Theory of optical long-baseline interferometry data reduction

F. Millour (OCA, Nice)
with some ideas and slides taken from
A. Merand, J.B. Lebouquin, O. Chesneau, C. Hummel, J. P. Berger, G. Perrin, etc.

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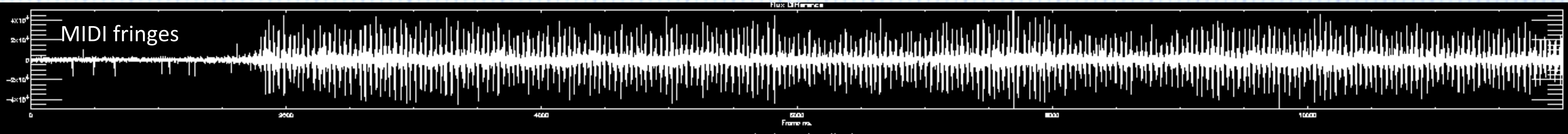

This course has 100+ slides

- Not an extensive review
- No pretty pictures
- 100% equations
- No fun (?)
- You will not be able to write your own software with it…

- But…
- You need to be critical!
  - DRS are often « black boxes »
  - Know the limitations
  - Consistency / inconsistency of results
- You need to understand the technique
  - Better observing strategies
  - Be able to interpret data

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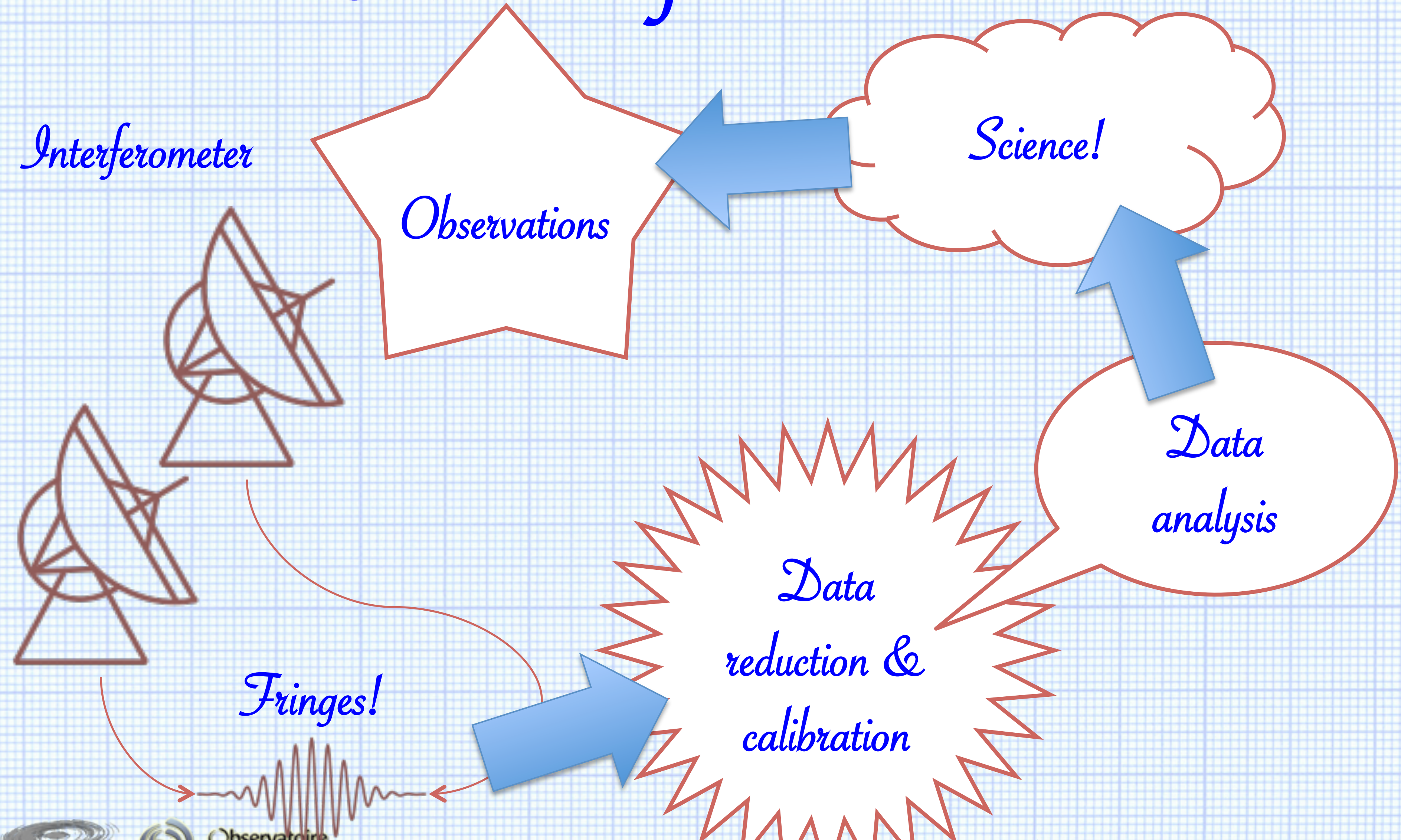
Context of this course



Outline

- Why do we care so much about data reduction?
  - What are we looking for?
  - What adversities are we fighting against?
- The interferometry observables
  - All the observables
  - Statistics
  - Calibration
- A few implementations
  - AMBER data reduction
  - MIDI data reduction
- Conclusions

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With a practical example:

Why do we care so much about data reduction?

1st of all, what are we looking for?

- ZVC*: complex degree of light coherence = normalized Fourier Transform of the source brightness
- Fringe = cosine modulation of light due to interferences

$$I(\delta_0) = I_0\left[1 + \mu \cos\left(\phi - 2\pi\frac{\delta_0}{\lambda}\right)\right]$$

- The fringe contrast (μ) & phase (ϕ), or fringe visibility ($V = \mu e^{i\phi}$) at the recombination point measures this complex degree of light coherence

*For dummies, ZVC means: « Zernicke & van Cittert Theorem »

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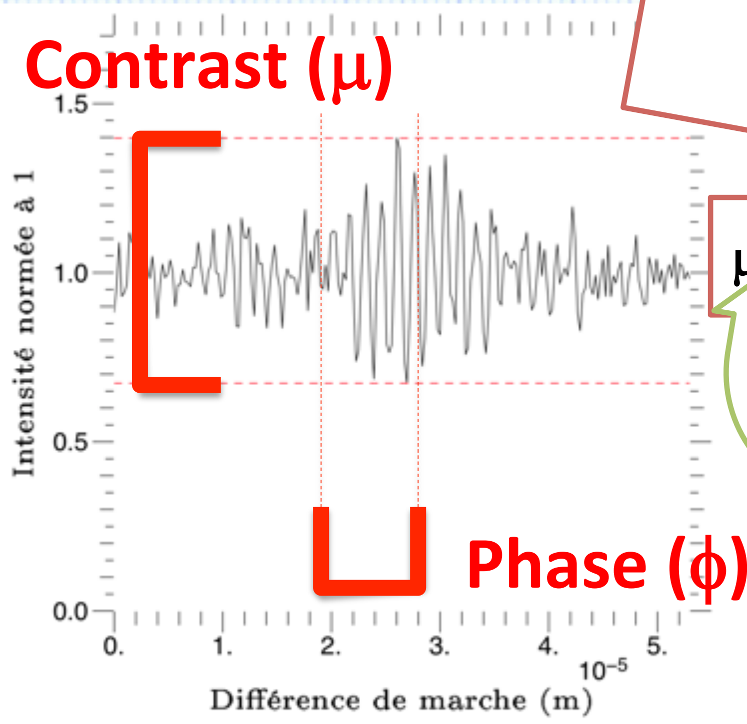
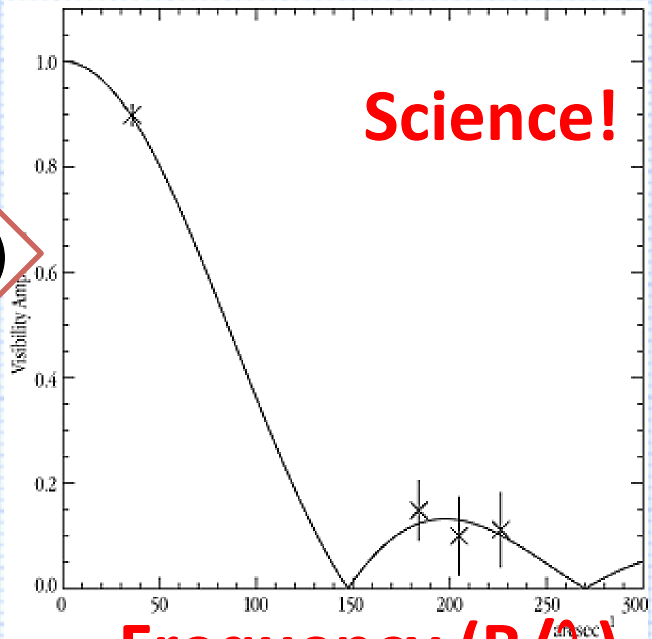
1st of all, what are we looking for?

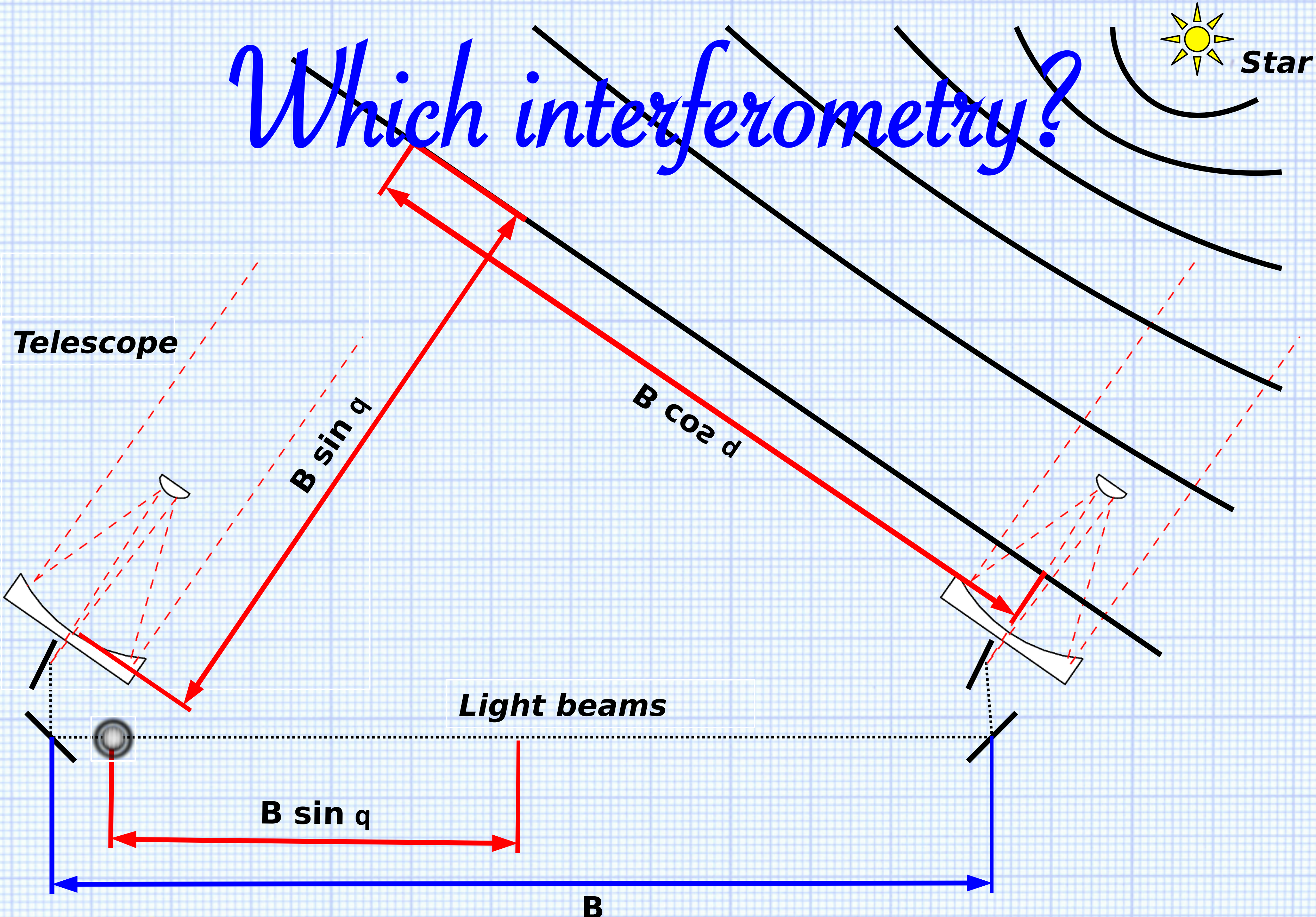


$\mu e^{i\phi}$ = TF[object](B/λ)

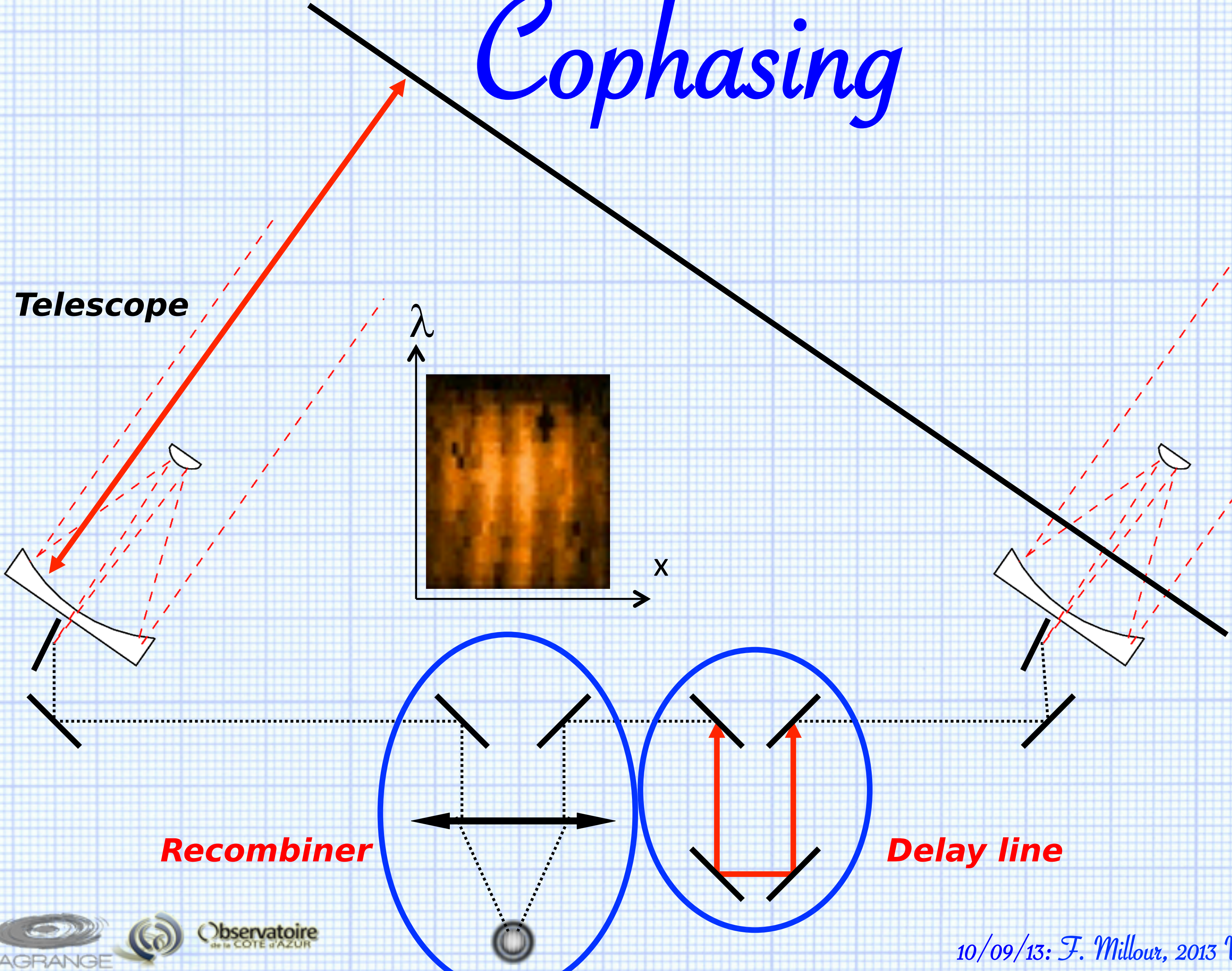
This course is about how do we get **μ** *and* **ϕ**



Which interferometry?



Cophasing



Delay lines

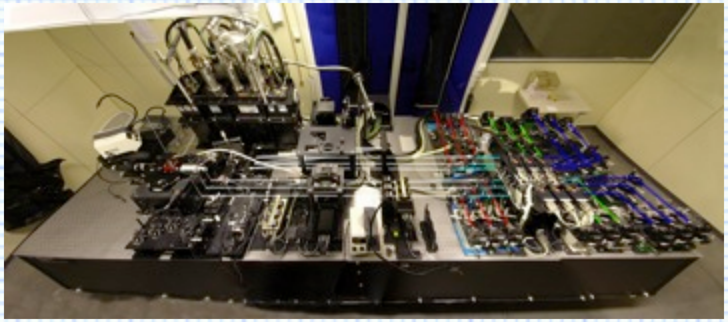
http://www.eso.org

http://www.mro.nmt.edu/Projects/interferometer.htm

Recombiners

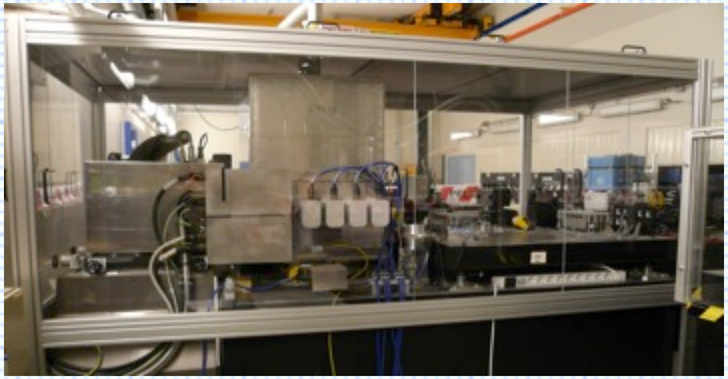
PIONIER

- 4 télescopes
- Bande H (1.65μm)
- Large bande



AMBER

- 3 télescopes
- J, H & K simultanés (1-2μm)
- Résolutions spectrales R=35, 1500 & 12000



MIDI

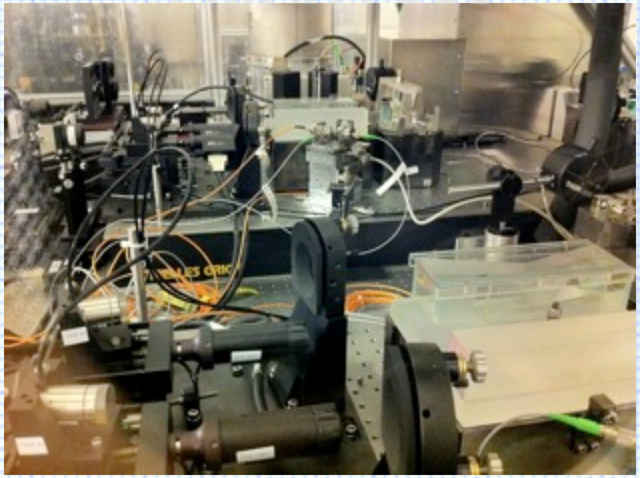
- 2 télescopes
- Bande N (8-13μm)
- Résolutions spectrales R=30 & 300

PRIMA

- 2 télescopes
- Bande K (1.65μm)
- Astrométrie

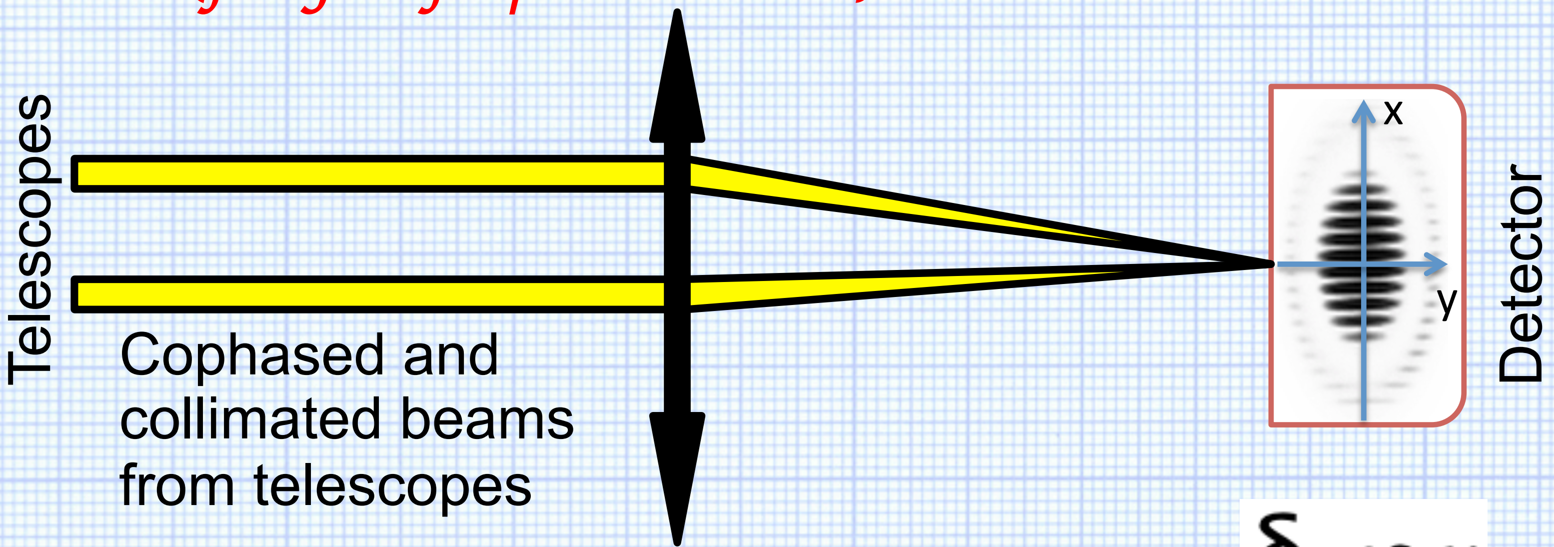
PRIMA DDLs

PRIMA FSUs



Multiaxial recombination

- Overlap the beams with a tilt to produce a variation of OPD (fringes of equal thickness)

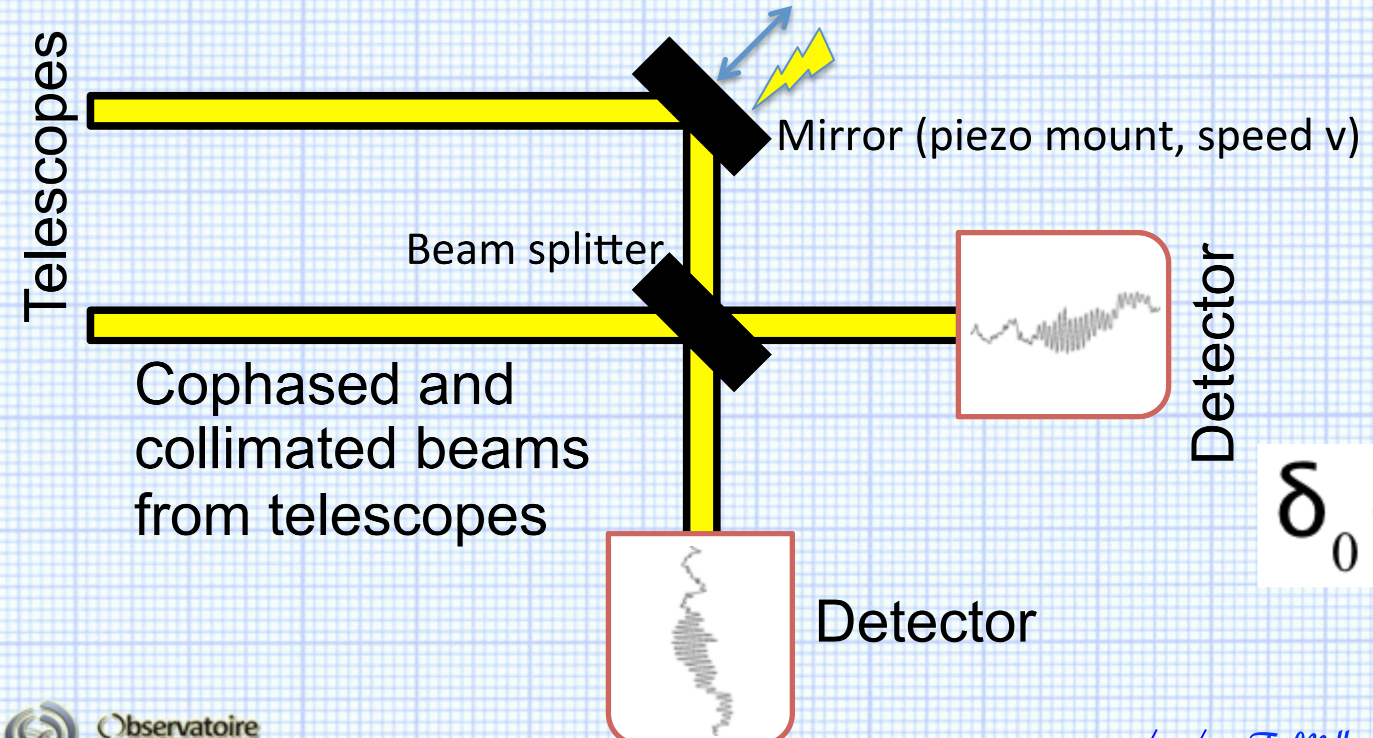


$$\delta_0 \propto x$$

$$I(\delta_0) = I_0\left[1 + \mu\cos\left(\phi - 2\pi\frac{\delta_0}{\lambda}\right)\right]$$

Coaxial recombination

- Overlap the beams on top of each other. OPD is varied with an input piston (fringes of equal path)



$$\delta_0 \propto v\, t$$

1st of all, what are we looking for?

- An interferometer produces
  - a lot of data with
  - tons of noise

Example: a MIDI file (1mn) weights 100Mb

Max. compression rate: 8%

- A DRS aims a getting the best results out of all this noise

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What are the issues?

Fringe signal has a simple expression:

$$I(\delta_0) = I_0\left[1 + \mu\cos\left(\phi - 2\pi\frac{\delta_0}{\lambda}\right)\right]$$

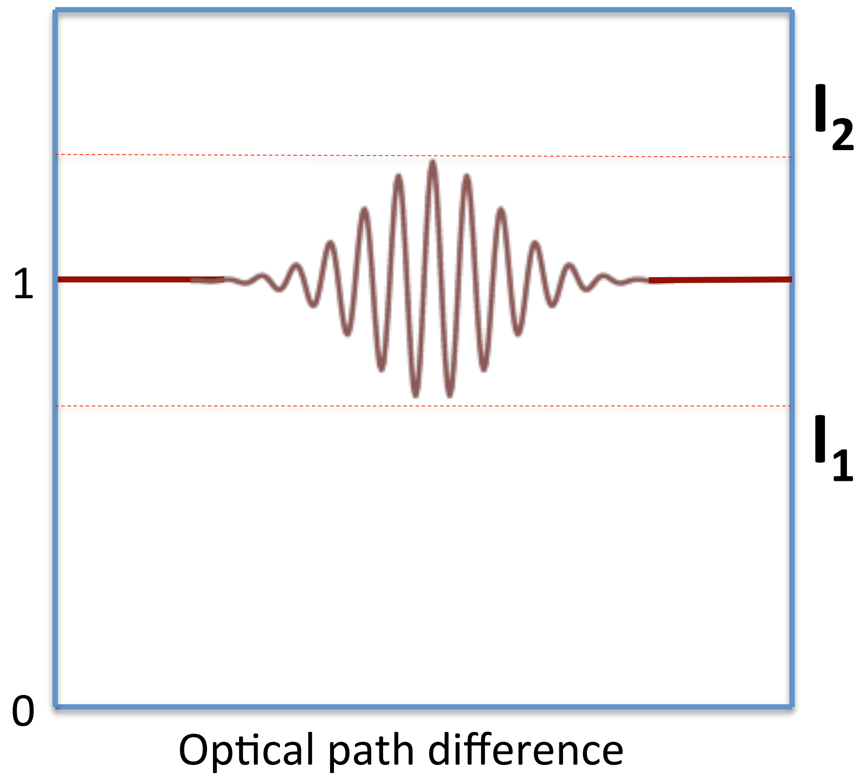
Visibility can be estimated linearly:

$$\Re(V) = I(0) - 1 \qquad \Im(V) = I(\lambda/4) - 1$$

So, there are no issues…

What are the issues?

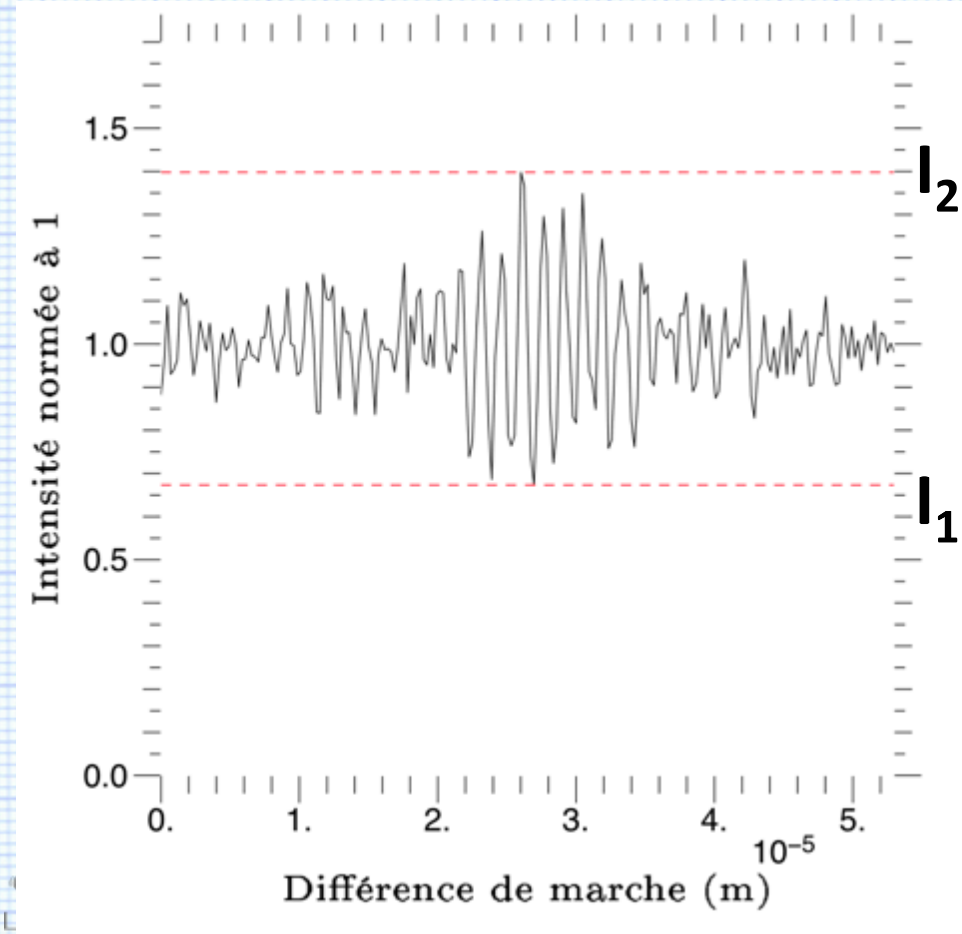
Fake data



$$\mu = \frac{I_2 - I_1}{I_2 + I_1}$$

What are the issues?

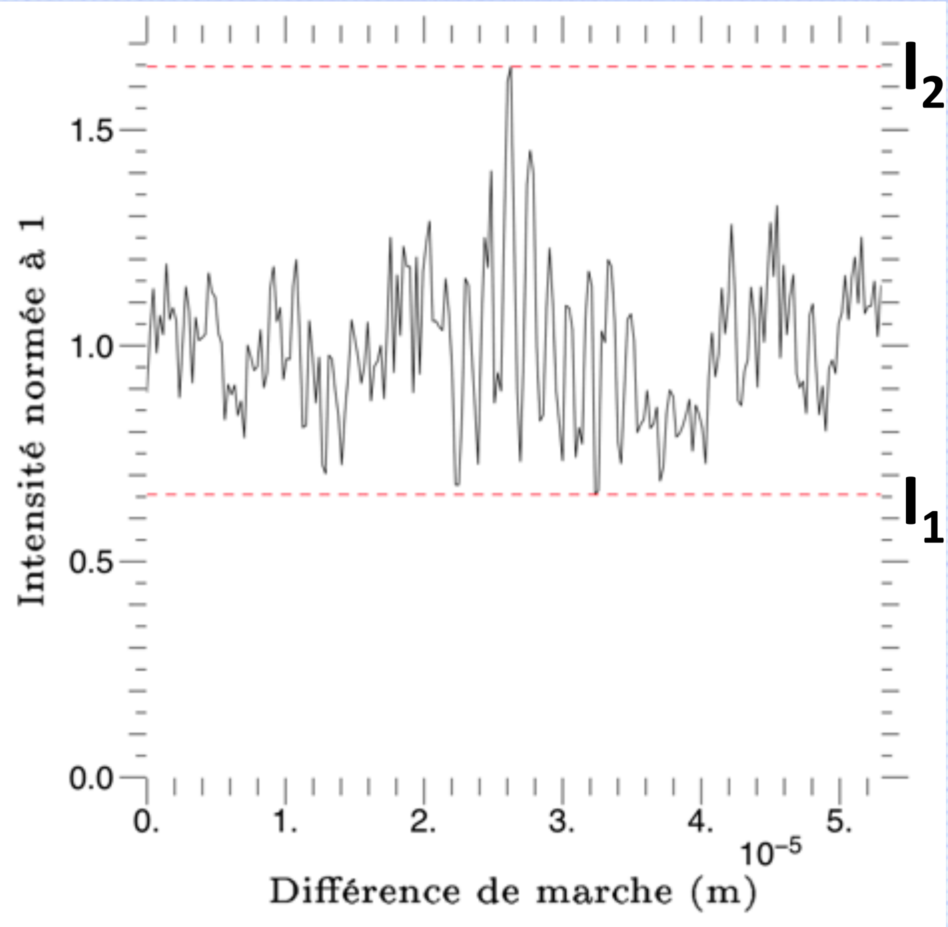
Real data (processed)



$$\mu \approx \frac{I_2 - I_1}{I_2 + I_1}$$

What are the issues?

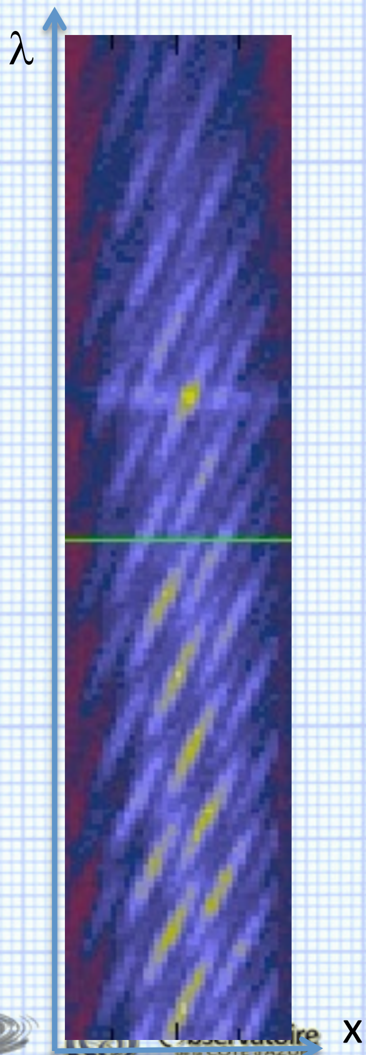
Real data (raw)



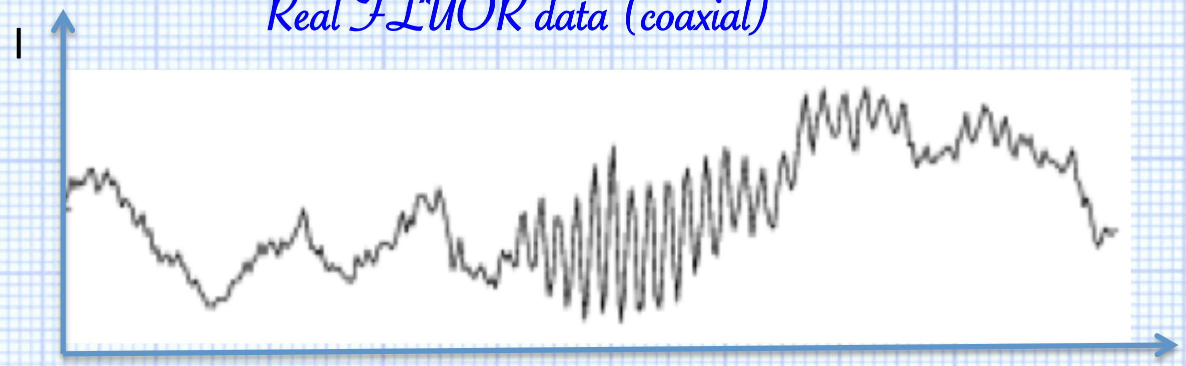
$$\mu \neq \frac{I_2 - I_1}{I_2 + I_1}$$

Real data look like this:

Real AMBER data (multiaxial)



Real FLUOR data (coaxial)



What means

- Multiaxial?
- Coaxial?

What are the issues?

Real fringes have a complicated expression:

$$I(\delta_0) = I_0\left[1 + \mu\cos\left(\phi - 2\pi\frac{\delta_0}{\lambda}\right)\right]$$

What are the issues?

Real fringes have a complicated expression:

$$I(\delta_0,t)=\frac{I_a(t)+I_b(t)}{2}+\sqrt{I_a(t)I_b(t)}\,e^{-\sigma^2_{turb}(t)}\cdot\mathrm{sinc}\left(2\pi\frac{\delta_0+\delta(t)}{R\lambda}\right)\cdot\mu\cos\left(\phi-2\pi\frac{\delta_0+\delta(t)}{\lambda}\right)+n_b(t)+\sigma(t)$$

1. Photometry unbalance
2. Jitter
3. Fringe motion
4. Spectral decoherence
5. Additive bias
6. Additive noise

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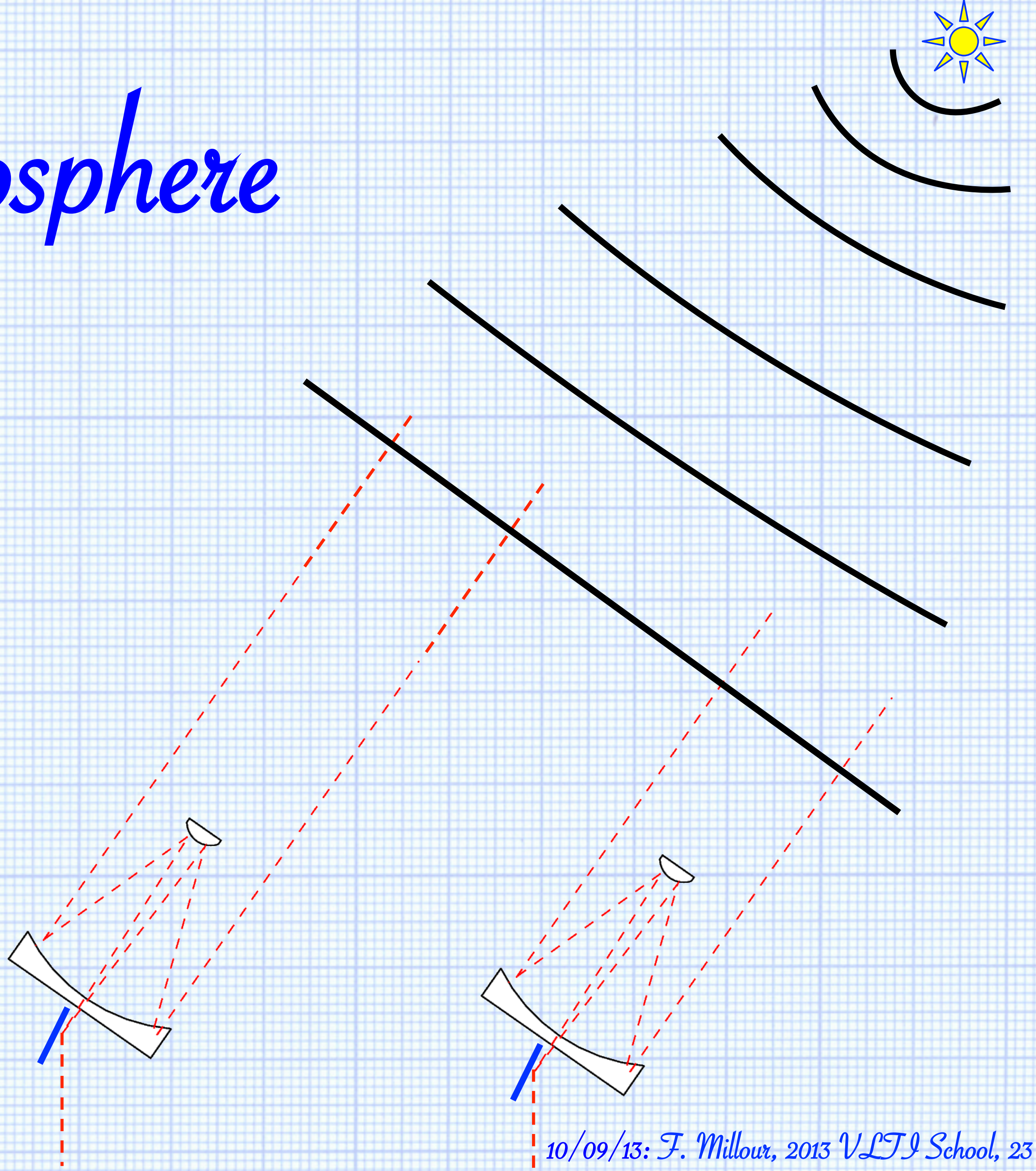
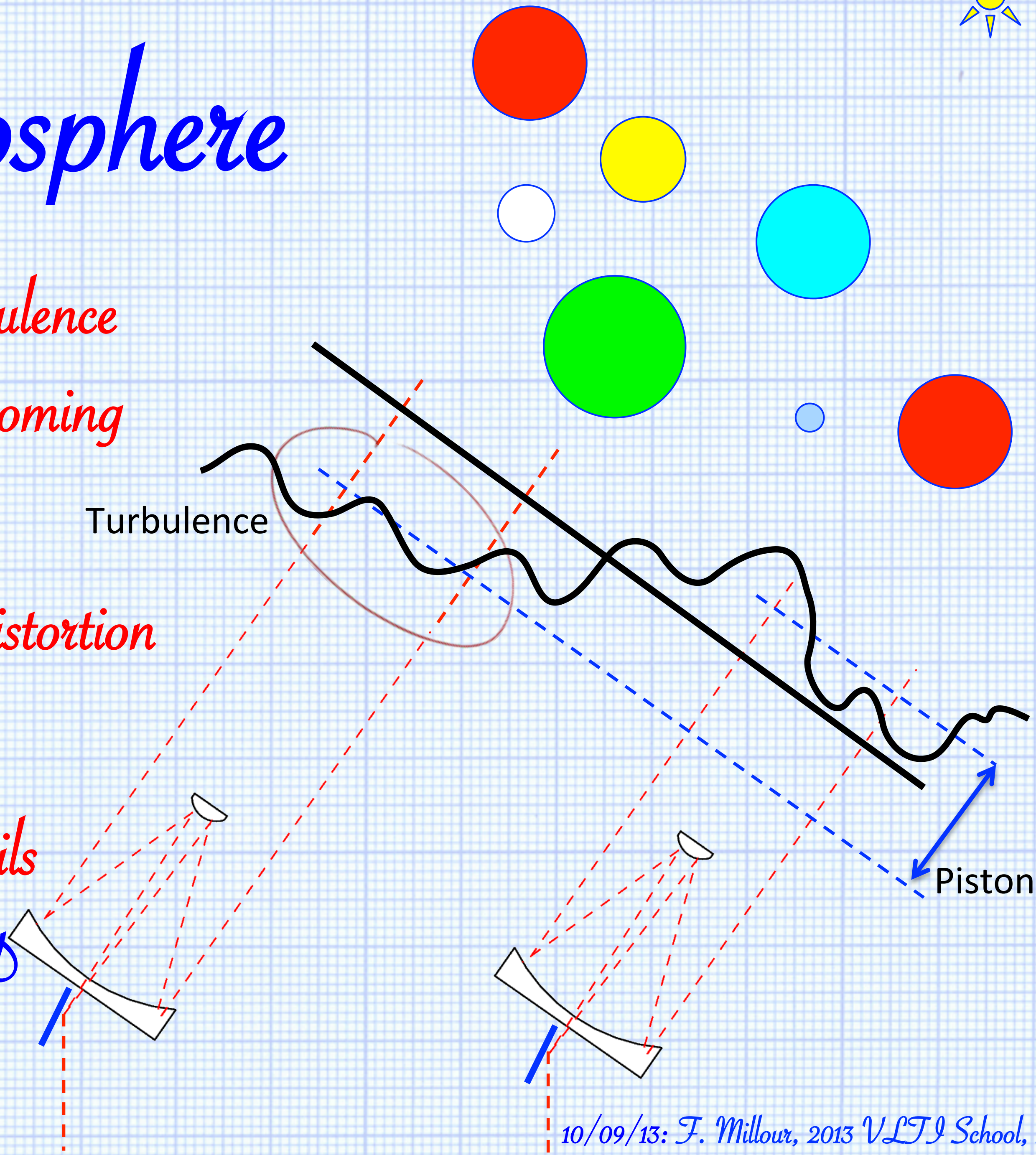
The atmosphere



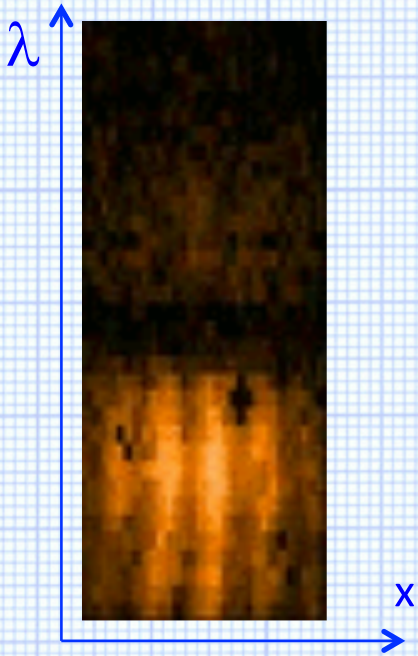
The atmosphere

- Atmospheric turbulence cells distort the incoming wavefront
- Pupil wavefront distortion
  - ➢Turbulence
- Shift between pupils
  - ➢Piston or OPD



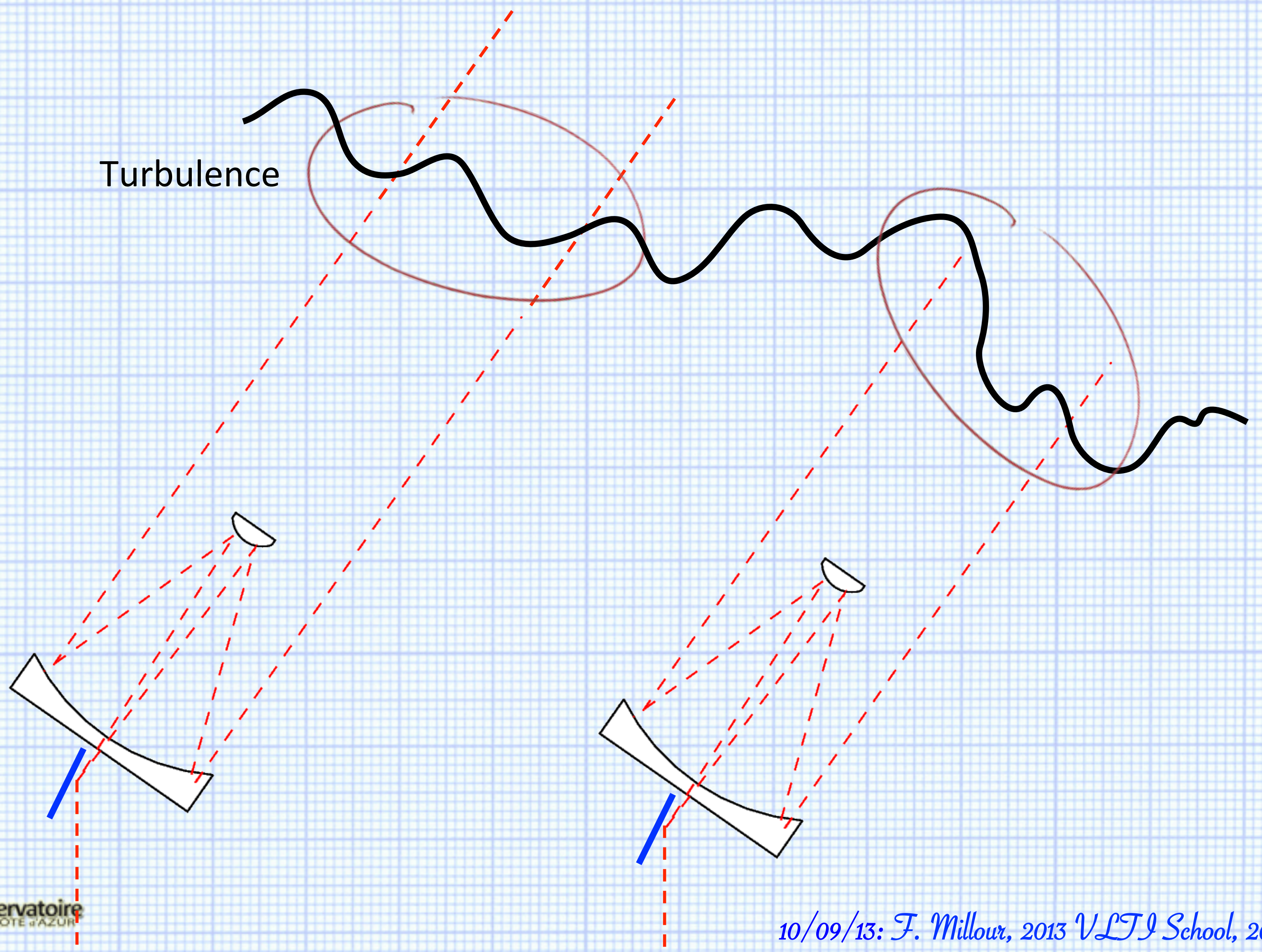
The piston creates 2 effects

- Fringe motion
  - Time-dependent phase shift of the fringes
    ➔ Fringe phase is lost!
- Fringe blurring
  - Contrast loss due to finite integration time
    ➔ Fringe amplitude is lost!

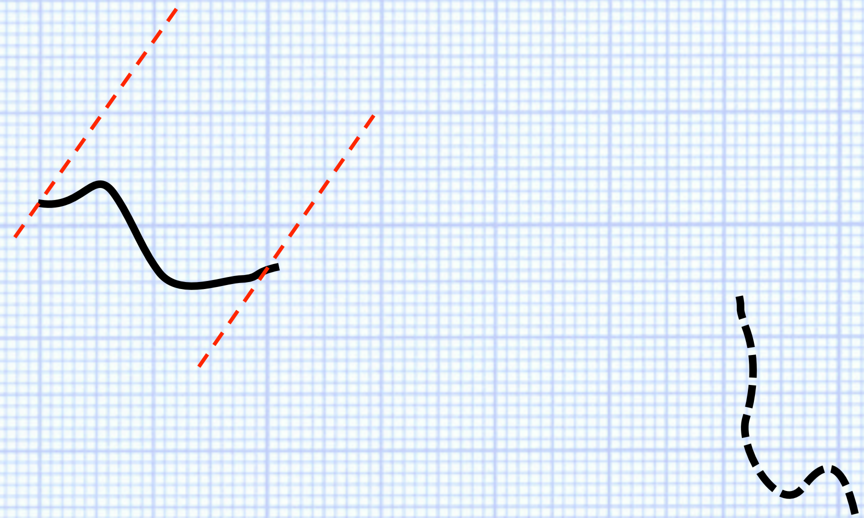


$$I(\delta_0, t) = e^{-\sigma^2_{\text{jitter}}(t)} \mu \cos\left(\phi - 2\pi \frac{\delta_0 + \delta(t)}{\lambda}\right)$$

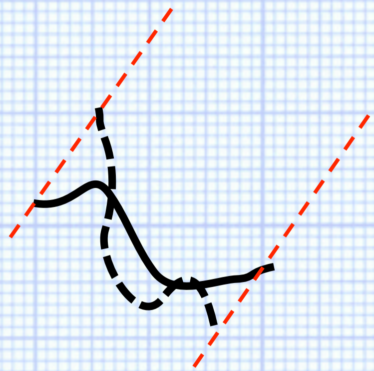
The turbulence



The turbulence



The turbulence



$$\rho_{turb} = e^{-\sigma^2_{turb}}$$

- Visibility reduced by wavefront variance over pupil
  - If turbulence small ➔ small effect (IR interferometry)
  - Reduce telescopes size (SUSI, NPOI)
  - Use adaptive optics (better solution)
  - Use another technique to flatten the wavefront

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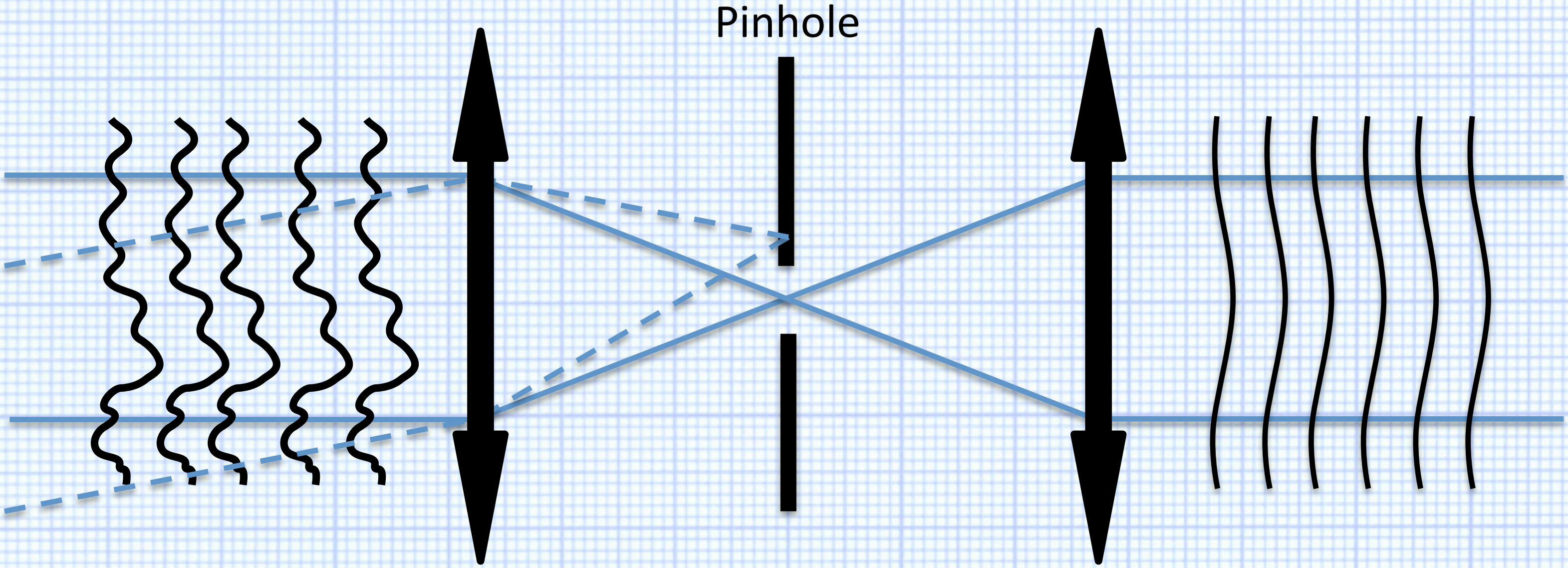
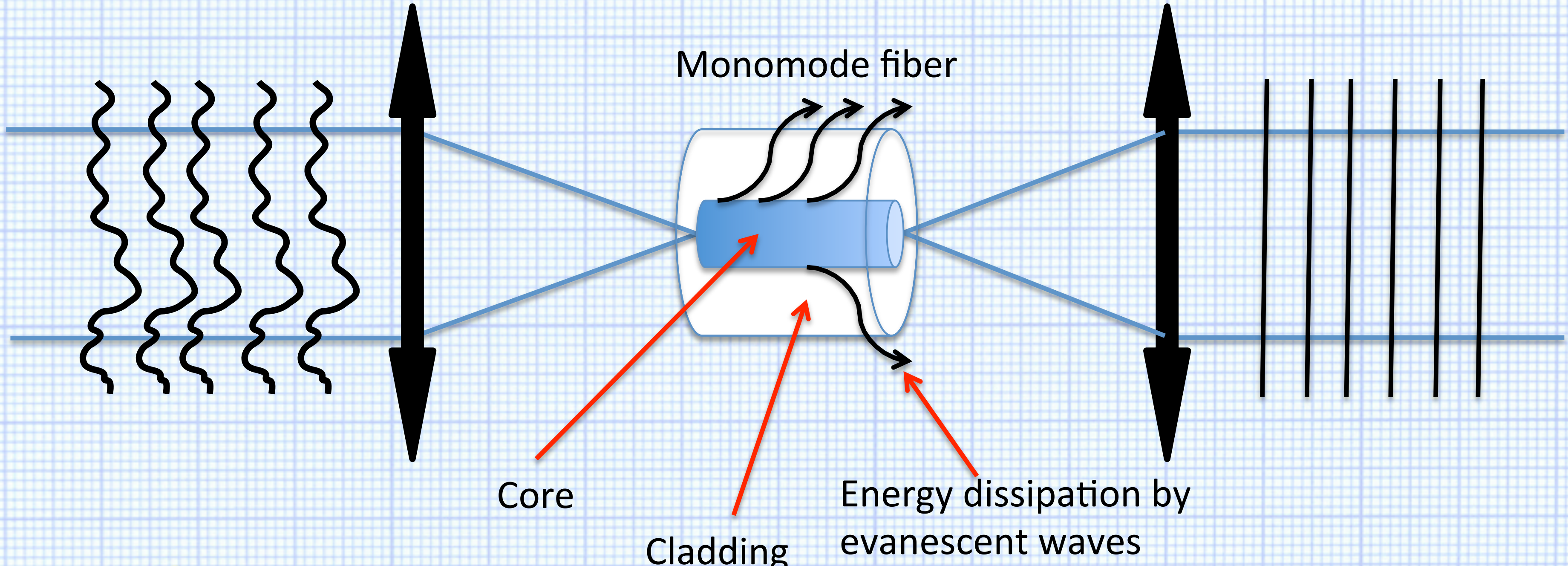
Modal filtering

- *A pinhole placed in an afocal system filters out wavefront corrugations*



Modal filtering

- A monomode optical fiber does the work even better
  - The corrugated part of the wavefront is rejected by the fiber
  - Corrugated wavefront ➔ flux variations



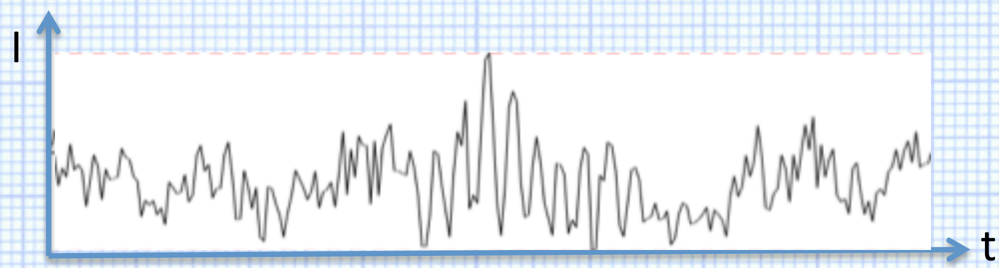
Modal filtering

$$I \sim \frac{I_a + I_b}{2} + \sqrt{I_a I_b} \cdot e^{-\sigma^2_{turb}} \cdot \mu \cos\left(\phi - 2\pi \frac{\delta_0}{\lambda}\right)$$

- *is transformed into*

$$I \sim \frac{I_a(t) + I_b(t)}{2} + \sqrt{I_a(t) I_b(t)} \cdot 1 \cdot \mu \cos\left(\phi - 2\pi \frac{\delta_0}{\lambda}\right)$$

That's why we have signals looking like that:



Remember:
OPD = vt
in coaxial

What are the issues? Photometry unbalance

$$I(\delta_0) = I_0\left[1 + \mu\cos\left(\phi - 2\pi\frac{\delta_0}{\lambda}\right)\right]$$

- In case of unbalanced beams, the interferogram becomes:

$$I(\delta_0) = \frac{I_a + I_b}{2} + \sqrt{I_a I_b}\,\mu\cos\left(\phi - 2\pi\frac{\delta_0}{\lambda}\right)$$

- Photometry is variable (scintillation, alignment, filtering):

$$I(\delta_0, t) = \frac{I_a(t) + I_b(t)}{2} + \sqrt{I_a(t) I_b(t)}\,\mu\cos\left(\phi - 2\pi\frac{\delta_0}{\lambda}\right)$$

- Instantaneous contrast becomes biased by:

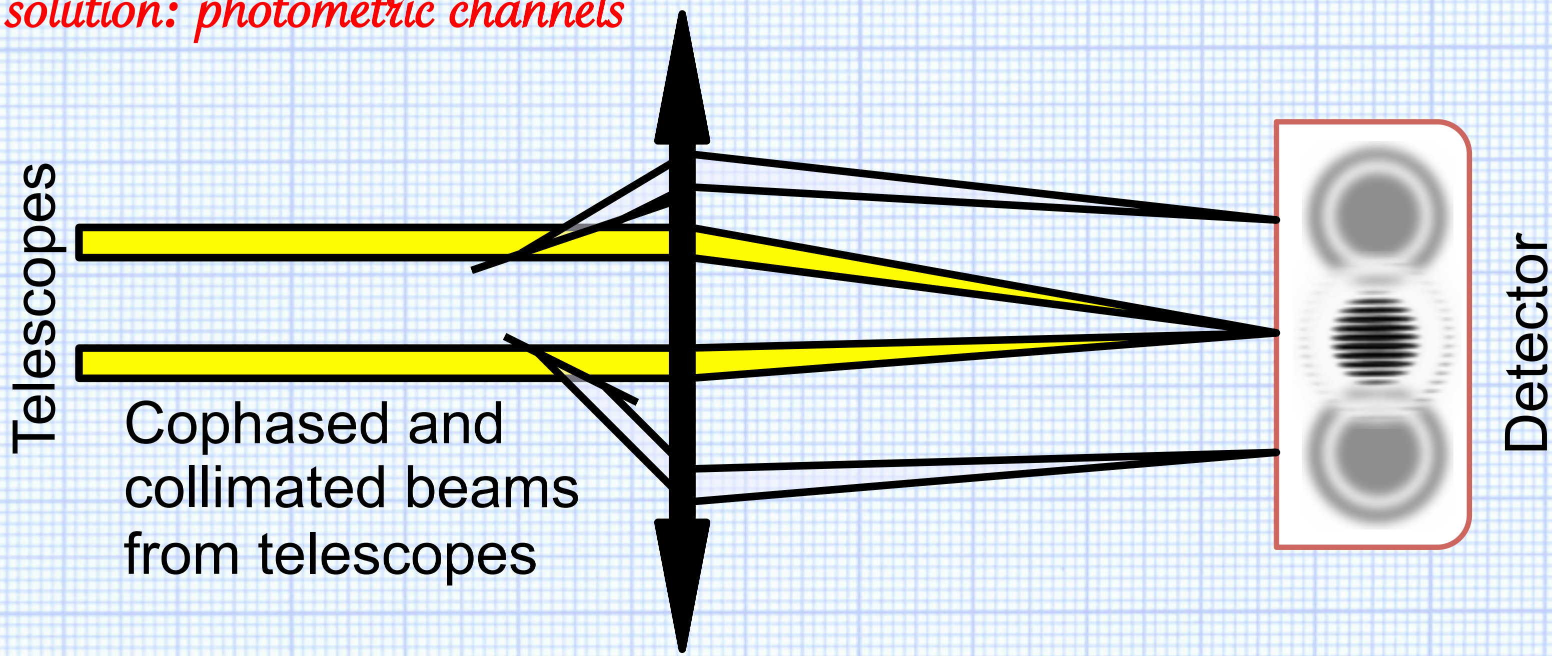
$$\frac{2\sqrt{I_a I_b}}{I_a + I_b}$$

= 0.94 if $I_a = 2\,I_b$

= 0.57 if $I_a = 10\,I_b$

What are the issues? Photometry unbalance

The solution: photometric channels

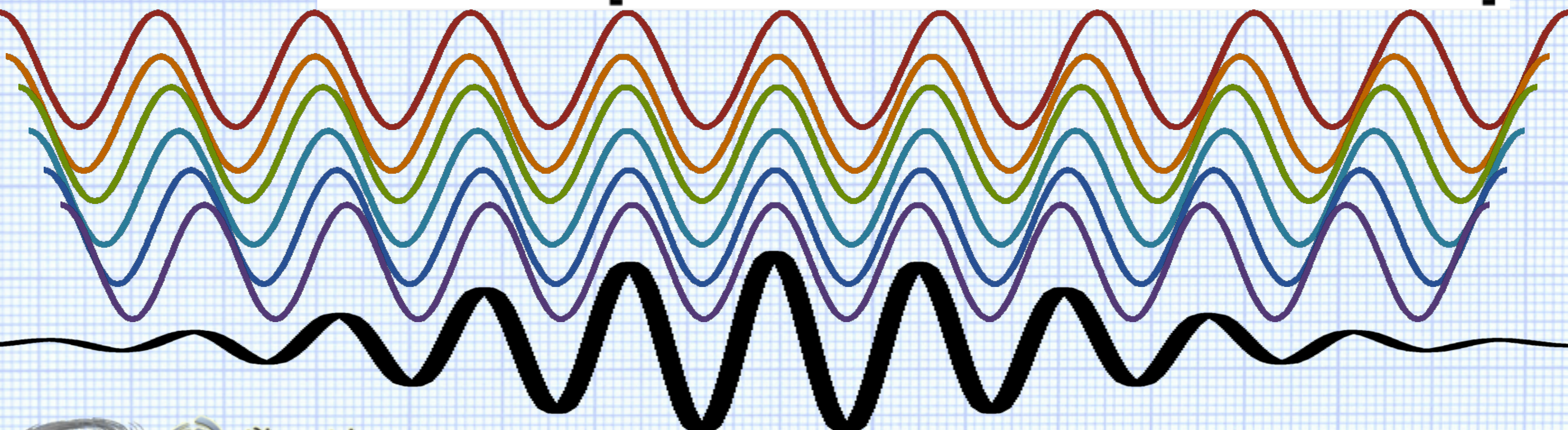


- Measure I_a and I_b using shutters before or after taking fringes
- Monitor photometries simultaneously

What are the issues? Spectral decoherence

- Fringes are not exactly cosine due to spectral bandwidth

$$I(\delta_0) = I_0 \left[1 + \int_{\lambda_1}^{\lambda_2} \mu \cos\left(\phi - 2\pi \frac{\delta_0}{\lambda} \right) d\lambda \right]$$

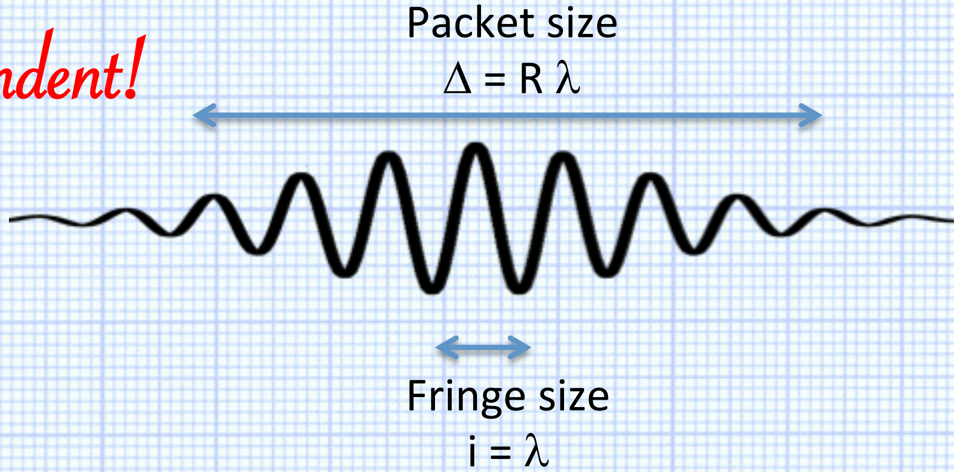


What are the issues? Spectral decoherence

- With a square filter:

$$I(\delta_0) = I_0\left[1 + \mathrm{sinc}\left(2\pi\frac{\delta_0}{R\lambda}\right)\cdot\mu\cos\left(\phi - 2\pi\frac{\delta_0}{\lambda}\right)\right]$$

- Fringe contrast is OPD-dependent!
- How to cope with that?
  - Be at OPD 0 !
  - Increase spectral resolution R



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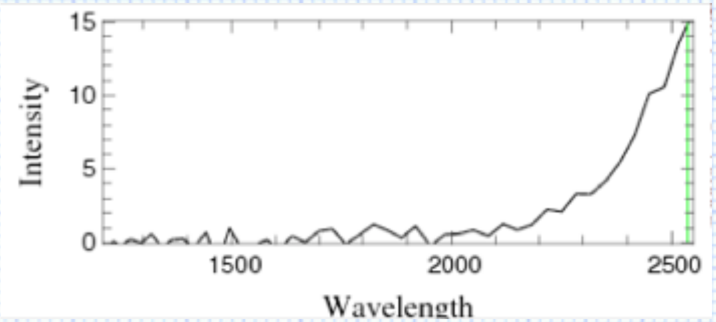
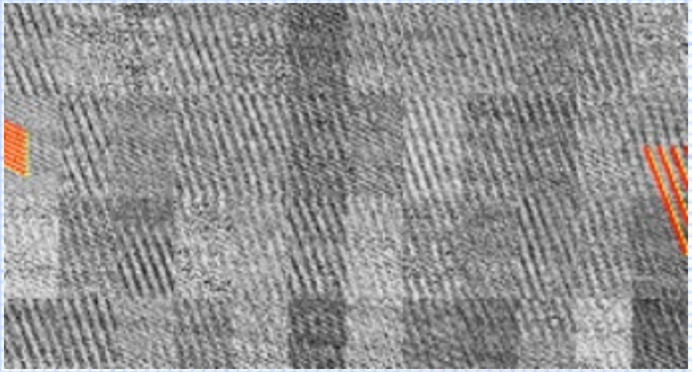
What are the issues? Biases

- A bias is some additive value with non-zero mean
- Examples:
  - Detector bias
  - Thermal background
  - EM detector perturbations
  - Photon noise bias
- How to cope with it?
  - Estimate it and subtract it!

AMBER: Sky brightness



AMBER: Dark exposure



Spurious fringes induced by electromagnetic disturbances (Li Causi et al. 2007)

What are the issues? Additive noises

- A noise is some additive value with a zero mean
- Examples:
  - Photon noise from the source
  - Photon noise from thermal background
  - Detector noise
- How to cope with it?
  - Statistics!
  - Error estimates!

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Summary

Real fringes have a complicated expression:

$$I(\delta_0,t)=\frac{I_a(t)+I_b(t)}{2}+\sqrt{I_a(t)I_b(t)}\cdot e^{-\sigma^2_{turb}(t)}\cdot \mathrm{sinc}\left(2\pi\frac{\delta_0+\delta(t)}{R\lambda}\right)\cdot\mu\cos\left(\phi-2\pi\frac{\delta_0+\delta(t)}{\lambda}\right)+n_b(t)+\sigma(t)$$

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A note about « visibility »

- « Visibility » is often referred as the fringe contrast & not the complex visibility of the object
- The measured visibility is not the visibility of the object:
  - Instrument's response is not 100% (polarization, vibrations)
  - Atmosphere affects fringe contrast (jitter, turbulence)
- From now on, « visibility » means uncalibrated fringe contrast (to make it simple…)

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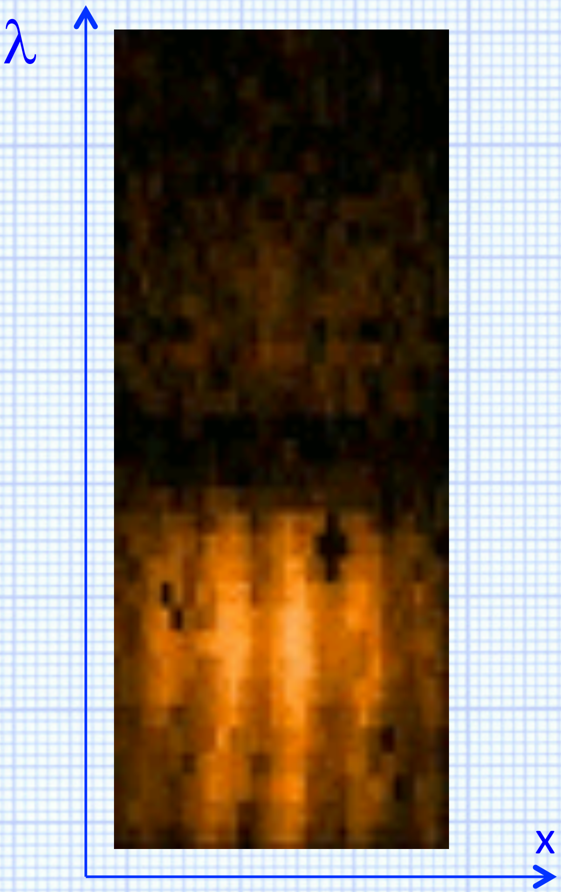
All the observables



Complex coherent flux:

$$C^{a,b} = \sqrt{I^a I^b} \cdot \mu_{\text{inst+atm}} \cdot \mu_{\text{object}}^{a,b}$$

Visibility:

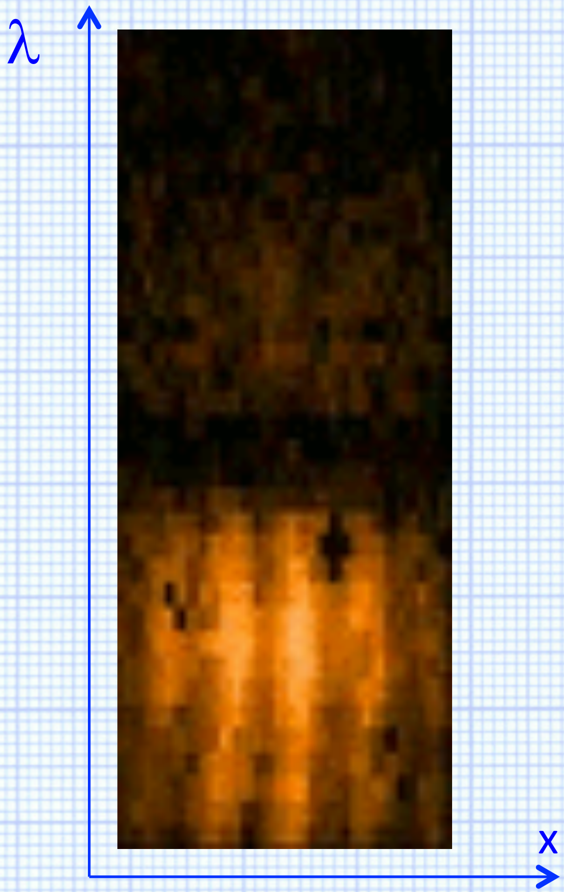
$$\mu_{\text{object}}^{a,b} = \frac{C^{a,b}}{\sqrt{I^a I^b} \cdot \mu_{\text{inst+atm}}}$$

~~**Phase:**~~

$$\phi_{\text{object}}^{a,b} = \arg\left(C^{a,b}\right)$$

All the observables

In real life:



Complex coherent flux:

$$C^{a,b} = \sqrt{I^a I^b} \cdot \mu_{\text{inst+atm}} \cdot \mu^{a,b}_{\text{object}}$$

Spectrum

Visibility squared

Differential phase

Closure phase

Phase reference

Differential visibility

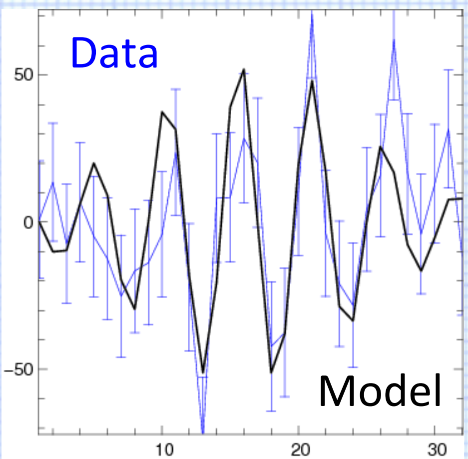
Coherent (or linear) visibility

“differential closure phase”

Closure amplitude

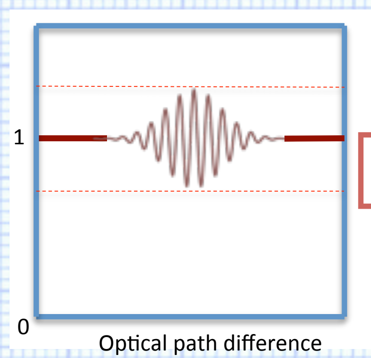
How do we get coherent flux?

- **Image-plane method(s)**
  - Image space fringe-fitting
    - ABCD, P2VM
      - We get directly R & I of the coherent flux

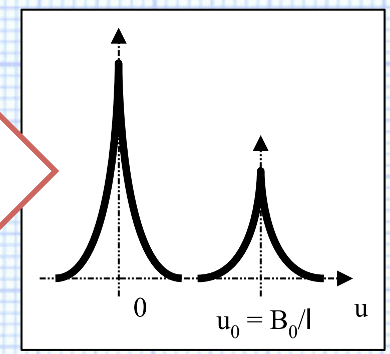


P2VM → R & I

- **Fourier-plane method(s)**
  - Fringes look like a cosine
    - signature is a single peak in the Fourier plane
      - Amplitude of the peak = coherent flux
      - Phase of the peak = phase



FT →



ABCD vs Fourier

- Image-plane method(s)
  - Strong a priori (model of the fringes)
  - Extra data needed to build fringe model
  - Optimized: the fringe packet is modelized using the instrument itself

- Fourier-plane method(s)
  - No a priori except « fringes look like a cosine »
  - Extra data needed to integrate fringe peak
  - Not optimized: a fringe packet is not really a sine wave

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Visibility estimator

- Coherent flux:

$$C^{a,b} = \sqrt{I^a I^b} \cdot \mu_{\mathrm{inst+atm}} \cdot \mu^{a,b}_{\mathrm{object}}$$

- Visibility:

$$\mu^{a,b}_{\mathrm{object}} = \frac{\left|C^{a,b}\right|}{\sqrt{I^a I^b} \cdot \mu_{\mathrm{inst+atm}}}$$

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Visibility estimator

$$\mu_{\text{object}}^{a,b} = \frac{\left|C^{a,b}\right|}{\sqrt{I^a I^b} \cdot \mu_{\text{inst+atm}}}$$

- Modulus
- Division
- Product
- Square root

Amplitude of a complex number

$$V = \mu e^{i\phi}, \langle n \rangle = 0$$

$$\begin{aligned} V' &= V + n \\ |V'| &= |V + n| \\ \langle |V'| \rangle &= \langle |V + n| \rangle ?? \end{aligned}$$

$$\begin{aligned} \langle |V'|^2 \rangle &= \langle |V + n|^2 \rangle \\ &= \langle |V|^2 \rangle + \langle 2\Re[V\, n] \rangle + \langle |n|^2 \rangle \\ &= |V|^2 + 2\Re[V \langle n \rangle] + \langle |n|^2 \rangle \\ &= |V|^2 + \langle |n|^2 \rangle \end{aligned}$$

- Transforms a zero-mean noise into a bias
  - Correction = estimating the bias. Here, bias= variance of the noise

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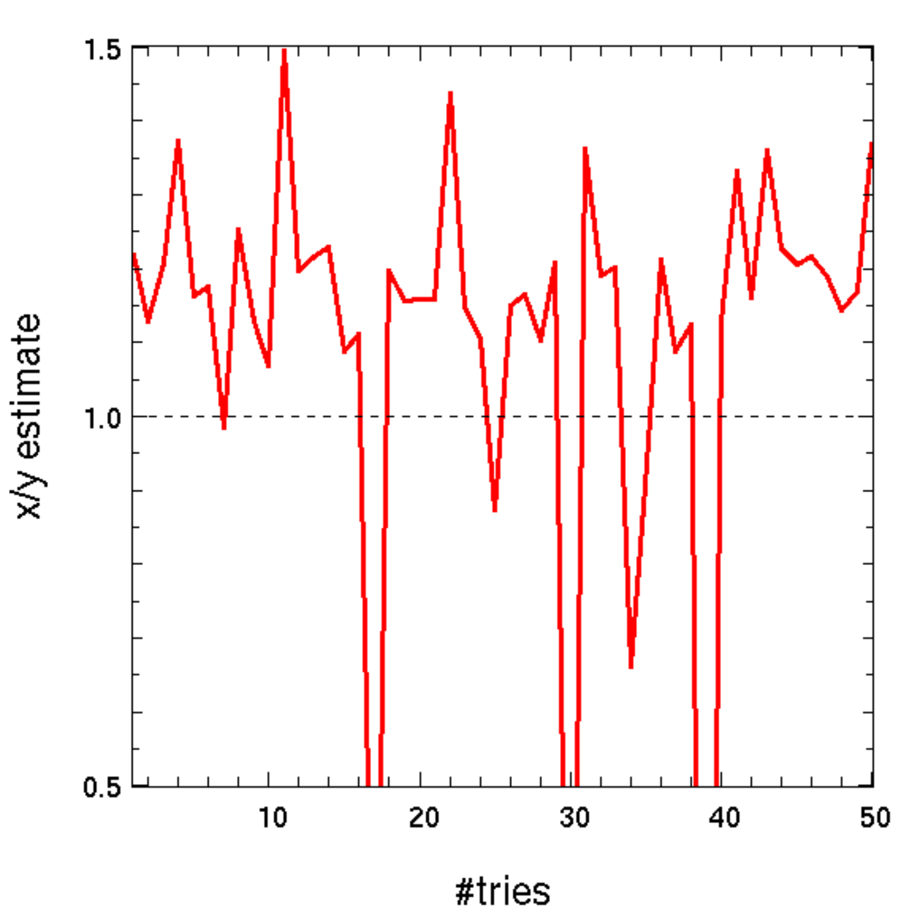
Division of 2 numbers

Let $x = \alpha + n_1$ *and* $y = \beta + n_2$, $<x>=<y>=3$, $\sigma n_1 = \sigma n_2 = 1$

- *How to average* $z = x/y$?
- *Let's try with* $z_1 = <x/y>$ *(1000 samples)*



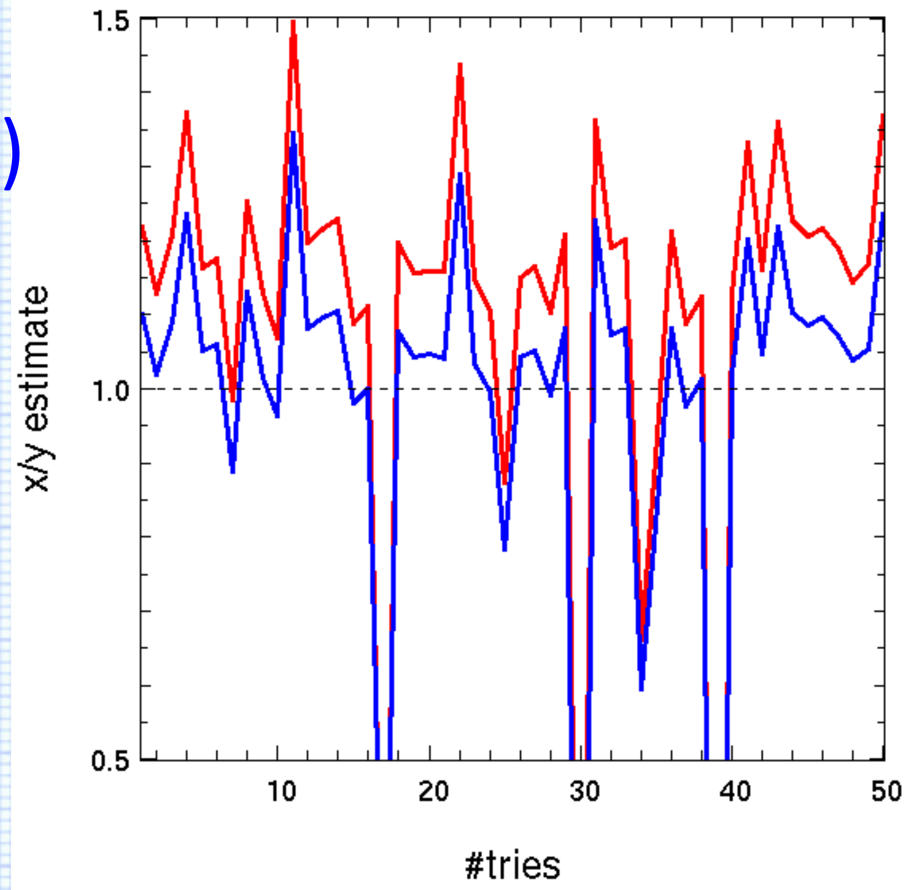
Such estimate is highly biased!

Bias depends on the noise!

Division of 2 numbers

- Solution 1
  - De-bias the estimator

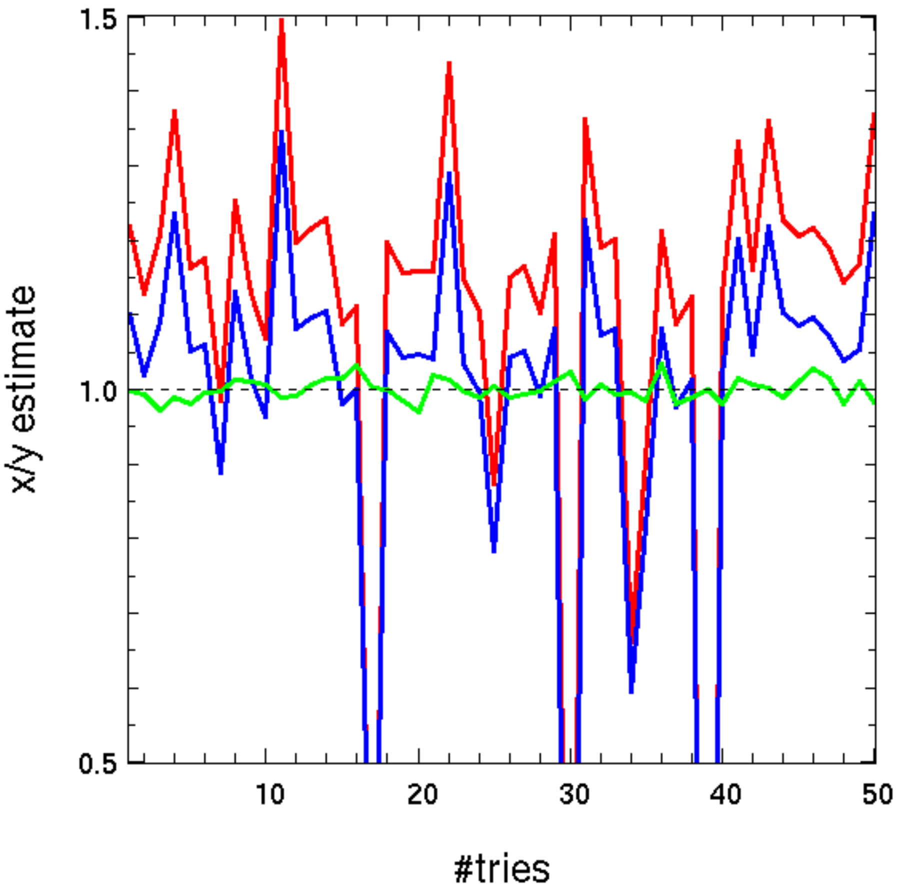
$z_2 = \langle x/y \rangle / (1 + \sigma y / \langle y \rangle^2)$



Division of 2 numbers

- Solution 2
  - Use an unbiased estimator

$z_3 = \langle x \rangle / \langle y \rangle$



You used to fear dividing by zero?

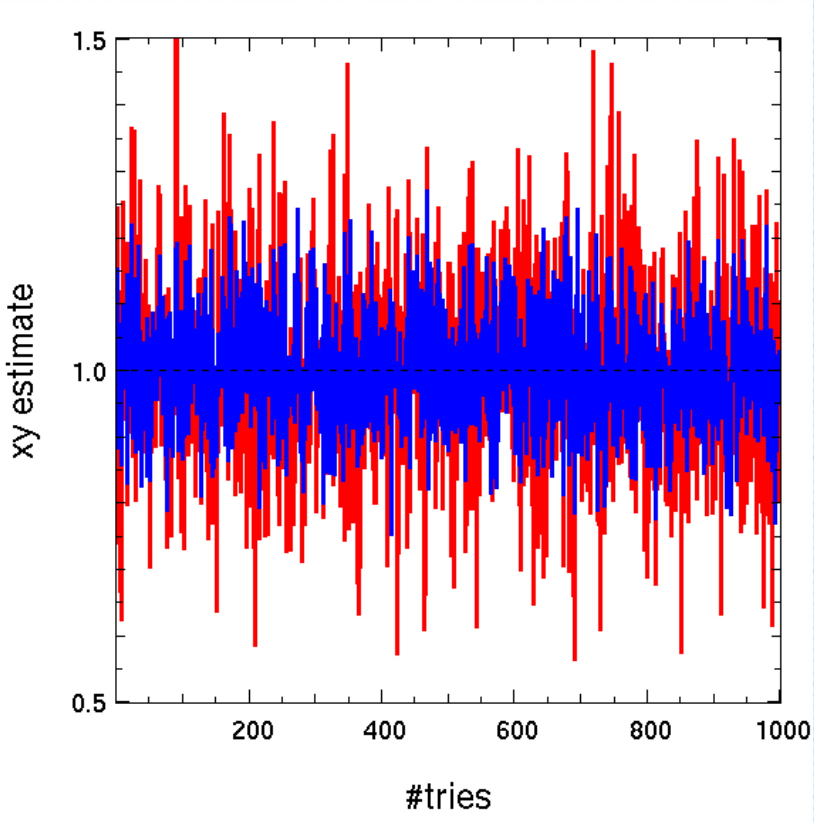
Now fear dividing by a noisy variable!

Multiplication of 2 numbers

- *Be careful when multiplying 2 random variables!*

$x = \alpha + n_1$ and $y = \beta + n_2$

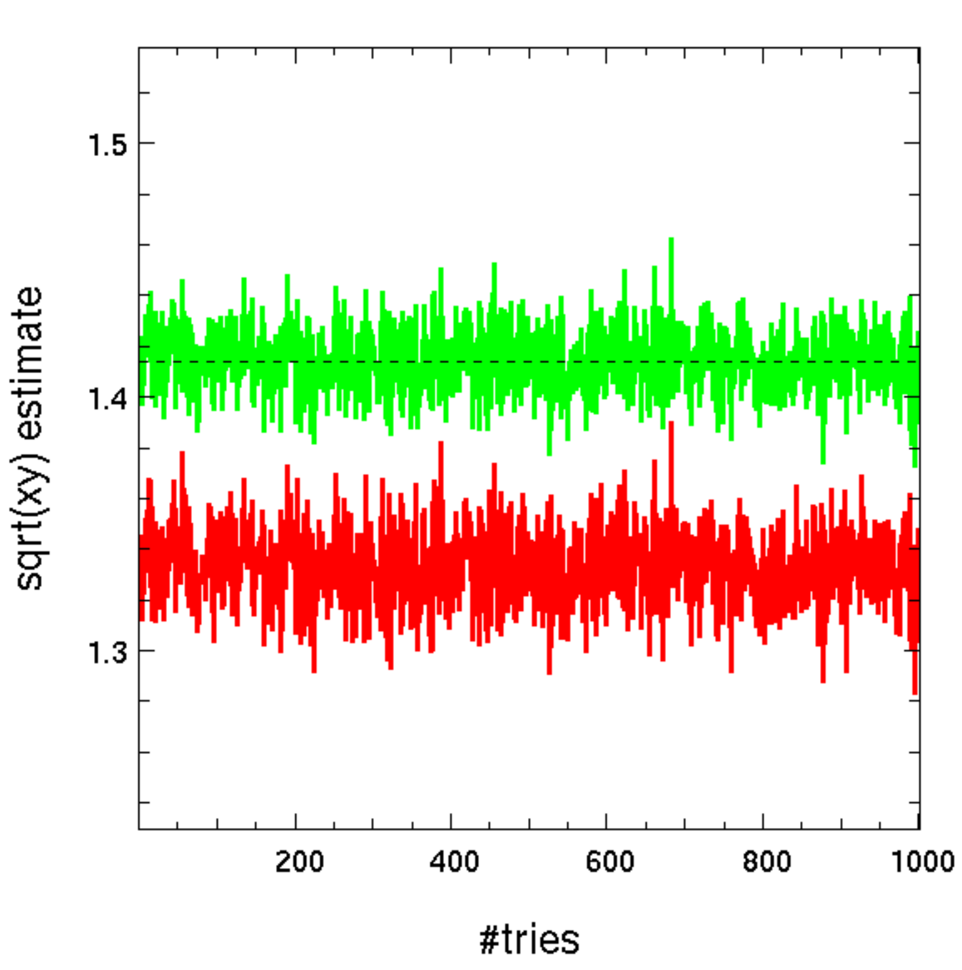
$z_1 = \langle xy \rangle$



$z_2 = \langle x \rangle \langle y \rangle$

Square root of 2 numbers

$x = \alpha + n_1$ and $y = \beta + n_2$



$z_1 = \langle \mathrm{sqrt}(x) \rangle * \langle \mathrm{sqrt}(y) \rangle$

$z_2 = \mathrm{sqrt}(\langle x \rangle \langle y \rangle)$

Visibility estimator recipe

- Go for squared visibility! Avoid pitfalls!
  - Extract $|C^{a,b}|$ (coherent flux) for each frame
  - Estimate I^a and I^b for each frame
  - Estimate noise variance $<|n|^2>$
  - Calculate $\mu^2 = <|V|^2>$ by
    $(<|C^{a,b}|^2> - <|n|^2>) / <I^a> <I^b>$

raw squared visibility

- And then?
  - Calibrate!

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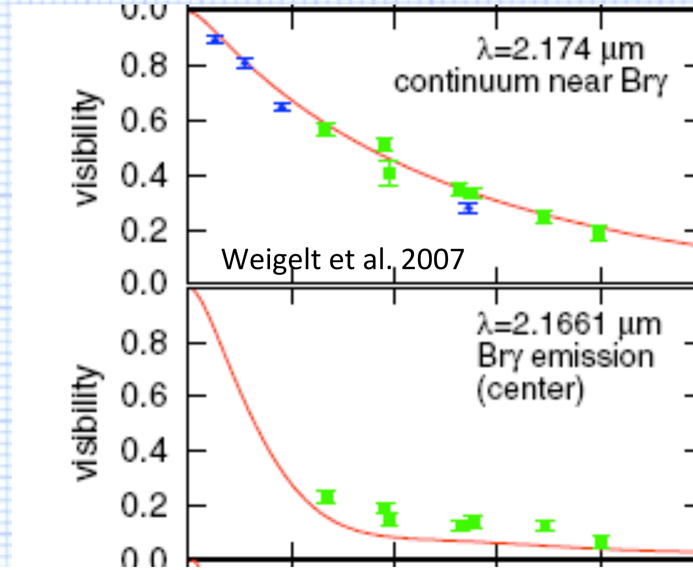
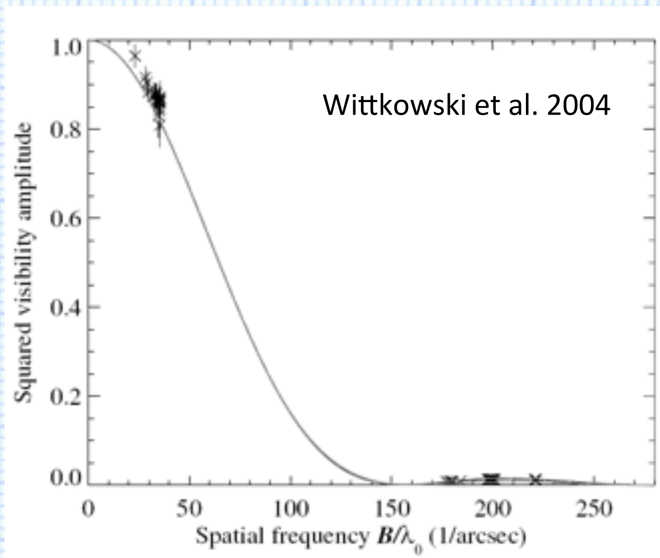
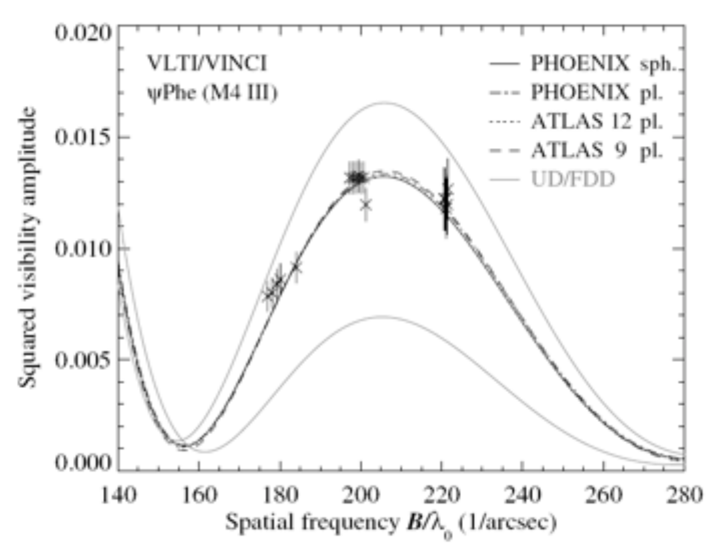
A few examples: circular objects

- *Visibility measures typical size of the object*
  - *The bigger the object, the lower the visibility*
  - *A bounce in visibility is a sign of a sharp edge in the image*
  - *A modulation of visibility is a sign of binarity*
- *η Car observed with* **AMBER**
- *ψ Phe observed with* **VINCI**

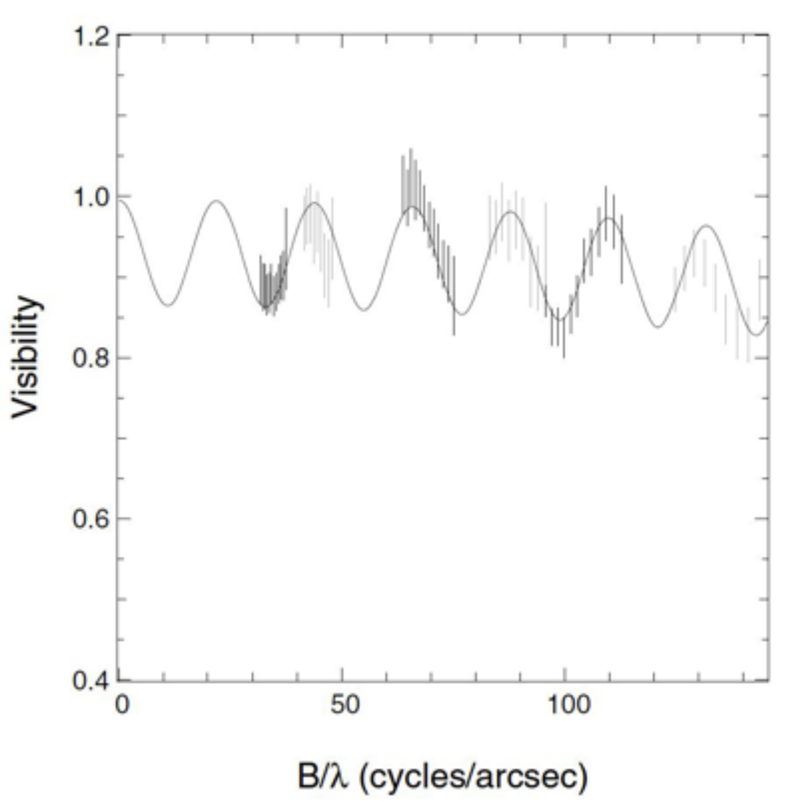






A few examples: binaries

δ Cen



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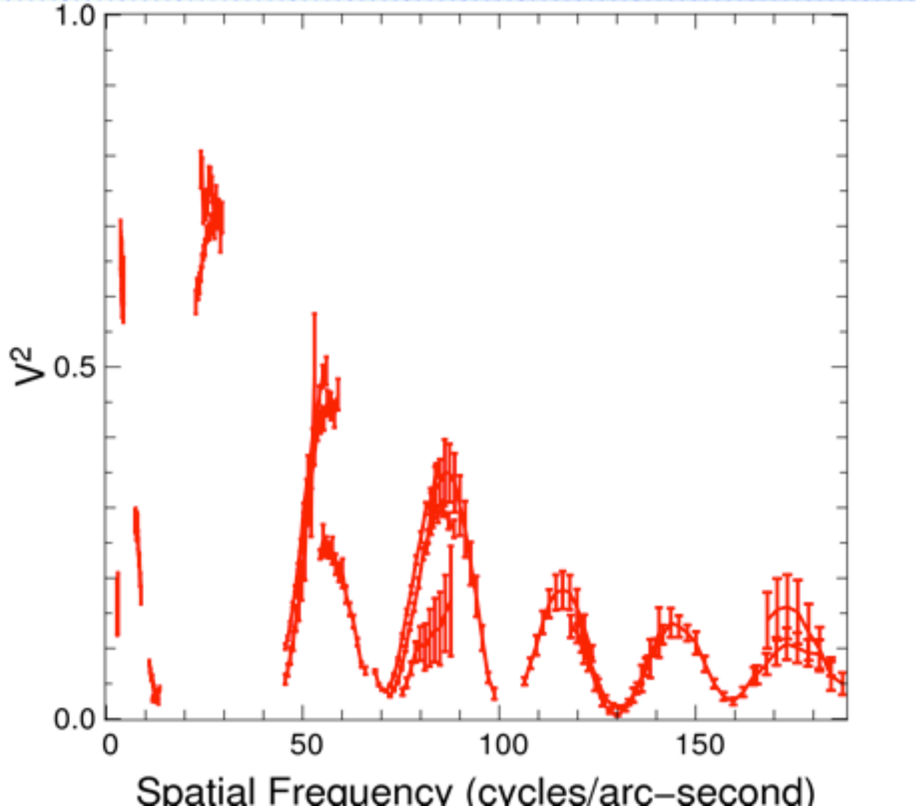
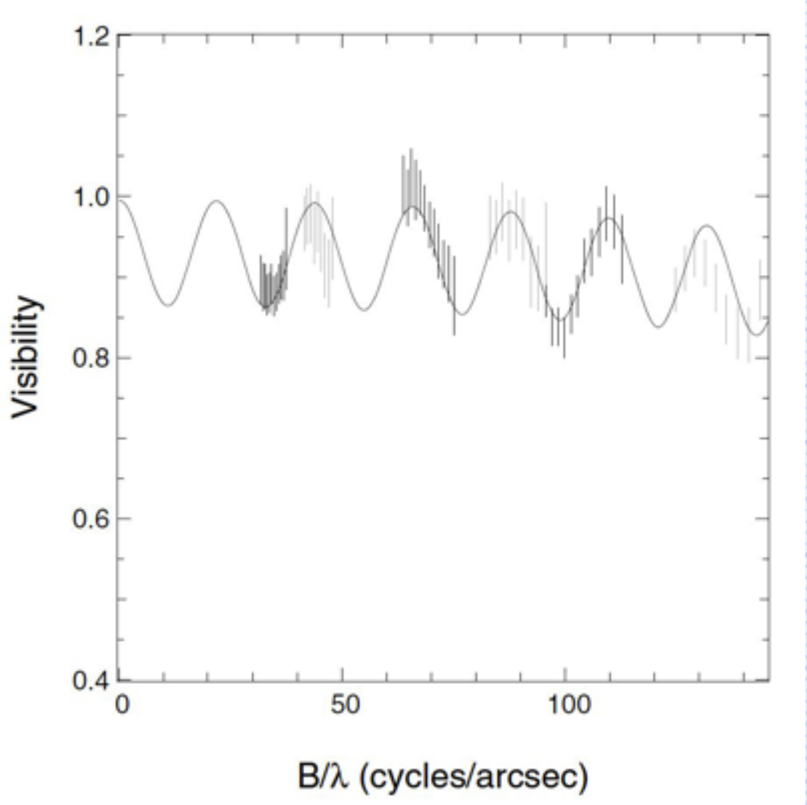


A few examples: binaries

δ Cen



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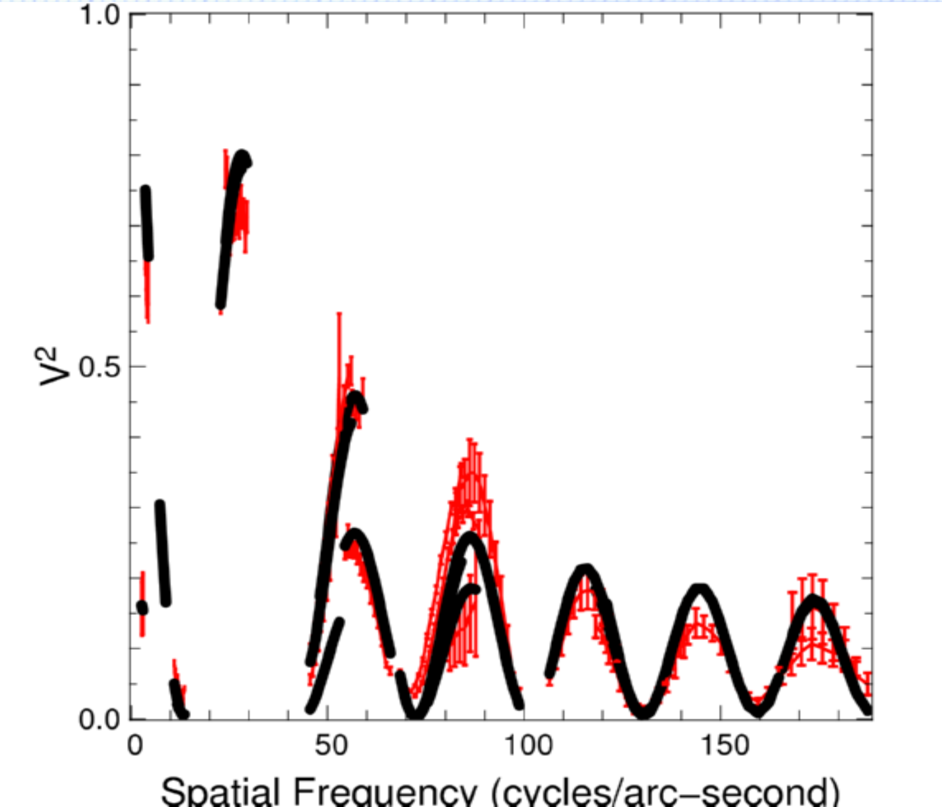
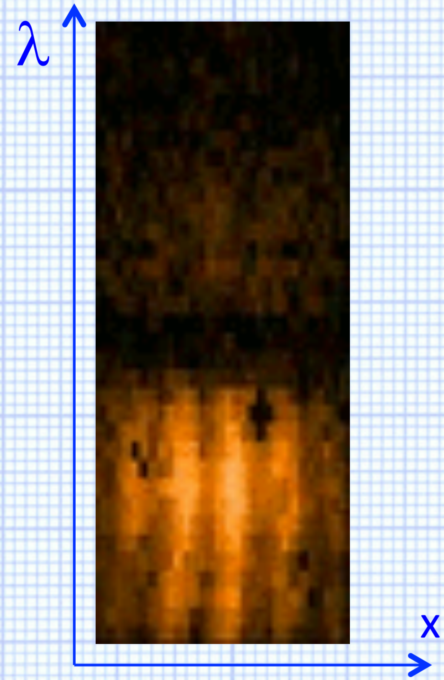


A few examples: what can go wrong?

What about phase?

Remember, due to the atmosphere:

- Fringe motion
  - Time-dependent phase shift of the fringes
    ➔ Fringe phase is lost!

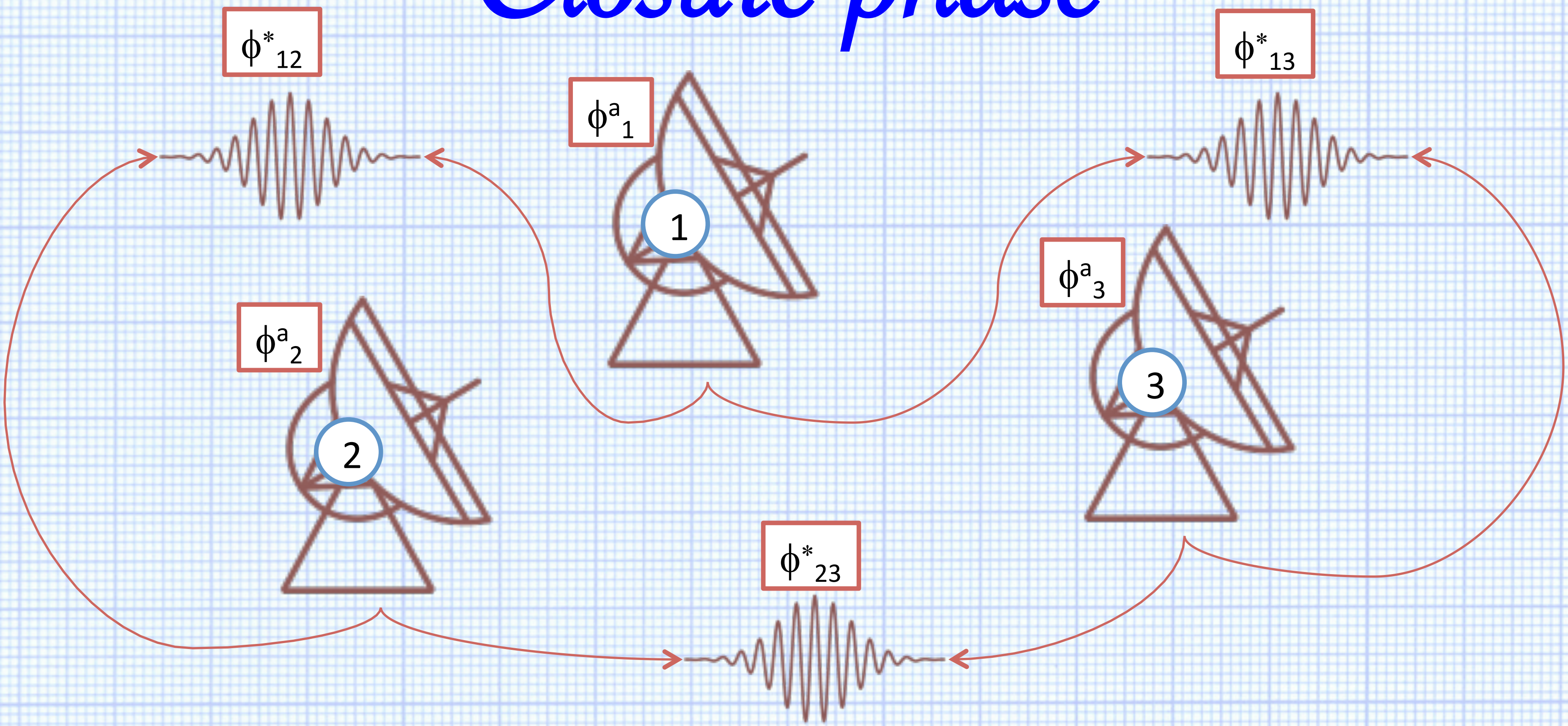


$$I(\delta_0, t) = e^{-\sigma^2_{\mathrm{jitter}}(t)} \mu \cos\left(\phi - 2\pi \frac{\delta_0 + \delta(t)}{\lambda}\right)$$

What about phase?

- Phases are lost in long-baseline interferometry
- How to work that around?
  - Get a phase which do not need a reference
    - Closure phase
  - Find a way to reference the phase (set the « zero phase »)
    - « Phase reference »: use a reference star close-by
    - « Differential phase »: use a wavelength close-by

Closure phase



$$\Phi_{123} = \phi^*_{12} + \cancel{\phi^a_2} - \cancel{\phi^a_1} + \phi^*_{23} + \cancel{\phi^a_3} - \cancel{\phi^a_2} + \phi^*_{31} + \cancel{\phi^a_1} - \cancel{\phi^a_3}$$

$$\Phi_{123} = \phi^*_{12} + \phi^*_{23} + \phi^*_{31}$$

Closure phase

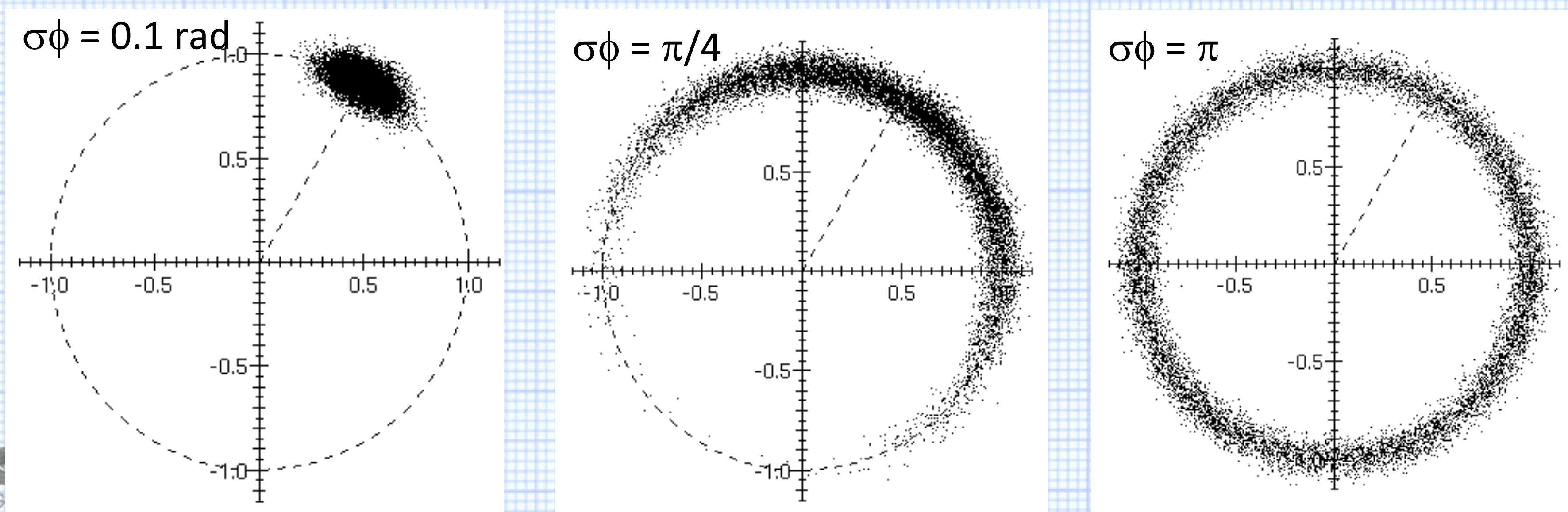
Closure phase cannot be obtained with phases sums!

why?

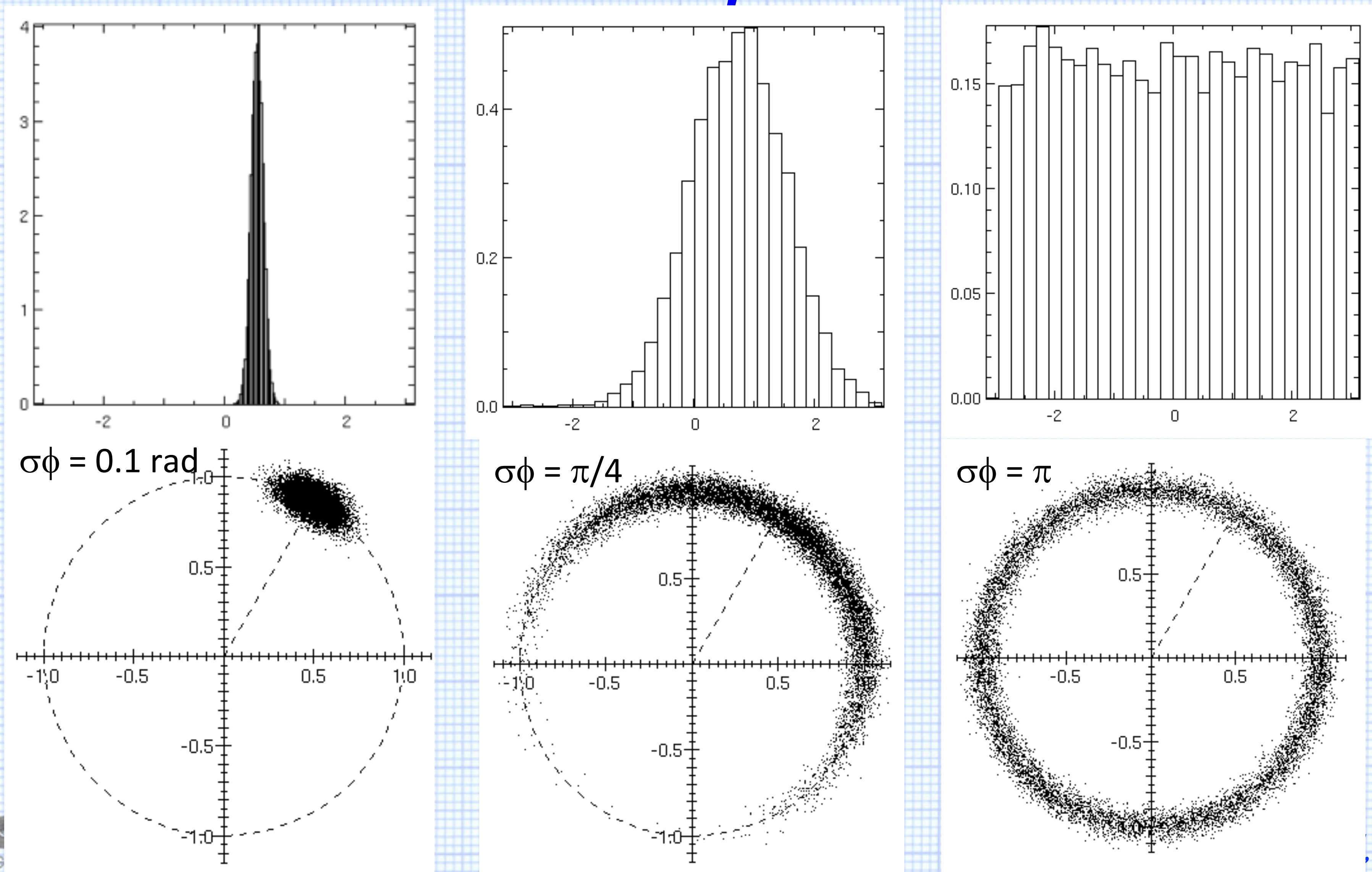
Noise!

Additive noises produce a phase wrapping

wrapped noisy phases have a top-hat distribution, when noise variance is high



Closure phase



Closure phase

- Closure phase cannot be obtained with phases sums!
- Stay in complex plane to avoid phase wrapping:
  - Bispectrum $<C_{12}C_{23}C_{31}>$
    - Phase of the bispectrum = closure phase
    - Amplitude of the bispectrum = $V_{12}V_{23}V_{31}$

Closure phase example

- Closure phase measures asymmetries
  - A non-zero closure phase means asymmetries in the object
  - A zero closure phase means… nothing!
- Closure phase is not straightforward to interpret!