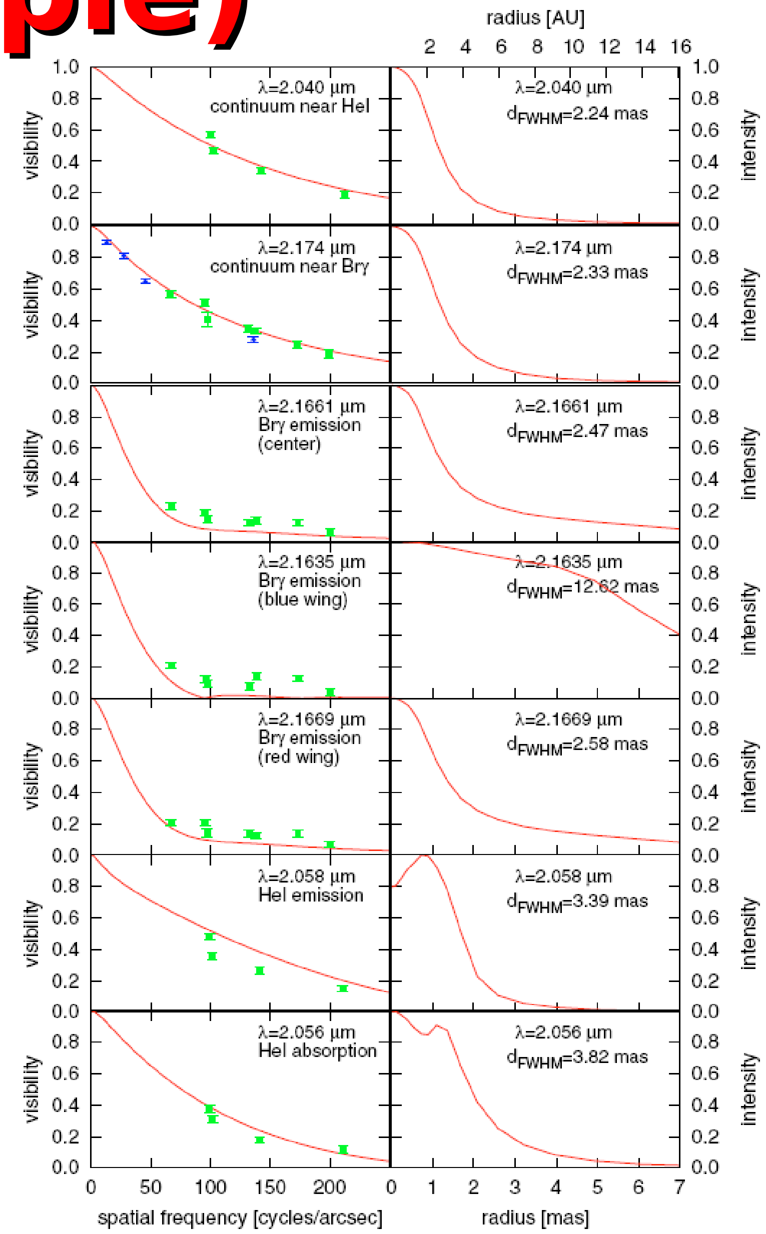
and

$$\rho = \sqrt{u^2 + v^2}$$

Circularly symmetric object (example)

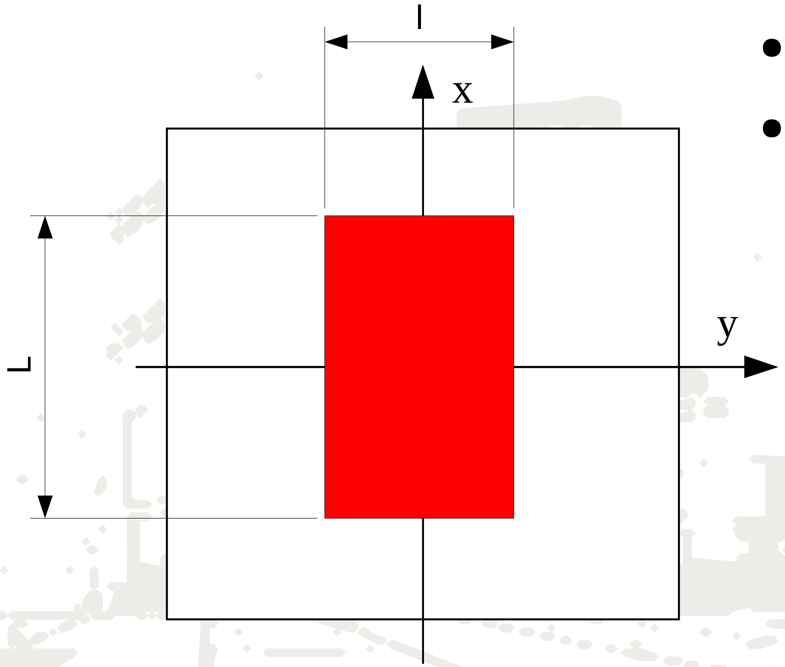
- Optically thick wind around η Car (Hillier models gives intensity profiles)

Weigelt et al., A&A 2007



Pixel

Basic brick of an image !

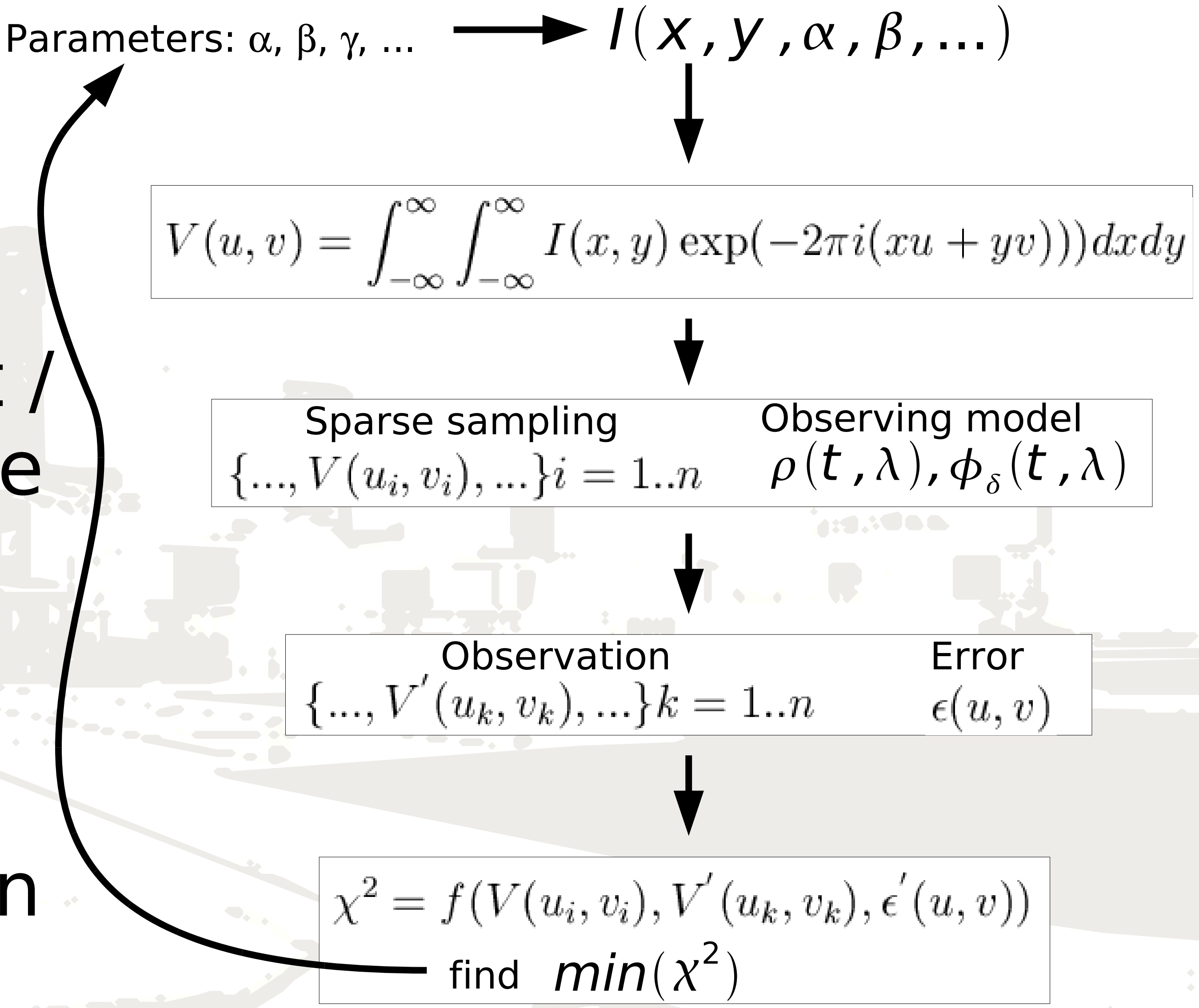


- I(x,y) = 1/lL if |x| < l and |y| < L
- I(x,y) = 0 otherwise

$$V = \frac{\sin(\pi x l)\sin(\pi y L)}{\pi^2 x y l L}$$

The modelling process

- Model
- Instrument / atmosphere
- Data
- Comparison



Parameters: α, β, γ, ... → $I(x, y, \alpha, \beta, \ldots)$

$$V(u,v) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} I(x,y)\exp(-2\pi i(xu+yv))dxdy$$

Sparse sampling

$\{\ldots, V(u_i, v_i), \ldots\} i = 1..n$

Observing model

$\rho(t, \lambda), \phi_\delta(t, \lambda)$

Observation

$\{\ldots, V'(u_k, v_k), \ldots\} k = 1..n$

Error

$\epsilon(u, v)$

$$\chi^2 = f(V(u_i, v_i), V'(u_k, v_k), \epsilon'(u, v))$$

find $min(\chi^2)$

Conclusion(s)

✓Visibility study without imaging can be efficient.

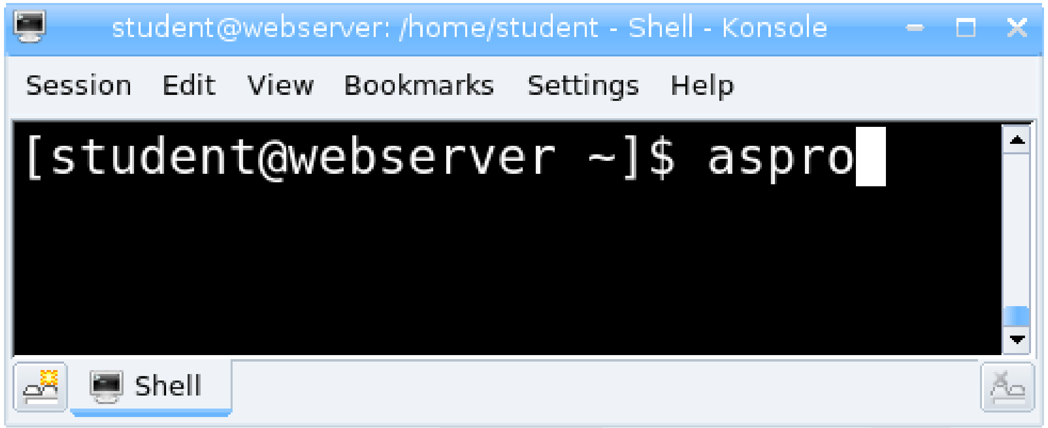
✓The (u,v) coverage strategy is different from imaging. Limited allocated time means (very) limited (u,v) points.

✓Use basic models in order to prepare your observation and determine what is the more constraining configuration.

✓Visibility space is the natural place to understand the errors of the final result.

✓Always start by describing your observations in terms of basic functions. It brings qualitative and quantitative information useful for further more detailed computations.

How to launch ASPRO (local installation)



```
[student@webserver ~]$ aspro
```

“FULL ASPRO INTERFACE”

Here you are !