Introduction to optical/IR interferometry

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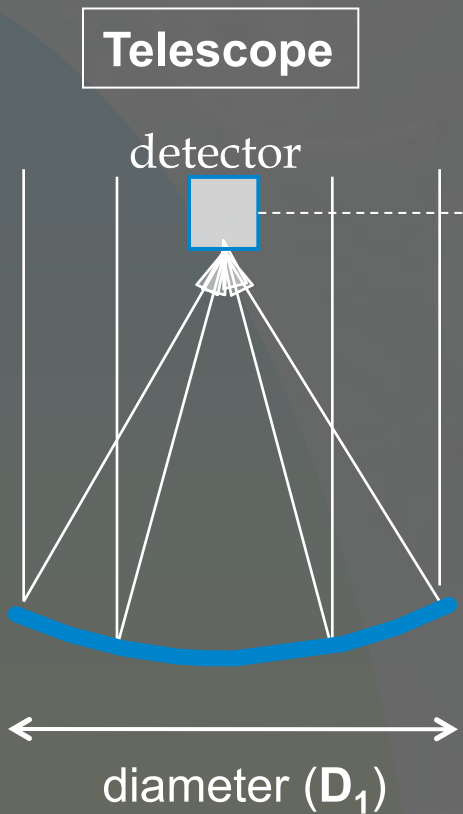
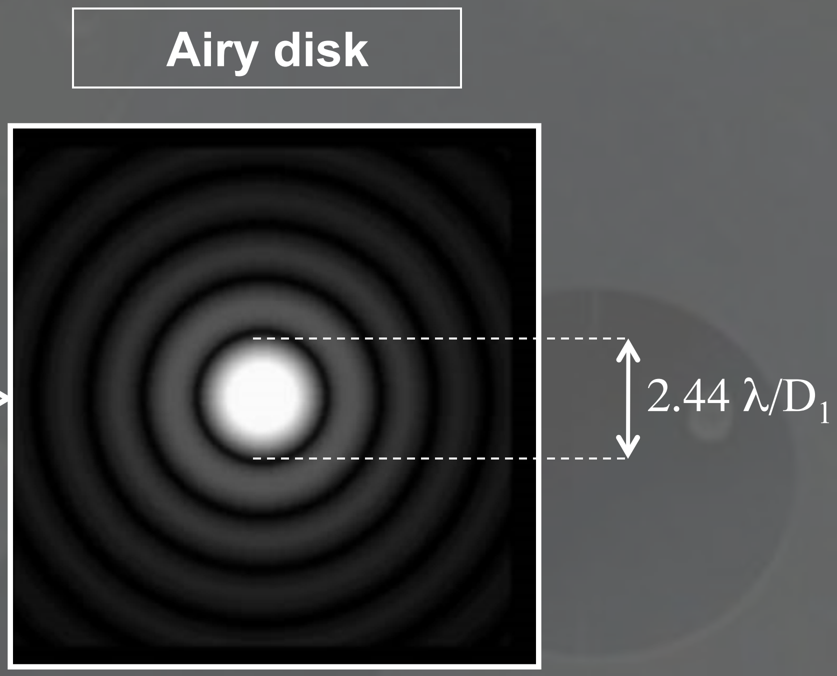
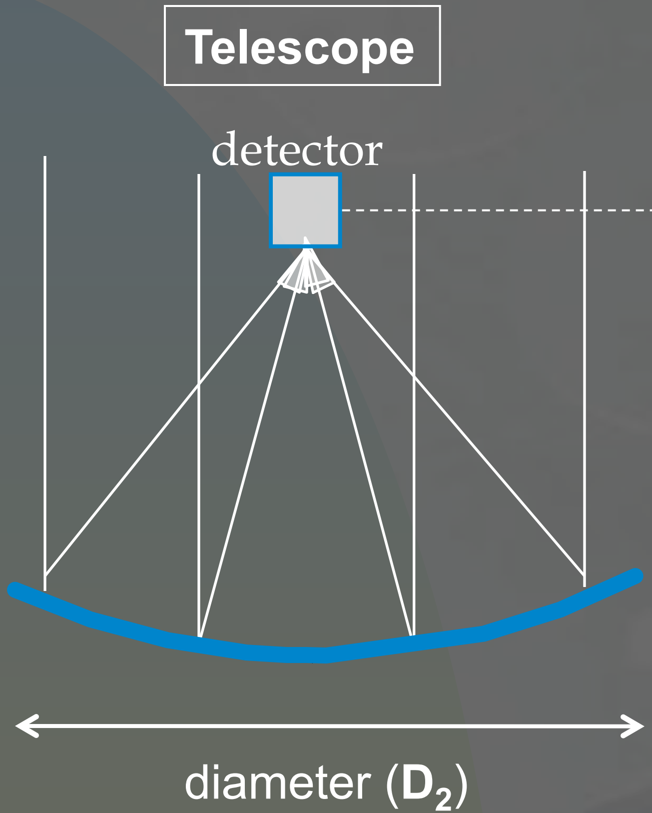
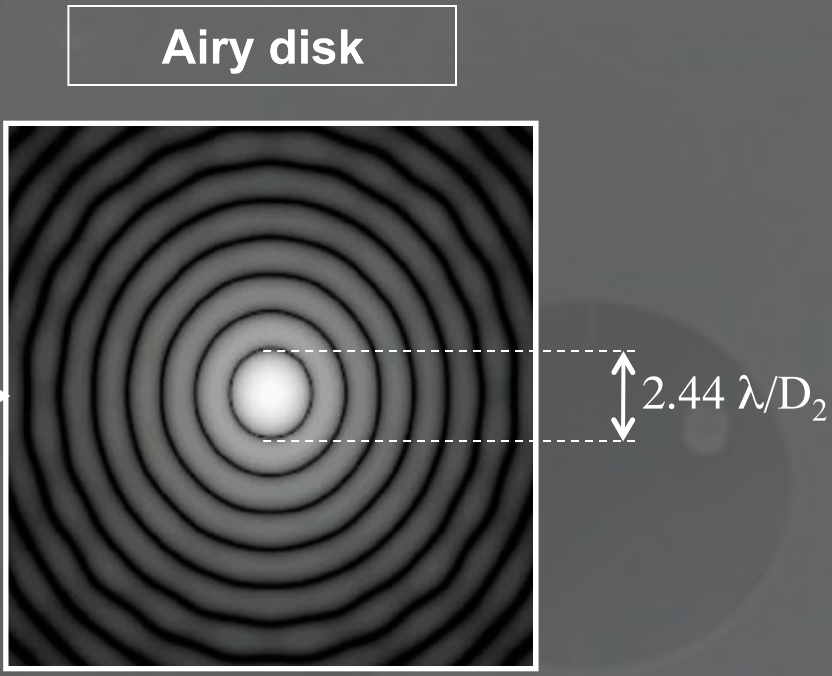


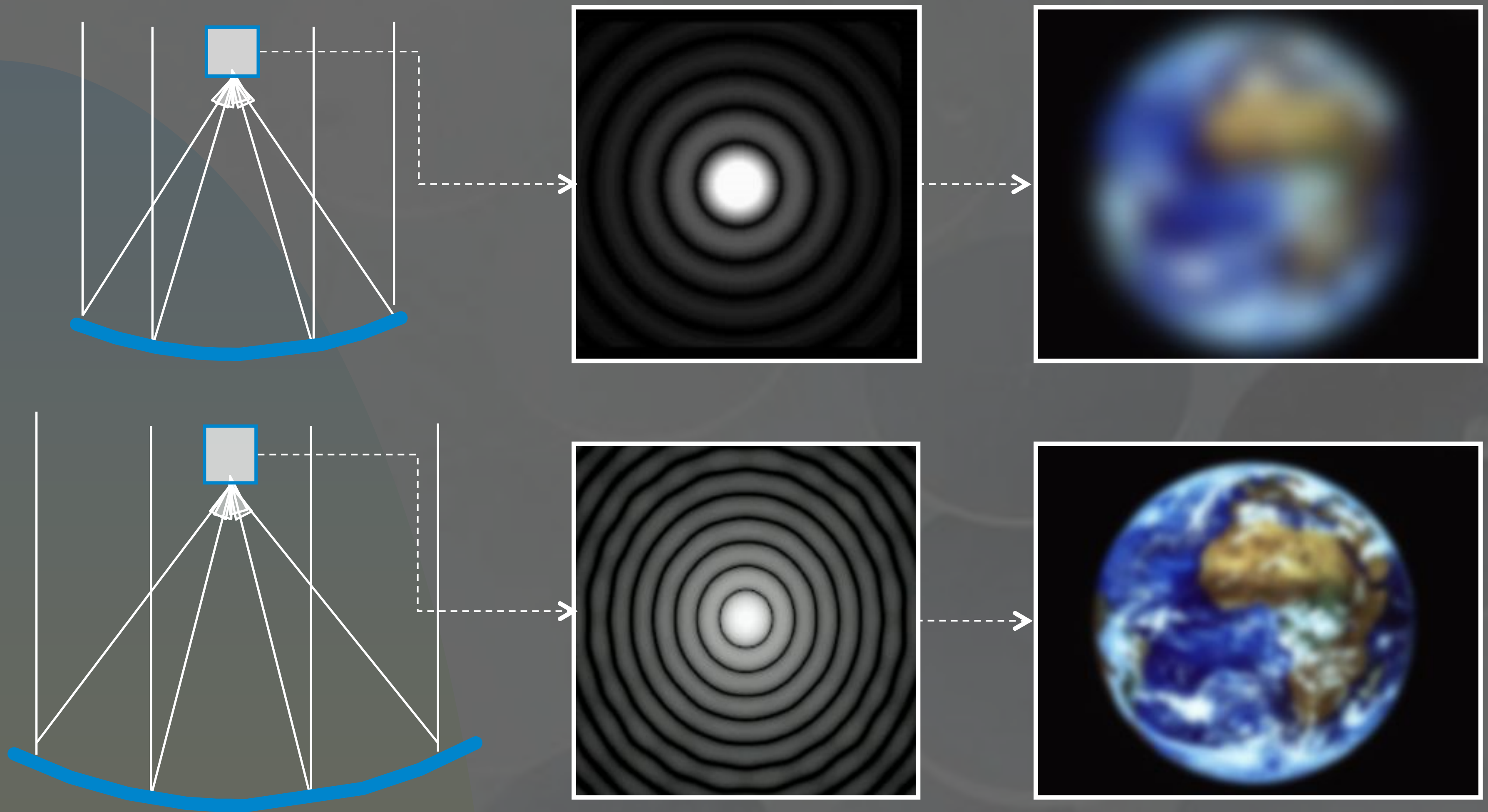


- The image of a star is like a dot!





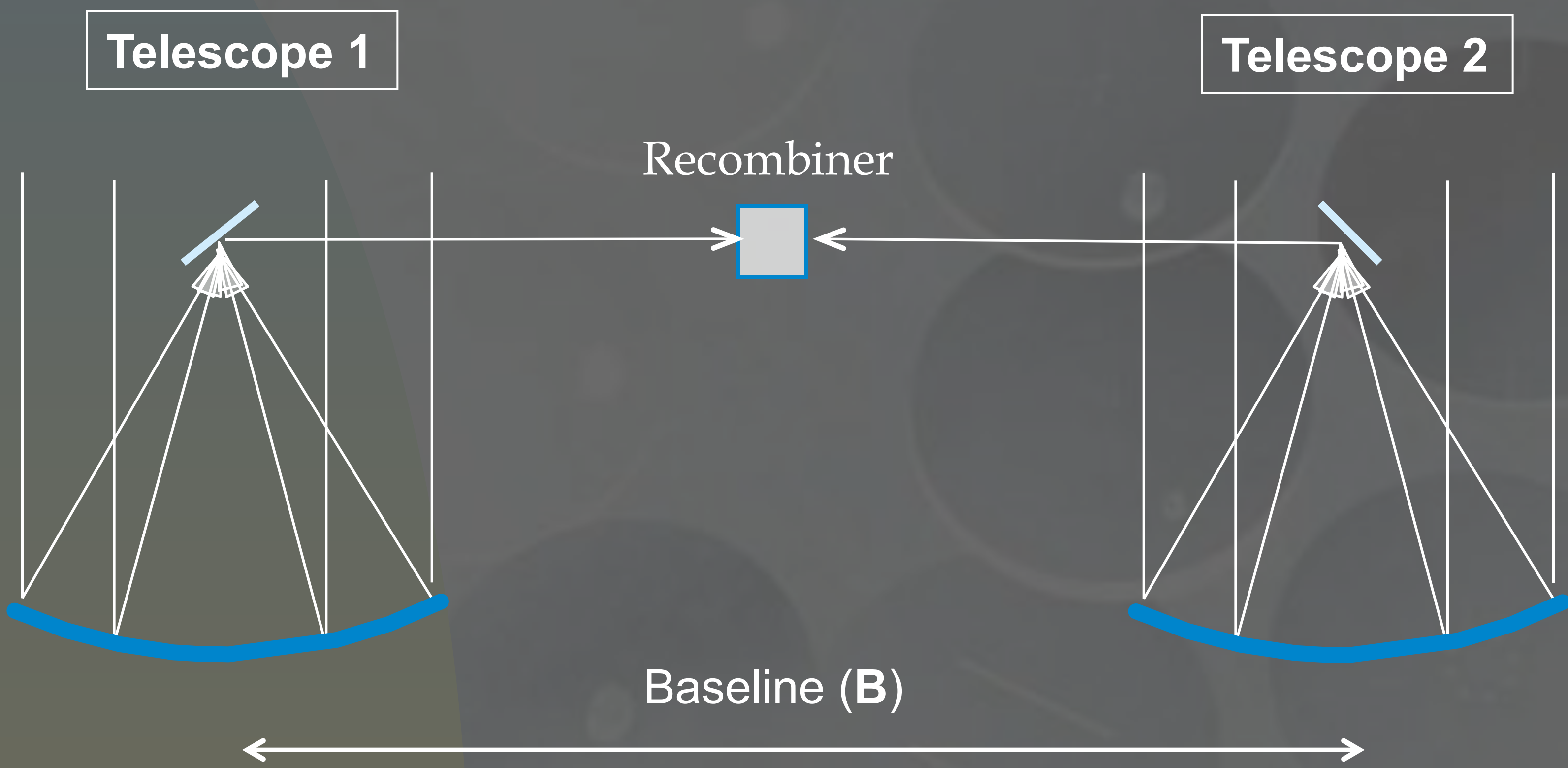
- The image of a star is still like a dot!

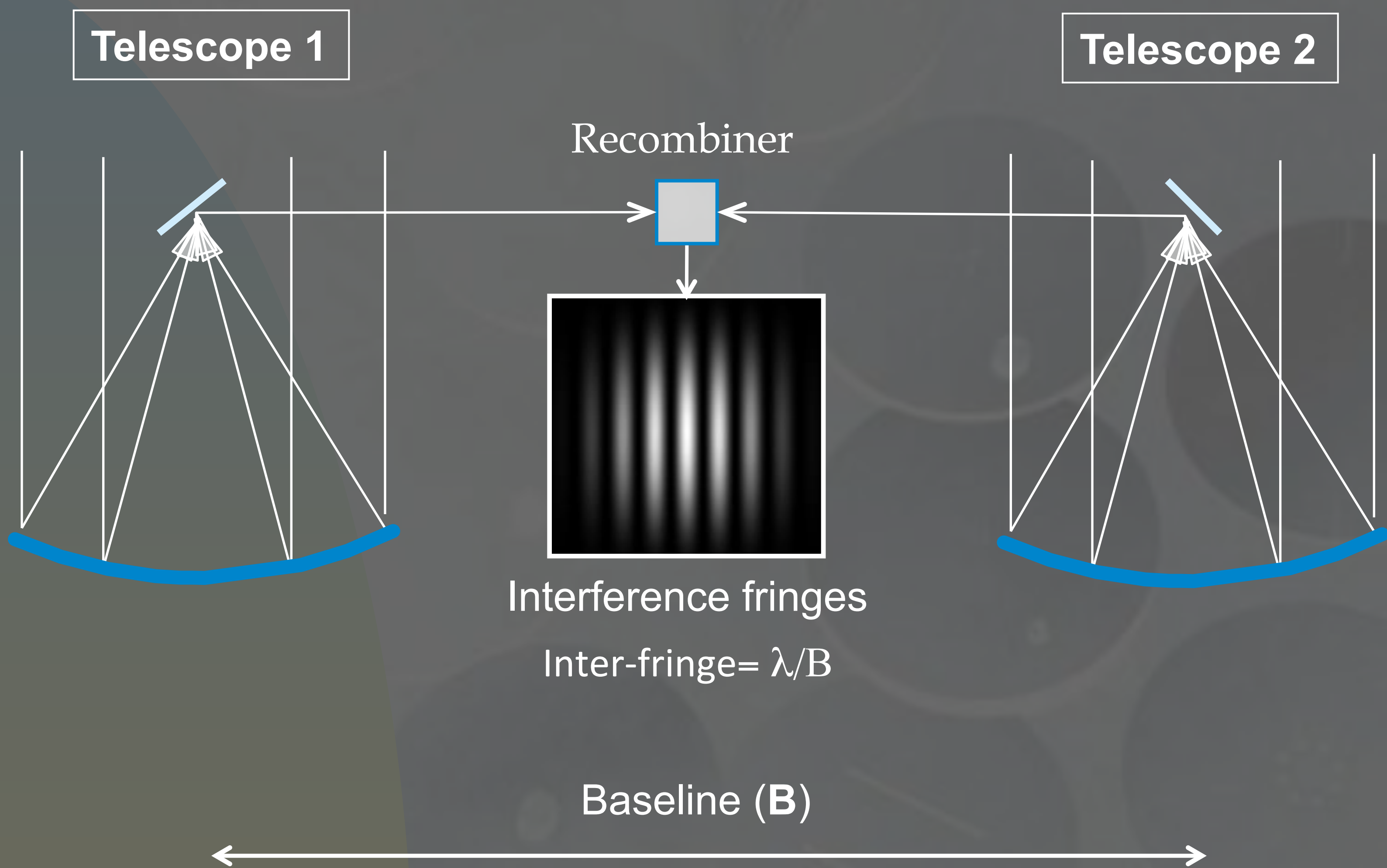


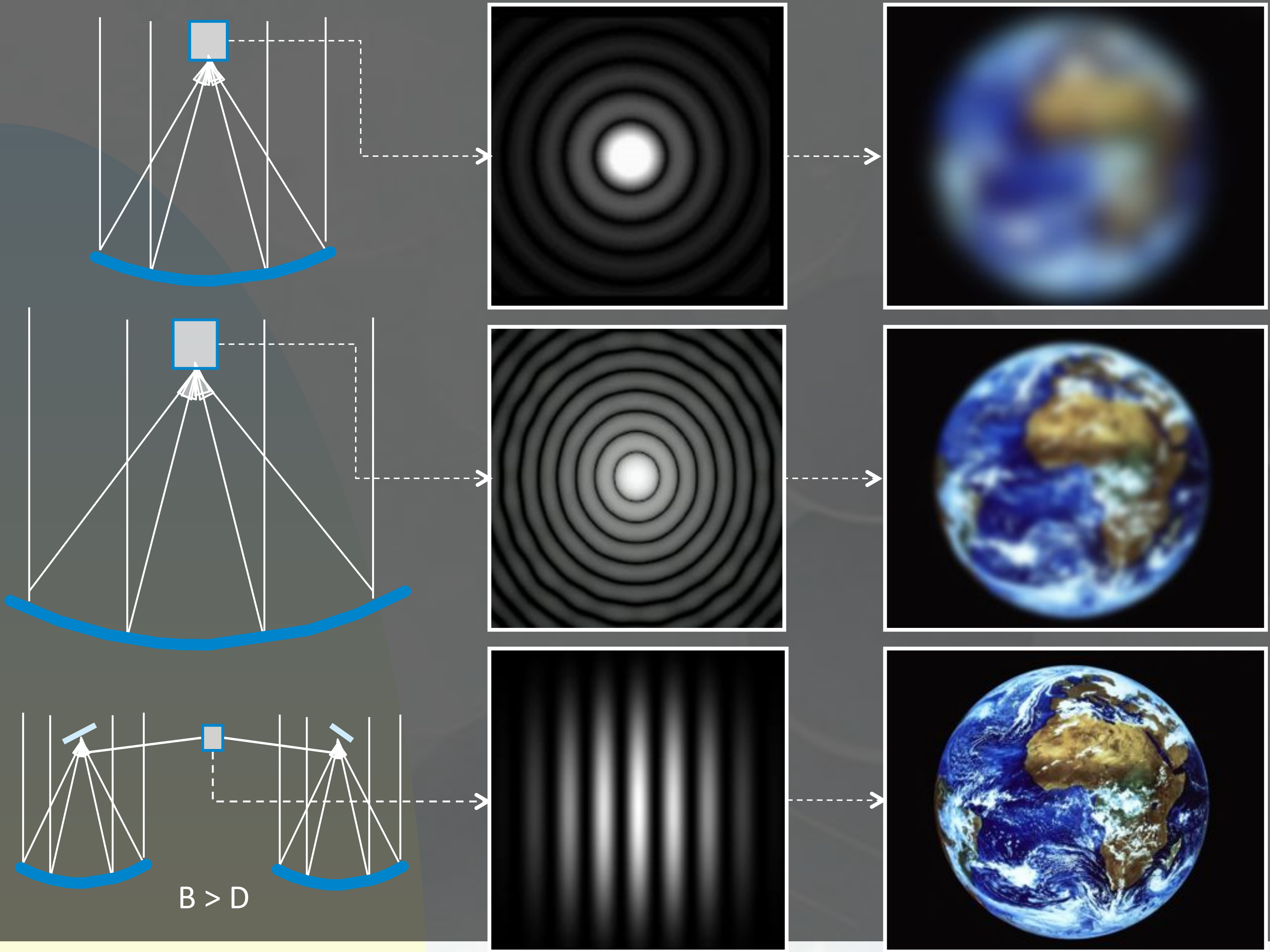
- Need for very large telescopes !!!

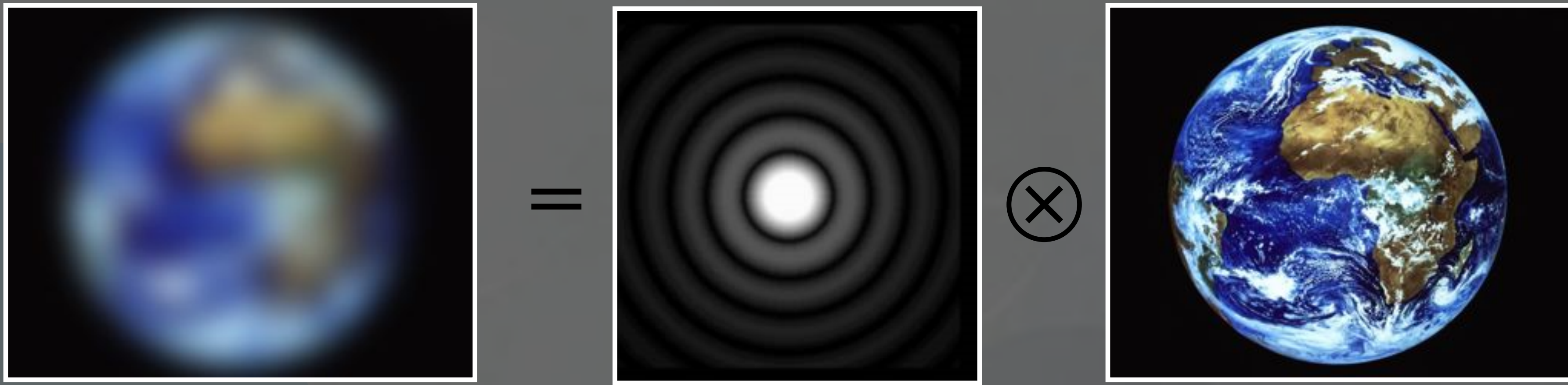
- H. Fizeau and E. Stephan (1868-1870):

  "In terms of angular resolution, two small apertures distant of B are equivalent to a single large aperture of diameter B"




Telescope 1
Telescope 2
Recombiner
Interference fringes
Inter-fringe= λ/B
Baseline (**B**)


B > D



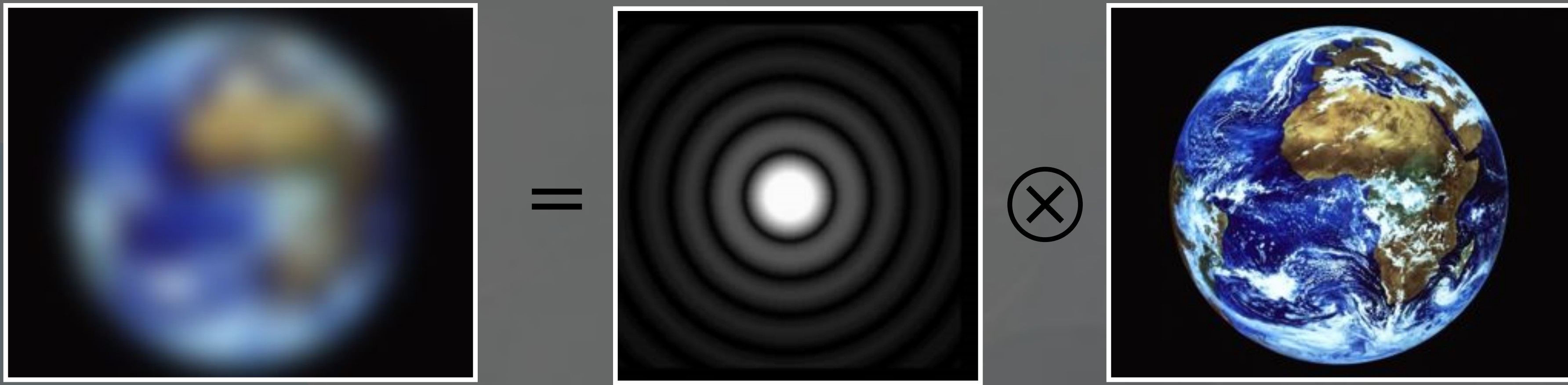
Convolution theorem

$$I(\zeta,\eta) = \iint PSF(\zeta-\zeta',\eta-\eta')\,O(\zeta',\eta')\,d\zeta'\,d\eta' = PSF(\zeta,\eta)\otimes O(\zeta,\eta)$$

$$TF(I(\zeta,\eta))(u,v) = TF(PSF(\zeta,\eta))(u,v)\cdot TF(O(\zeta,\eta))(u,v)$$

$u = B_u / \lambda, v = B_v / \lambda$

$$O(\zeta,\eta) = TF(-1)TF(O(\zeta,\eta)) = TF(-1)(TF(I(\zeta,\eta)) / TF(PSF(\zeta,\eta)))$$



Wierner Kitchen theorem

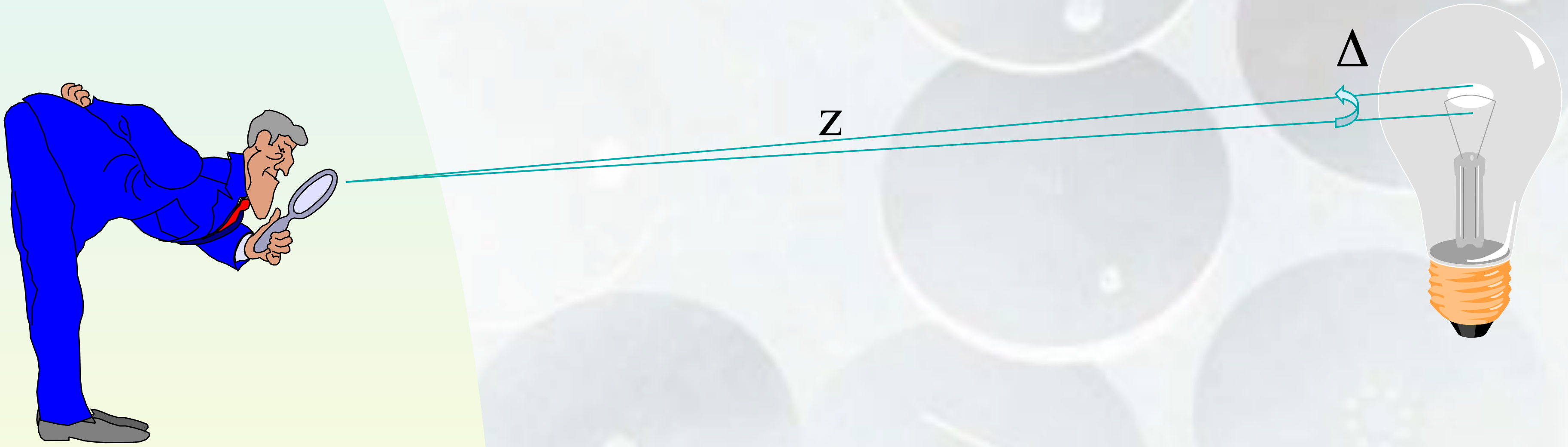
$$TF(PSF(\zeta,\eta))(u,v) = \iint A^{*}(x,y)\ A\ (x+u, y+v)\ dx\ dy$$

An introduction to optical/IR interferometry

- 1 Introduction
- 2 Reminders
- 3 Brief history of stellar diameter measurements
- 4 Interferometry with two independent telescopes
- 5 Light coherence (Zernicke-van Cittert theorem)
- 6 Examples of optical interferometers
- 7 Results
- 8 Three important theorems (Fundamental theorem, Convolution theorem and Wiener-Khintchin theorem)!

An introduction to optical/IR interferometry

- 1 Introduction



An introduction to optical/IR interferometry

- 1 Introduction

$$\rho = R / z \qquad (1.1)$$

$$F = f / \rho^2 \qquad (1.2)$$

$$T_{eff} = (F/\sigma)^{1/4} = (f / \sigma\, \rho^2)^{1/4} \qquad (1.3)$$

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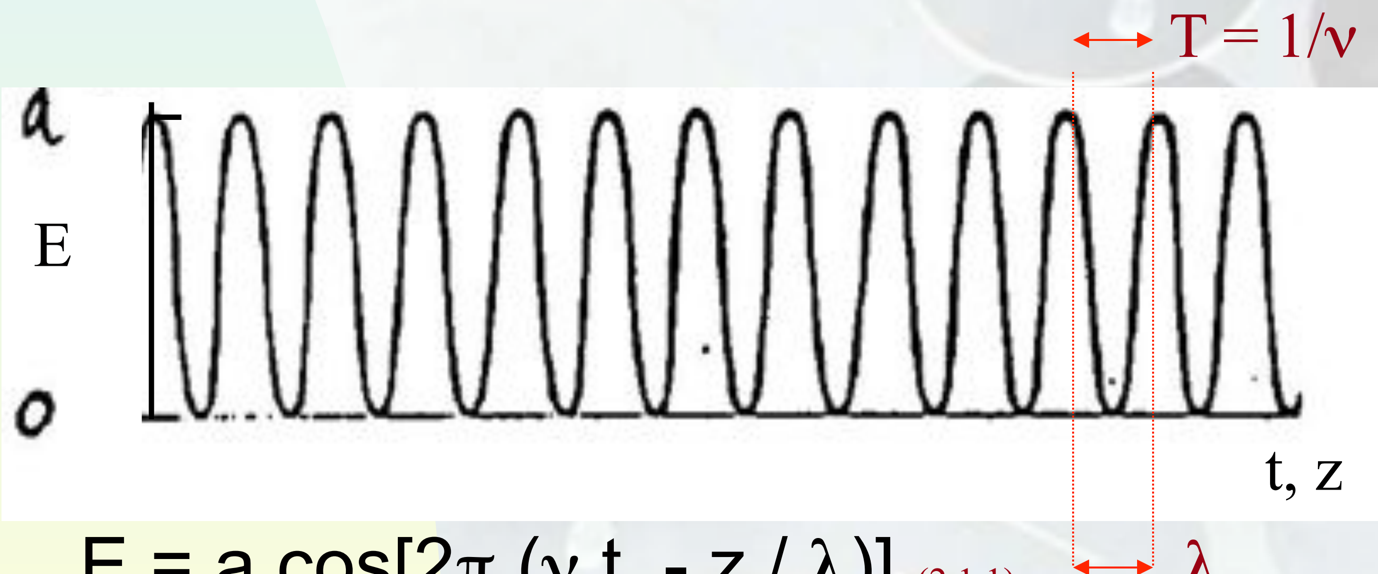
- 2 Reminders

- 2.1. Representation of an electromagnetic wave



$$E = a \cos[2\pi (\nu t - z / \lambda)] \quad (2.1.1)$$

where $\lambda = c\,T = c / \nu.$ (2.1.2)

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- 2.1. Representation of an electromagnetic wave

$E = \mathrm{Re}\{ a \exp[i2\pi(\nu t - z / \lambda)]\}$ (2.1.3)

$E = \mathrm{Re}\{ a \exp[-i \phi] \exp[i2\pi\nu t]\}$ (2.1.4)

where $\phi = 2\pi z / \lambda$. (2.1.5)

$E = a \exp[-i \phi] \exp[i2\pi\nu t]$ (2.1.6)

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- 2.1. Representation of an electromagnetic wave

$$E = A \exp[i2\pi\nu t] \qquad (2.1.7)$$

with $$A = a \exp[-i \phi] \qquad (2.1.8)$$

$\nu \sim 6\ 10^{14}$ Hz for $\lambda = 5000$ Å

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- 2.1. Representation of an electromagnetic wave

$$\langle E^2 \rangle = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{+T} E^2 dt \qquad (2.1.9)$$

$$\langle E^2 \rangle = a^2 / 2 \qquad (2.1.10)$$

$$\mathrm{I} = \mathrm{A}\,\mathrm{A}^* = |\mathrm{A}|^2 = \mathrm{a}^2 . \qquad (2.1.11)$$

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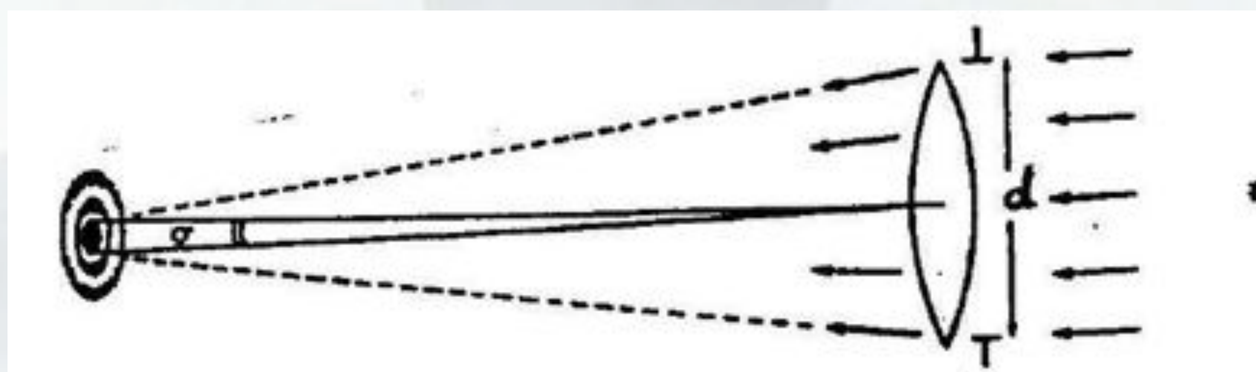
- 2.2. The Huygens-Fresnel principle

$$\sigma = 2.44\ \lambda / d$$

(2.2.1)



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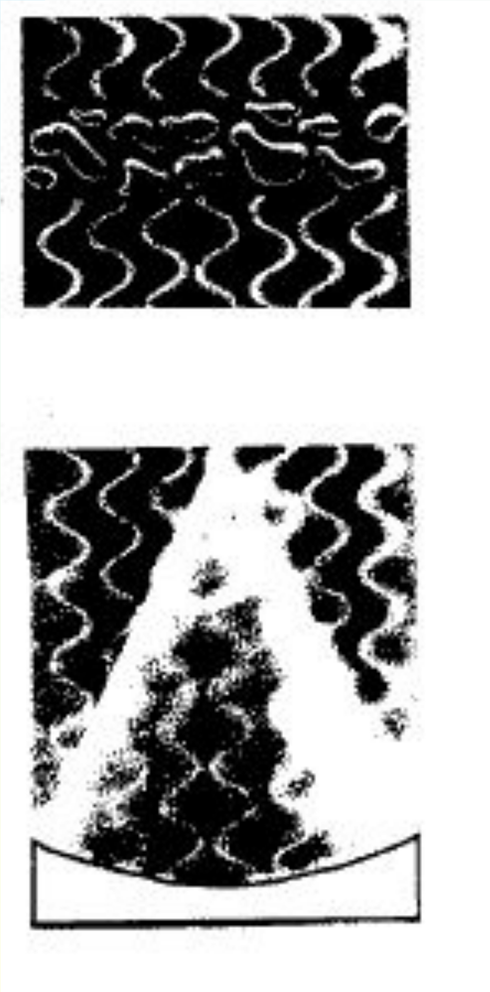
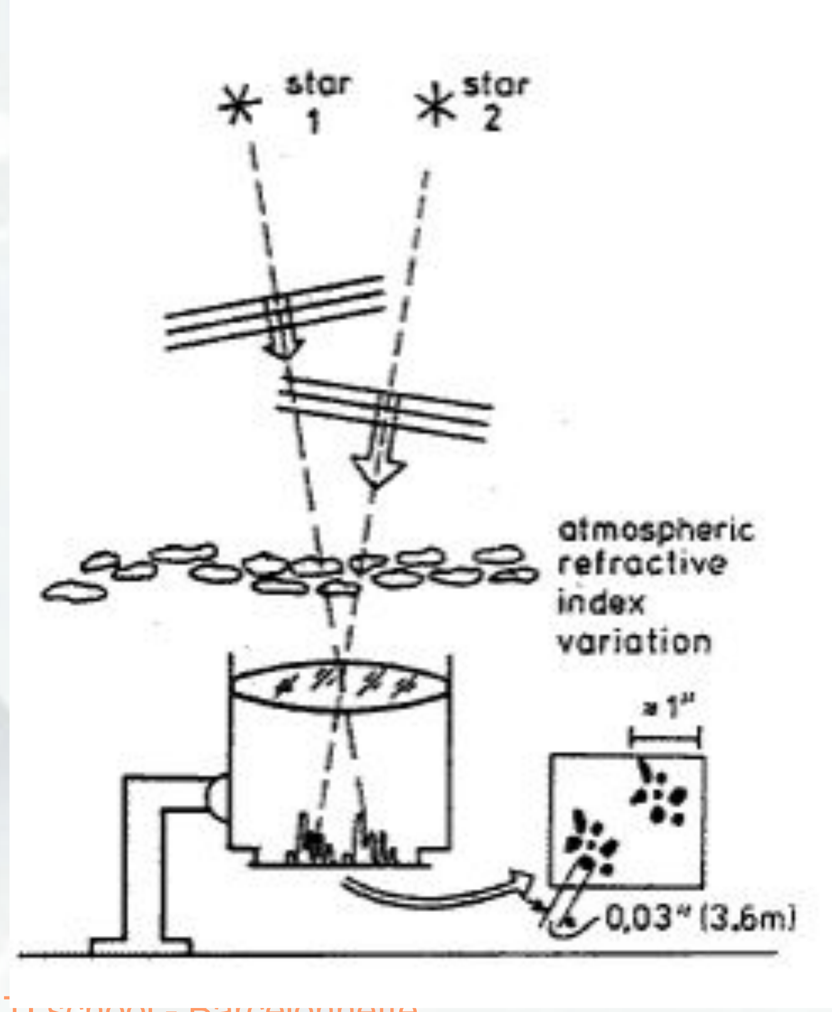
- 2.2. The Huygens-Fresnel principle





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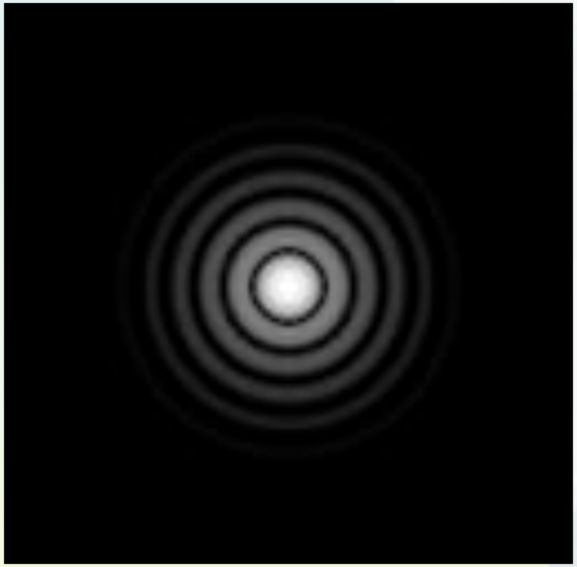
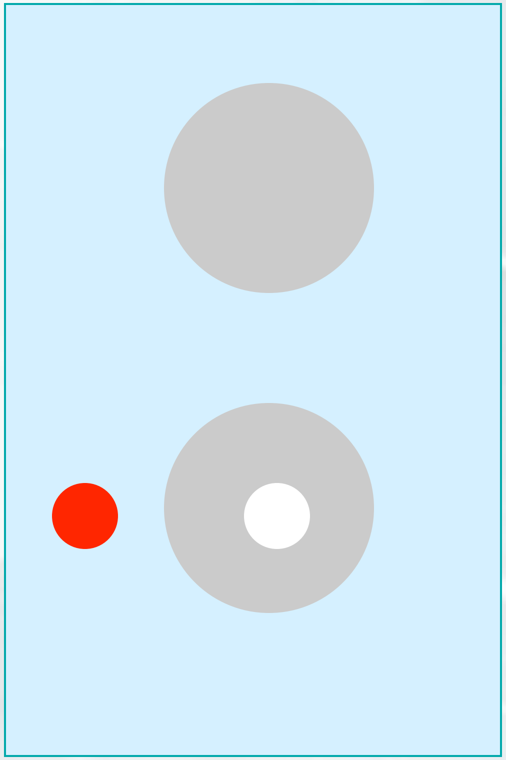
- 2.2. The Huygens-Fresnel principle





1st experiment!

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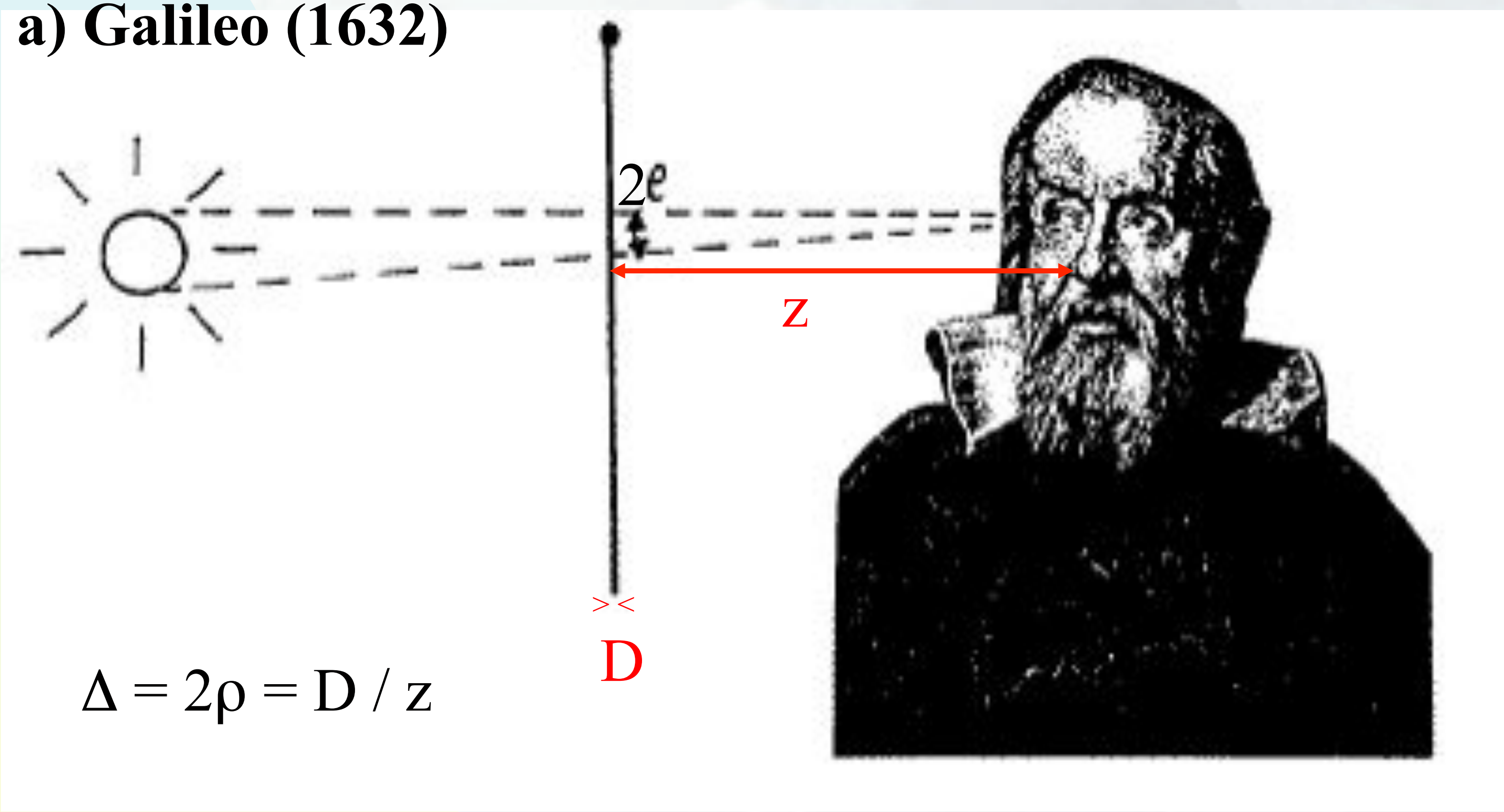
- 3 Brief history of stellar diameter measurements



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- 3 Brief history of stellar diameter measurements

b) Newton:

$$V_{\odot} - V = -5 \log (z / z_{\odot}) , \qquad (3.1)$$

$$\Delta = 2 R_{\odot} / z , \qquad (3.2)$$

$$\Delta \sim 2\ 10^{-3}{''}\ (8\ 10^{-3}{''}) . \qquad (3.3)$$

c) Fizeau-type interferometry

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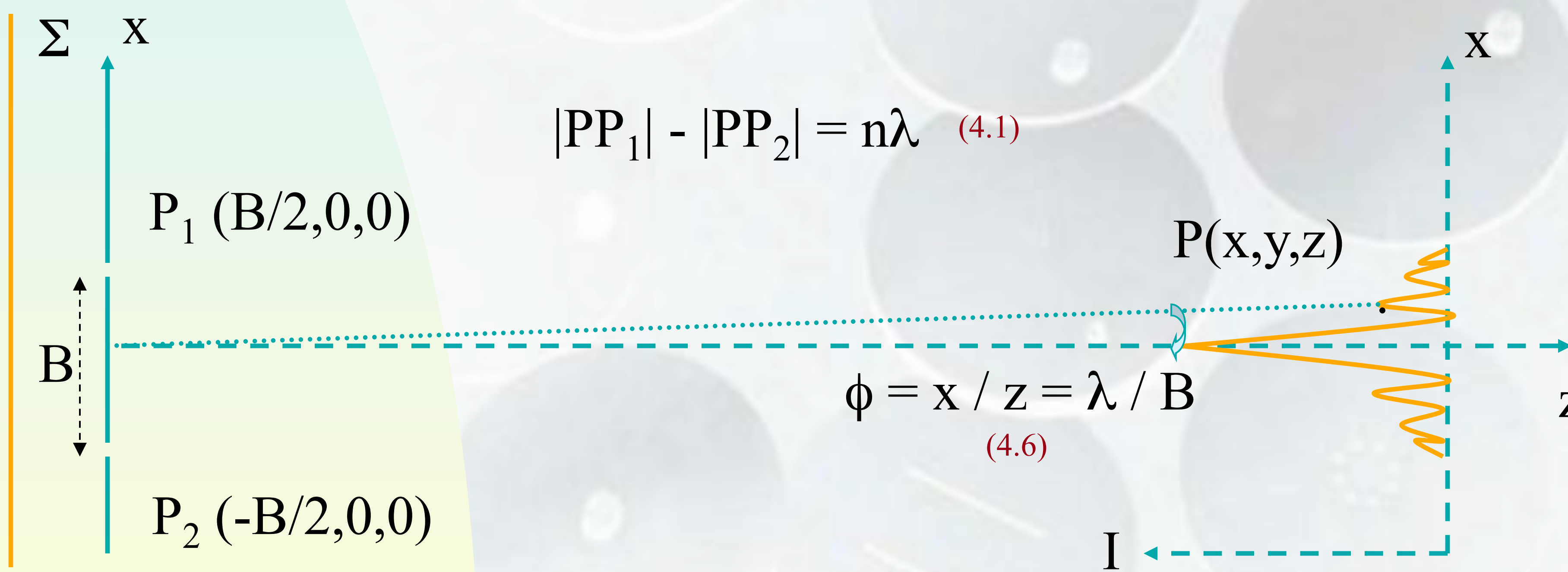
- 4 Interferometry with two independent telescopes

a) Young's double hole experiment (24-11-1803)



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- 4 Interferometry with two independent telescopes

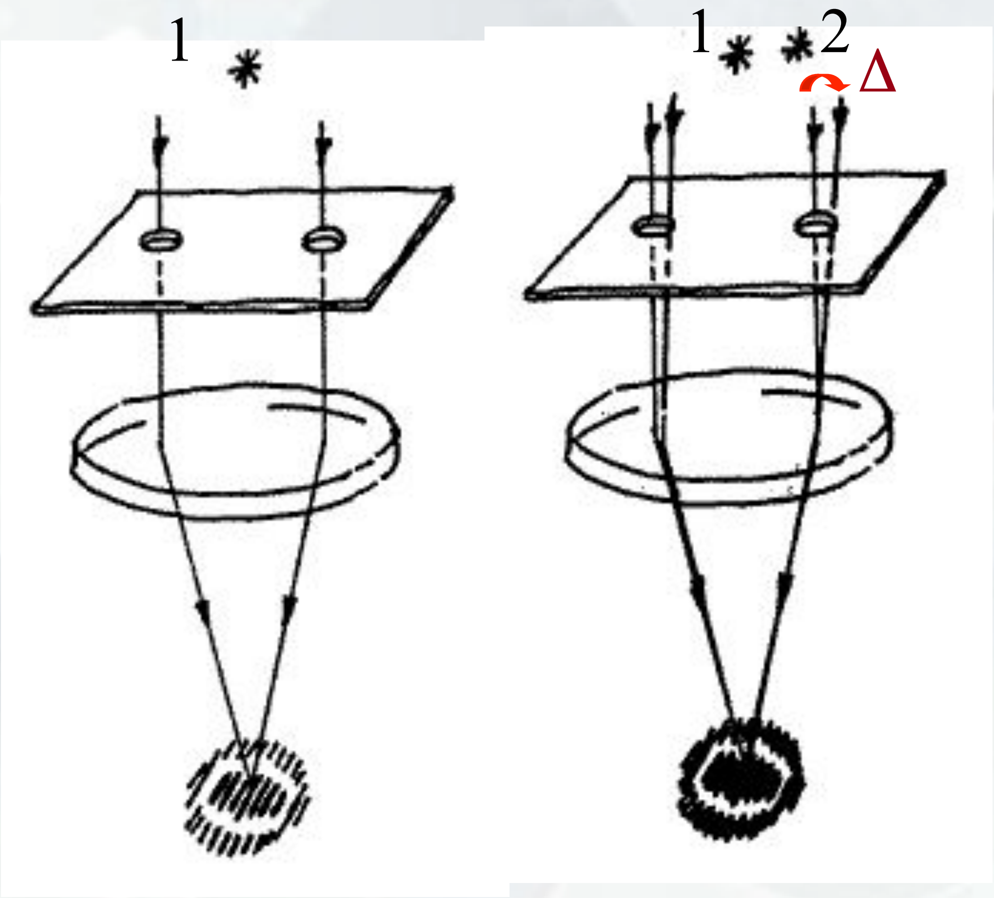
b) Fizeau … the father of stellar interferometry (1868)

If $\Delta \geq \phi/2 = \lambda / (2B)$, (4.7)

fringe disappearance!

Fringe visibility:

$$\upsilon = \left(\frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}} \right)$$



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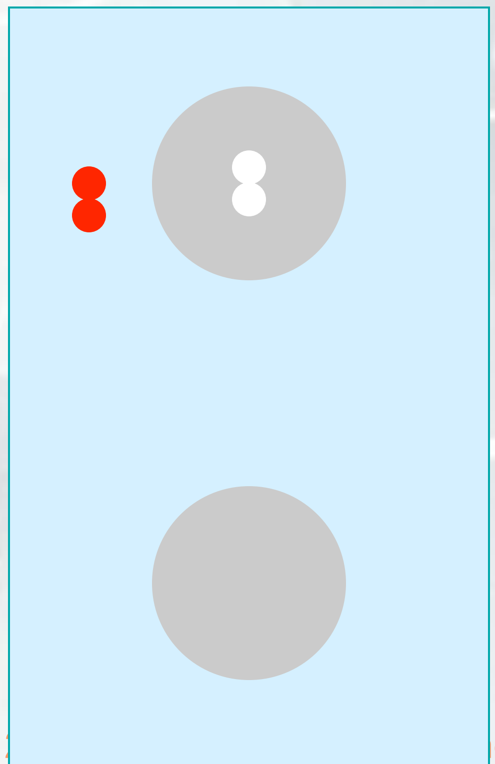
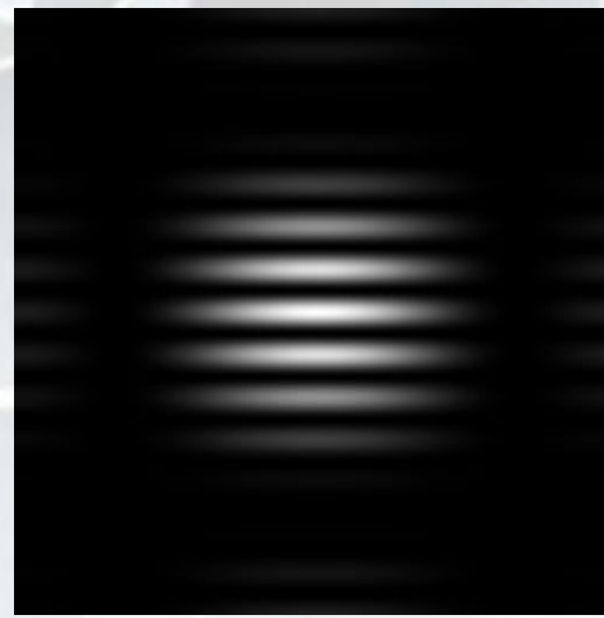
- 4 Interferometry with two independent telescopes

b) Fizeau … the father of stellar interferometry (1868)

2nd experiment!





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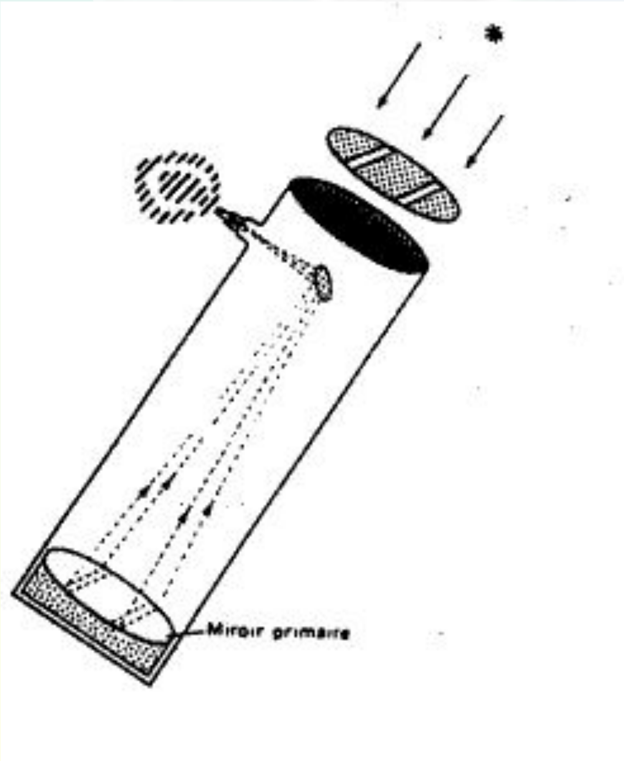
- 4 Interferometry with two independent telescopes

b) Fizeau … the father of stellar interferometry (1868)



Stéphan, 1873
$\Delta << 0{,}16''$

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- Marseille 80 cm telescope

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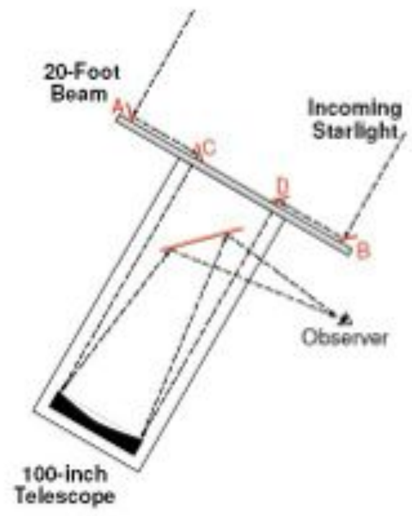
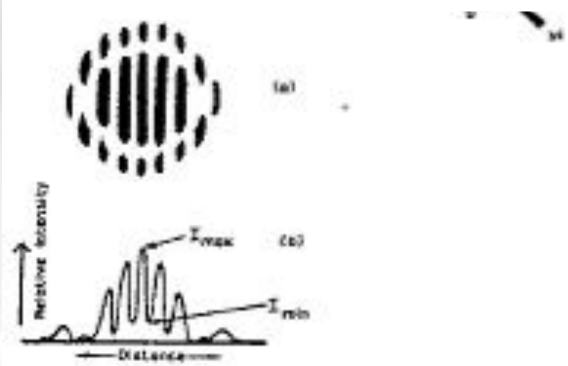
4 Interferometry with two independent telescopes

b) Fizeau … the father of stellar interferometry (1868)

- Michelson, 1890 (satellites of Jupiter)
- Michelson and Pease (1920)





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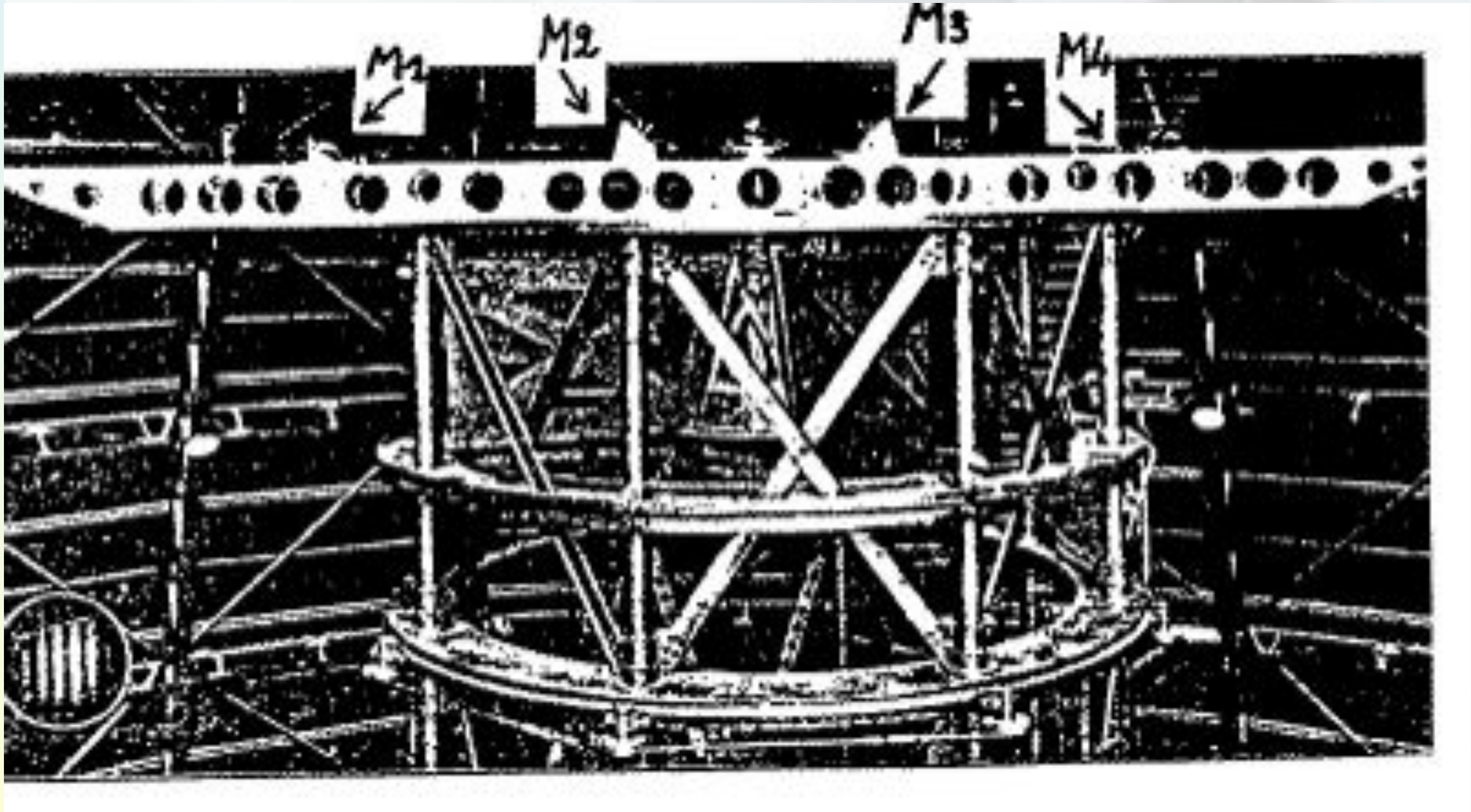
- 4 Interferometry with two independent telescopes

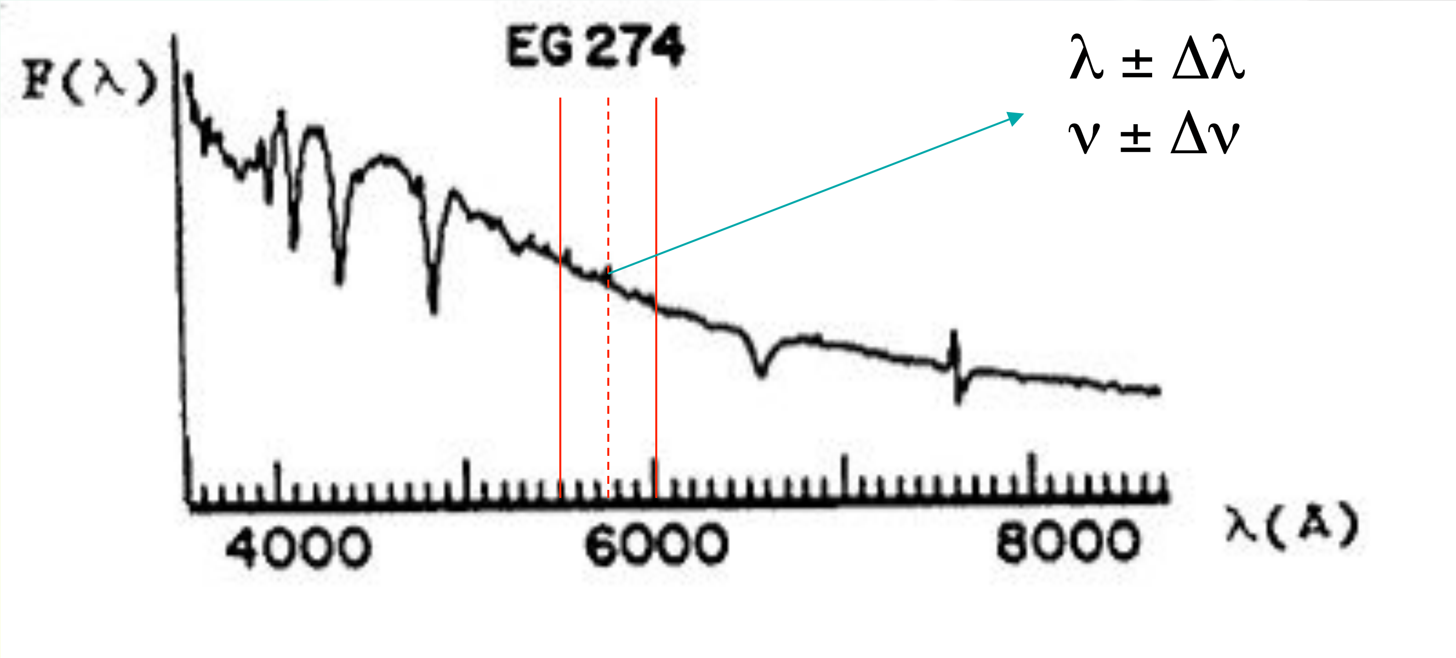
b) Fizeau … the father of stellar interferometry (1868)

- Anderson
- Brown and Twiss (1956)
- Radio Interferometry (1950)



An introduction to optical/IR interferometry

- 5 Light coherence
- **5.1 Quasi monochromatic light (waves and wave groups)**



An introduction to optical/IR interferometry

- 5 Light coherence
- **5.1 Quasi monochromatic light (waves and wave groups)**

$$I = \langle V(t) V^{*}(t) \rangle \qquad (5.1.1)$$

$$V(z,t) = \int_{\nu-\Delta\nu}^{\nu+\Delta\nu} a(\nu') \exp(i2\Pi(\nu' t - z/\lambda')) d\nu' \qquad (5.1.2)$$

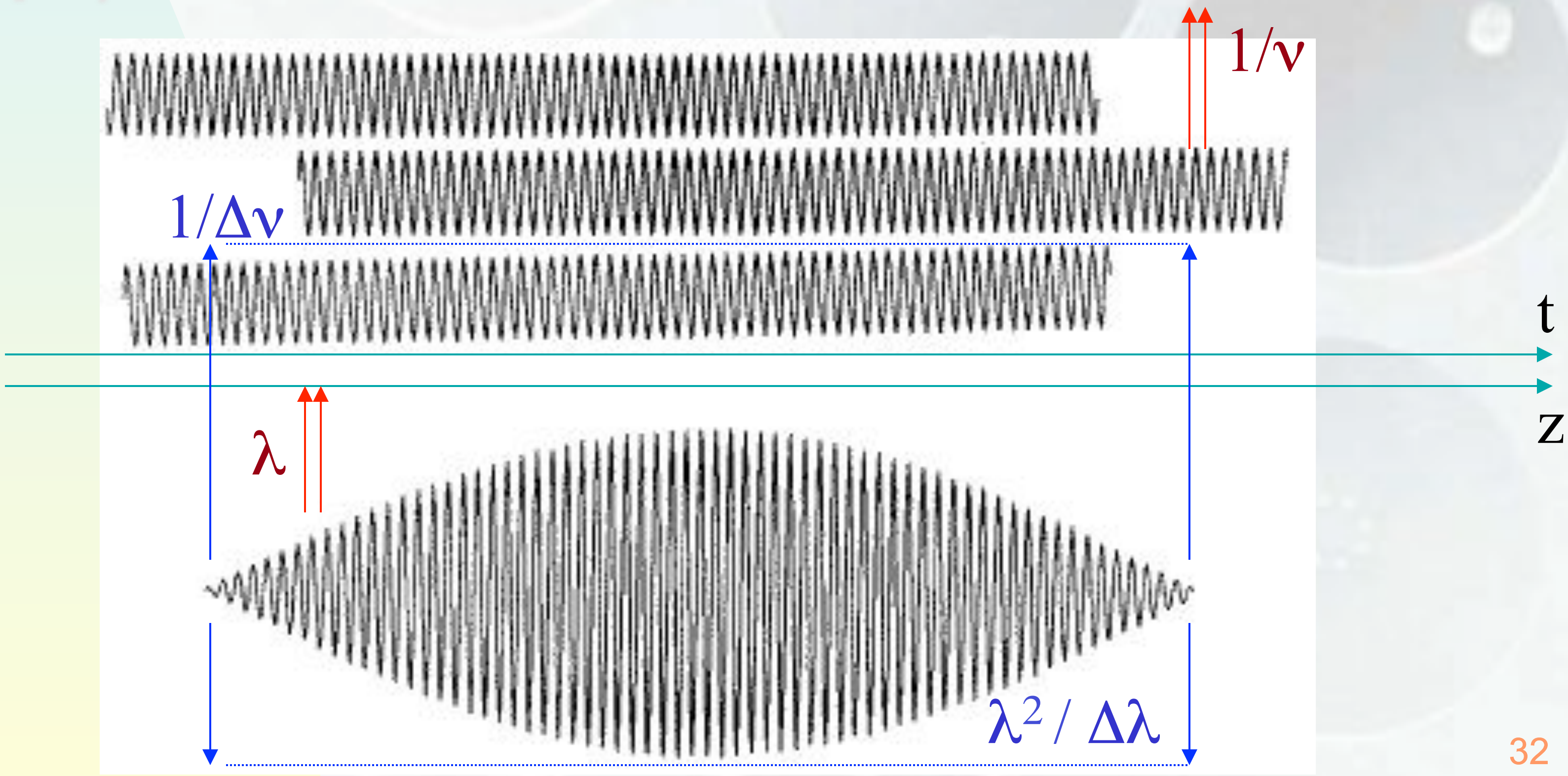
$\exp(-i2\Pi(\nu t - z/\lambda))$ $\exp(i2\Pi(\nu t - z/\lambda))$

$$V(z,t) = A(z,t) \exp(i2\Pi(\nu t - z/\lambda)) \qquad (5.1.3)$$

$$A(z,t) = \int_{\nu-\Delta\nu}^{\nu+\Delta\nu} a(\nu') \exp(i2\Pi((\nu' - \nu)t - z(1/\lambda' - 1/\lambda))) d\nu' \qquad (5.1.4)$$

An introduction to optical/IR interferometry

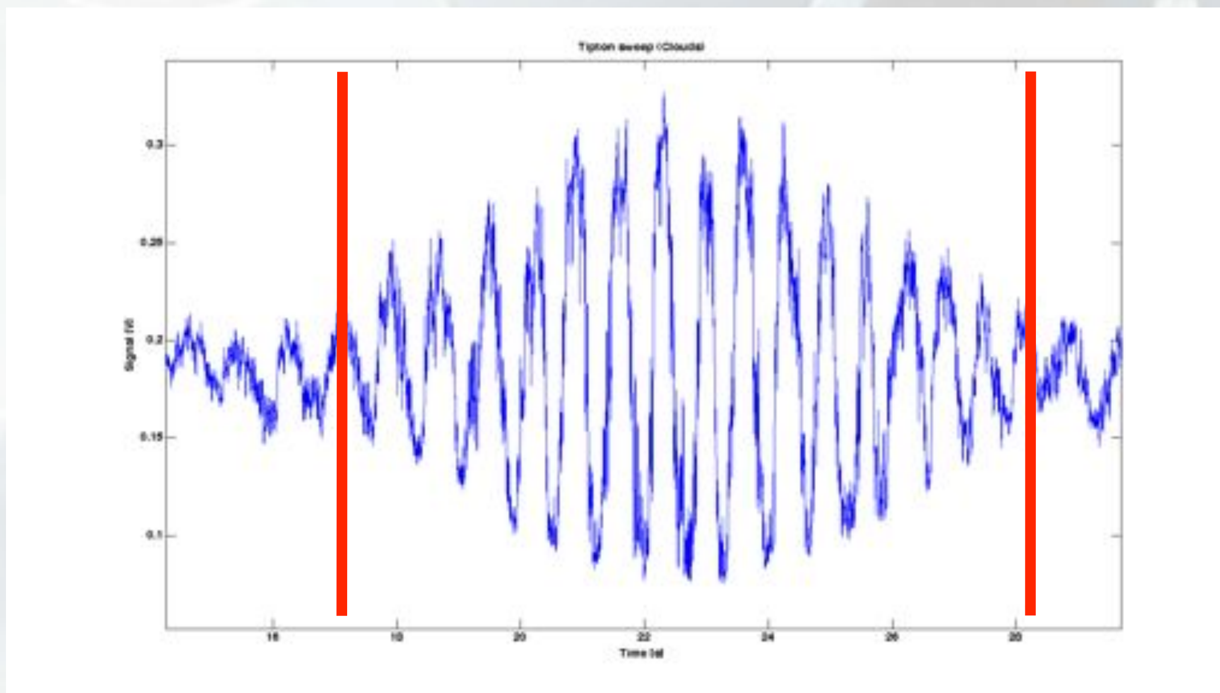
- 5 Light coherence
- **5.1 Quasi monochromatic light (waves and wave groups)**



An introduction to optical/IR interferometry

- 5 Light coherence
- **5.1 Quasi monochromatic light (waves and wave groups)**

λ_0 = 2.2μm
λ∈ [2.07 ; 2.33]μm
Δλ = 0.13μm

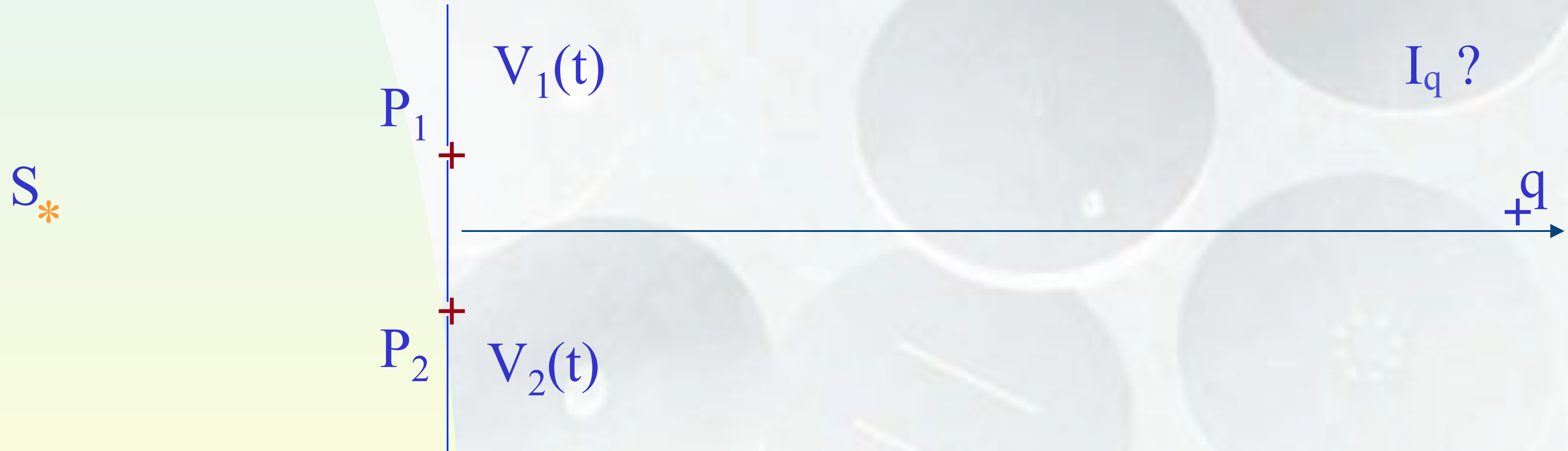


An introduction to optical/IR interferometry

- 5 Light coherence
- **5.2 Fringe visibility**

$$I_q = \langle V_q^*(t) V_q(t) \rangle \quad (5.2.1)$$

$$V_q(t) = V_1(t - t_{1q}) + V_2(t - t_{2q}) \quad (5.2.2)$$



$$V_q(t) = V_1(t) + V_2(t - \tau) \quad (5.2.3)$$

$$\tau = t_{2q} - t_{1q} \quad (5.2.4)$$

34

- 5 Light coherence
- **5.2 Fringe visibility**

$$I_q = I + I + 2\,I\ \mathrm{Re}\{\gamma_{12}(\tau)\} \qquad (5.2.5)$$

$$\gamma_{12}(\tau) = \langle V_1^*(t)\,V_2(t-\tau)\rangle / I \qquad (5.2.6)$$

$$\gamma_{12}(\tau) = \langle A_1^*(z,t)\ A_2(z,t-\tau)\rangle \exp(-i2\Pi\nu\tau) / I \qquad (5.2.7)$$

If $\tau \langle\langle 1/\Delta\nu$

$$\gamma_{12}(\tau) = |\gamma_{12}(\tau=0)| \exp(i\beta_{12} - i2\Pi\nu\tau) \qquad (5.2.8)$$

An introduction to optical/IR interferometry

- 5 Light coherence
- **5.2 Fringe visibility**

$$I_q = I + I + 2I\,|\gamma_{12}(0)|\cos\left(\beta_{12} - 2\Pi\nu\tau\right) \qquad (5.2.9)$$

$$\upsilon = \left(\frac{I_{max} - I_{min}}{I_{max} + I_{min}}\right) = |\gamma_{12}(0)| \qquad (5.2.10)$$

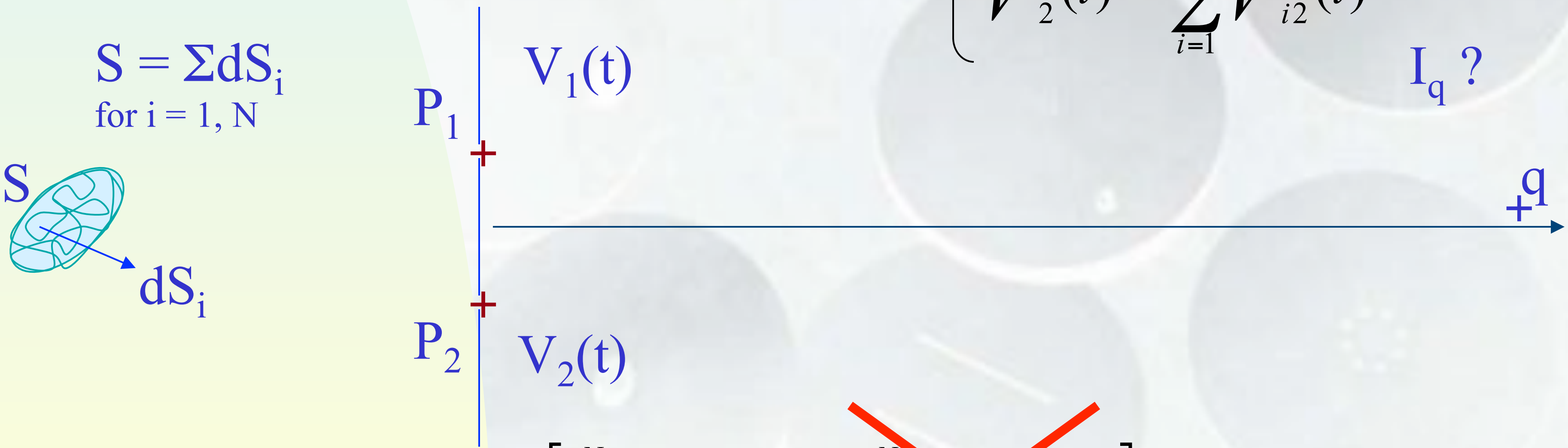
An introduction to optical/IR interferometry

- 5 Light coherence
- **5.3 Spatial light coherence**

?? $$\gamma_{12}(\tau = 0) = \langle V_1^*(t) V_2(t) \rangle / I \quad (5.3.1)$$

$$\begin{cases} V_1(t) = \sum_{i=1}^{N} V_{i1}(t) \\ V_2(t) = \sum_{i=1}^{N} V_{i2}(t) \end{cases} \quad (5.3.2)$$

$S = \Sigma dS_i$
for i = 1, N



$$\gamma_{12}(0) = \left[\sum_{i=1}^{N} \langle V_{i1}^* V_{i2} \rangle + \sum_{i \neq j}^{N} \langle V_{i1}^* V_{j2} \rangle \right] / I \quad (5.3.3)$$

An introduction to optical/IR interferometry

- 5 Light coherence
- **5.3 Spatial light coherence**

$$\gamma_{12}(0) = \left[\sum_{i=1}^{N} \langle V^*_{i1} V_{i2} \rangle + \sum_{i \neq j}^{N} \langle V^*_{i1} V_{j2} \rangle \right] / I \qquad (5.3.3)$$

$$\begin{cases} V_{i1}(t) = \left(a_i(t - r_{i1}/c)/r_{i1} \right) \exp\{ i2\Pi\nu(t - r_{i1}/c) \} & (5.3.4) \\ V_{i2}(t) = \left(a_i(t - r_{i2}/c)/r_{i2} \right) \exp\{ i2\Pi\nu(t - r_{i2}/c) \} \end{cases}$$

$$V^*_{i1}(t) V_{i2}(t) = \left| a_i(t - r_{i1}/c) \right|^2 / (r_{i1} r_{i2}) \exp\{ -i2\Pi\nu(r_{i2} - r_{i1})/c \} \qquad (5.3.5)$$

as long as: $$\left| r_{i1} - r_{i2} \right| \leq c/\Delta\nu = \lambda^2/\Delta\lambda = 1 \qquad (5.3.6)$$

An introduction to optical/IR interferometry

- 5 Light coherence
- **5.3 Spatial light coherence**

$$I(s)ds = \left|a_i(t - r/c)\right|^2 \quad (5.3.7)$$

$$\gamma_{12}(0) = \int_S \frac{I(s)}{r_1 r_2} \exp\left\{-i2\Pi\left(r_2 - r_1\right)/\lambda\right\} ds / I \quad (5.3.8)$$

!!! Theorem of Zernicke-van Cittert !!!

An introduction to optical/IR interferometry

- 5 Light coherence
- **5.3 Spatial light coherence**

$$|r_2 - r_1| = |P_2P_i - P_1P_i| = |-(X^2 + Y^2) / 2\,Z' + (X\,\zeta + Y\,\eta)| \quad (5.3.9)$$

where $\zeta = X' / Z'$ and $\eta = Y' / Z'$ (5.3.10)

An introduction to optical/IR interferometry

- 5 Light coherence
- **5.3 Spatial light coherence**

$$\gamma_{12}(0, X/\lambda, Y/\lambda) = \exp(-i\phi_{X,Y}) \frac{\iint_S I(\zeta,\eta)\exp\{-i2\Pi(X\zeta + Y\eta)/\lambda\}d\zeta d\eta}{\iint_S I(\zeta',\eta')d\zeta' d\eta'} \quad (5.3.11)$$

$$I'(\zeta,\eta) = I(\zeta,\eta) / \iint_S I(\zeta',\eta')d\zeta' d\eta' \quad (5.3.12)$$

Setting u = X/λ, v = Y/λ:

$$\gamma_{12}(0,u,v) = \exp(-i\phi_{u,v})\iint_S I'(\zeta,\eta)\exp\{-i2\Pi(u\zeta + v\eta)\}d\zeta d\eta \quad (5.3.13)$$

$$I'(\zeta,\eta) = \iint \gamma_{12}(0,u,v)\exp(i\phi_{u,v})\exp\{i2\Pi(\zeta u + \eta v)\}d(u)d(v) \quad (5.3.14)$$

5.4 Fourier transform (cf. Léna 1996)

5.4.1 Definitions:

$$TF\_f(s) = \int_{-\infty}^{\infty} f(x)\, e^{-2i\pi sx} dx, \qquad (5.4.1)$$

$$f(x) = \int_{-\infty}^{\infty} TF\_f(s)\, e^{2i\pi sx} ds, \qquad (5.4.2)$$

$$\int_{-\infty}^{\infty} \left| f(x) \right|^2 dx. \qquad (5.4.3)$$

5.4 Fourier transform (cf. Léna 1996)

5.4.1 Definitions: Generalisation:

$$TF\_f(\vec{w}) = \int_{-\infty}^{\infty} f(\vec{r})\, e^{-2i\pi \vec{r}\vec{w}}\, d\vec{r} \quad . \qquad (5.4.4)$$

5.4.2 Some properties:

a) Linearity:

TF_(af) = a TF_f, a $\in \Re$, a being a constant, (5.4.5)

TF_(f+g) = TF_f + TF_g. (5.4.6)

An introduction to optical/IR interferometry

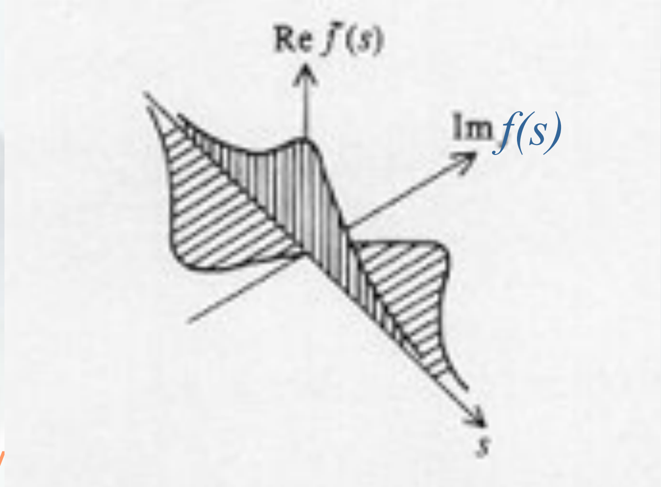
- **5.4 Fourier transform (cf. Léna 1996)**

5.4.2 Some properties: b) Symmetry & parity:

f(x) = P(x) + I(x), (5.4.7)

$$TF\_f(s) = 2\int_0^{\infty} P(x)\cos(2\pi xs)dx - 2i\int_0^{\infty} I(x)\sin(2\pi xs)dx \,. \quad (5.4.8)$$

Illustration of TF_f(s): f(x) is a real fonction. The real and imaginary parts of TF_f(s) are shown.



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- **5.4 Fourier transform (cf. Léna 1996)**

c) Similitude:

$$TF\_(f(x/a))(s) = |a|\ TF\_(f(x))(sa), \quad (5.4.9)$$

where $a \in \Re$, is a constant.

d) Translation:

$$TF\_(f(x - a))(s) = e^{-2i\pi as}\ TF\_(f(x))(s) \quad (5.4.10)$$

- **5.4 Fourier transform (cf. Léna 1996)**

e) Derivation:

$TF\_(df/dx)(s) = 2i\pi s\ TF\_f(s)$, $TF\_(d^nf/dx^n)(s) = (2i\pi s)^n\ TF\_f(s)$. (5.4.11)

5.4.3 Some important cases (one dimension):

a) Door function:

$\Pi(x) = 1$ if $x \in\ ]-1/2, 1/2[$, (5.4.12)

$= 0$ if $x \in\ ]-\infty, -1/2]$ or $x \in [1/2, \infty[$.

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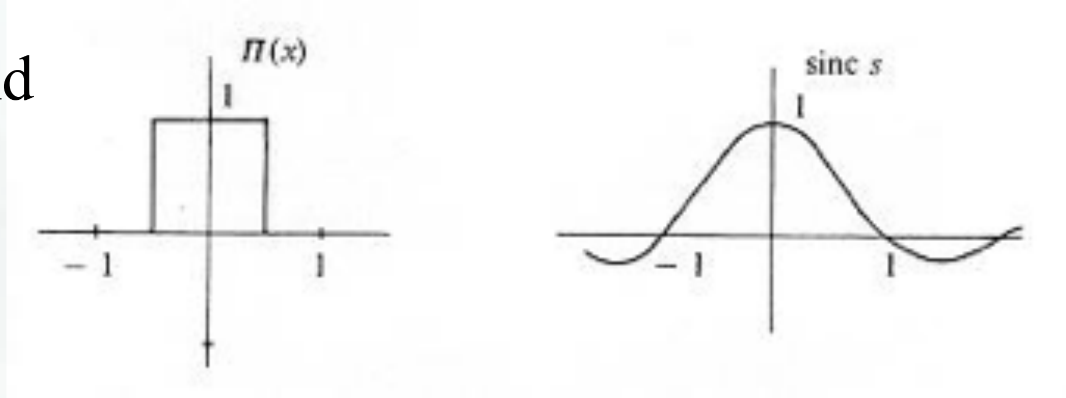
- **5.4 Fourier transform (cf. Léna 1996)**

The door function and its Fourier transform (cardinal sine)



$$TF\_ (\Pi(x))(s) = \mathrm{sinc}(s) = \sin(\pi s) / \pi s. \quad (5.4.13)$$

$$TF\_ (\Pi(x/a))(s) = |a|\ \mathrm{sinc}(as) = |a| \sin(\pi as) / \pi as. \quad (5.4.14)$$

5.4 Fourier transform (cf. Léna 1996)

b) Dirac distribution:

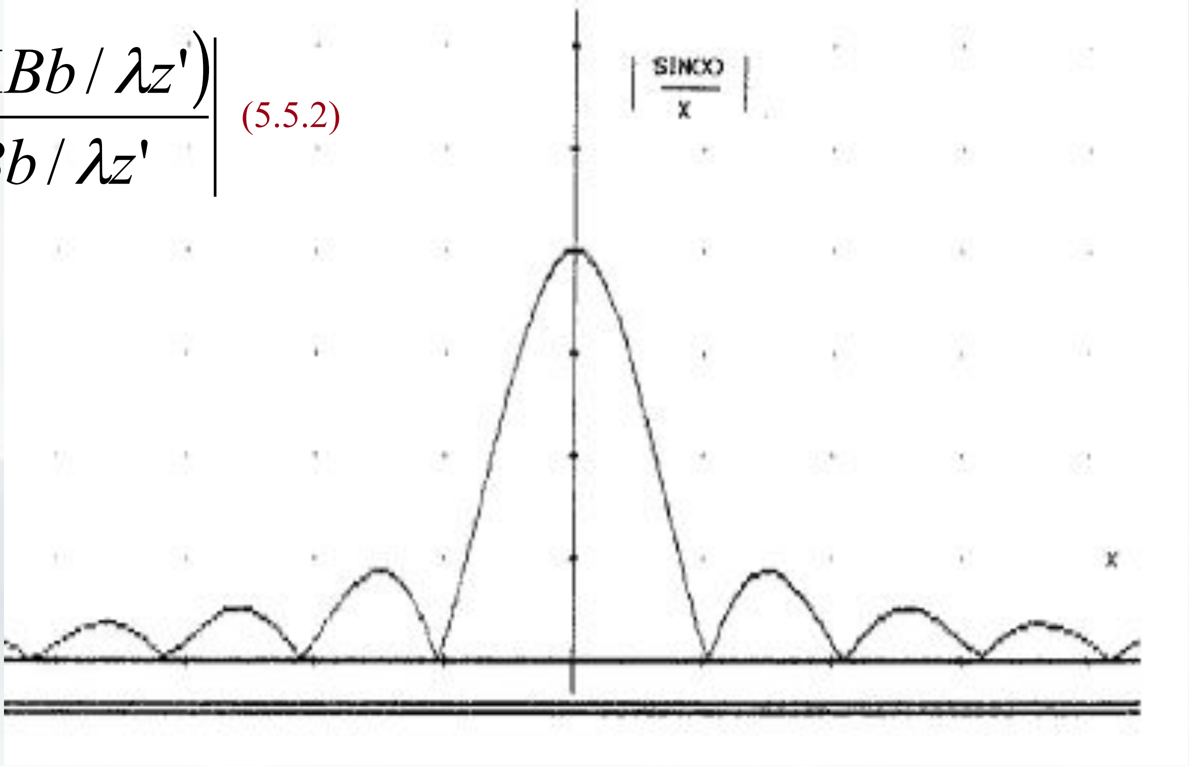
$$\delta(x) = \int_{-\infty}^{\infty} e^{2i\pi sx} ds \, . \qquad (5.4.15)$$

its Fourier transform is thus unity (= 1) in the interval]-∞, ∞[.

An introduction to optical/IR interferometry

- 5 Light coherence
- **5.5 Aperture synthesis**

$$\upsilon = \left|\gamma_{12}(0, B/\lambda)\right| = \left|\frac{\sin(\Pi Bb/\lambda z')}{\Pi Bb/\lambda z'}\right| \quad (5.5.2)$$



$$\Pi Bb/\lambda z' = \Pi \quad (5.5.3)$$

$\Delta \sim \lambda$ / B, for a (5.5.4) rectangular source.

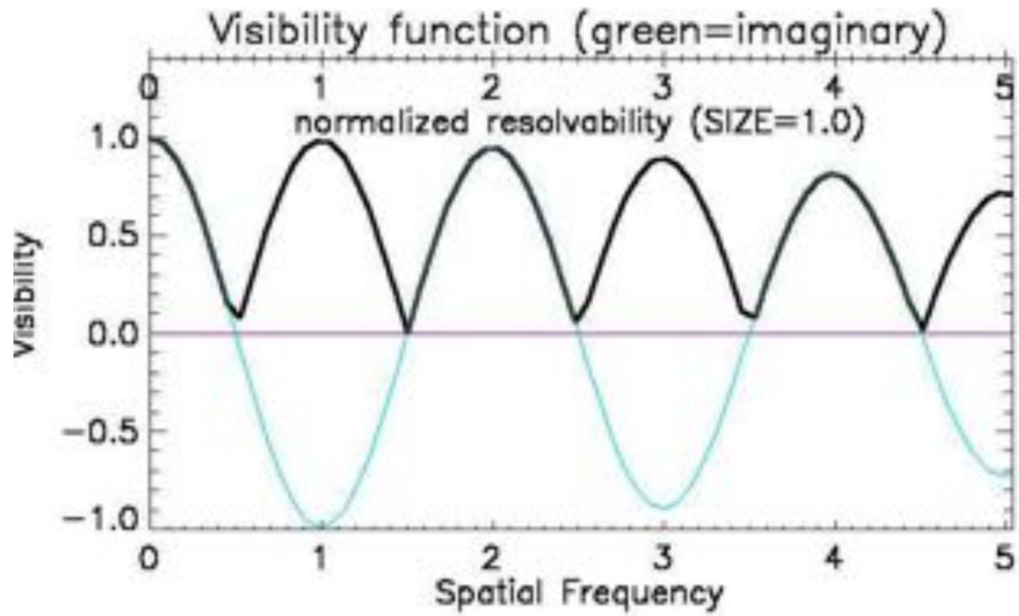
$\Delta \sim 1.22\ \lambda$ / B, for (5.5.5) a circular source !

An introduction to optical/IR interferometry

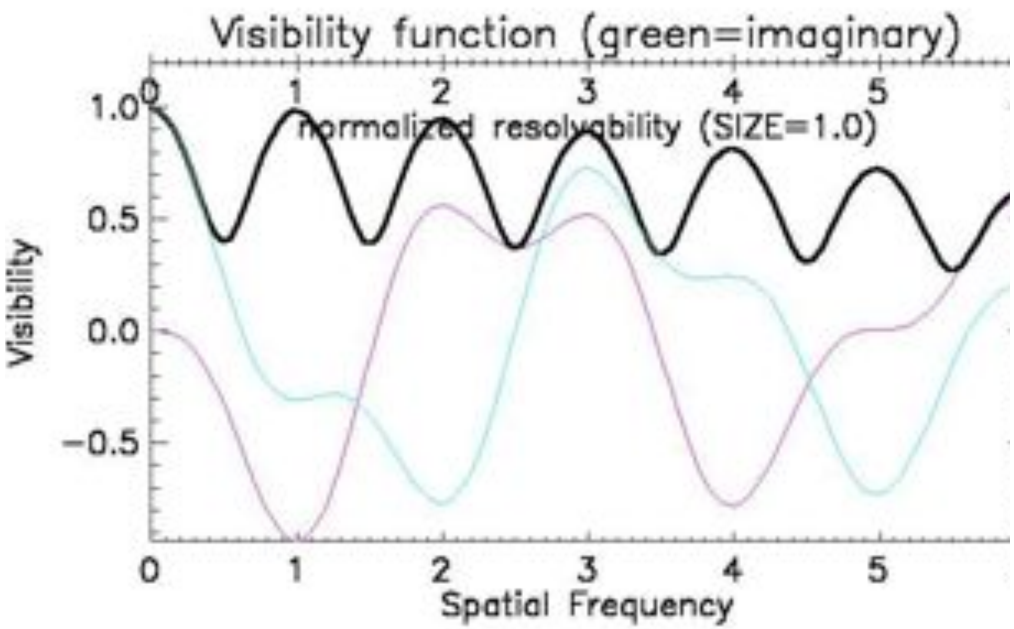
- 5 Light coherence
- **5.5 Aperture synthesis**

Exercises (…): point-like source?, double point-like source with a flux ratio = 1?, gaussian-like source?, uniform disk source?, …

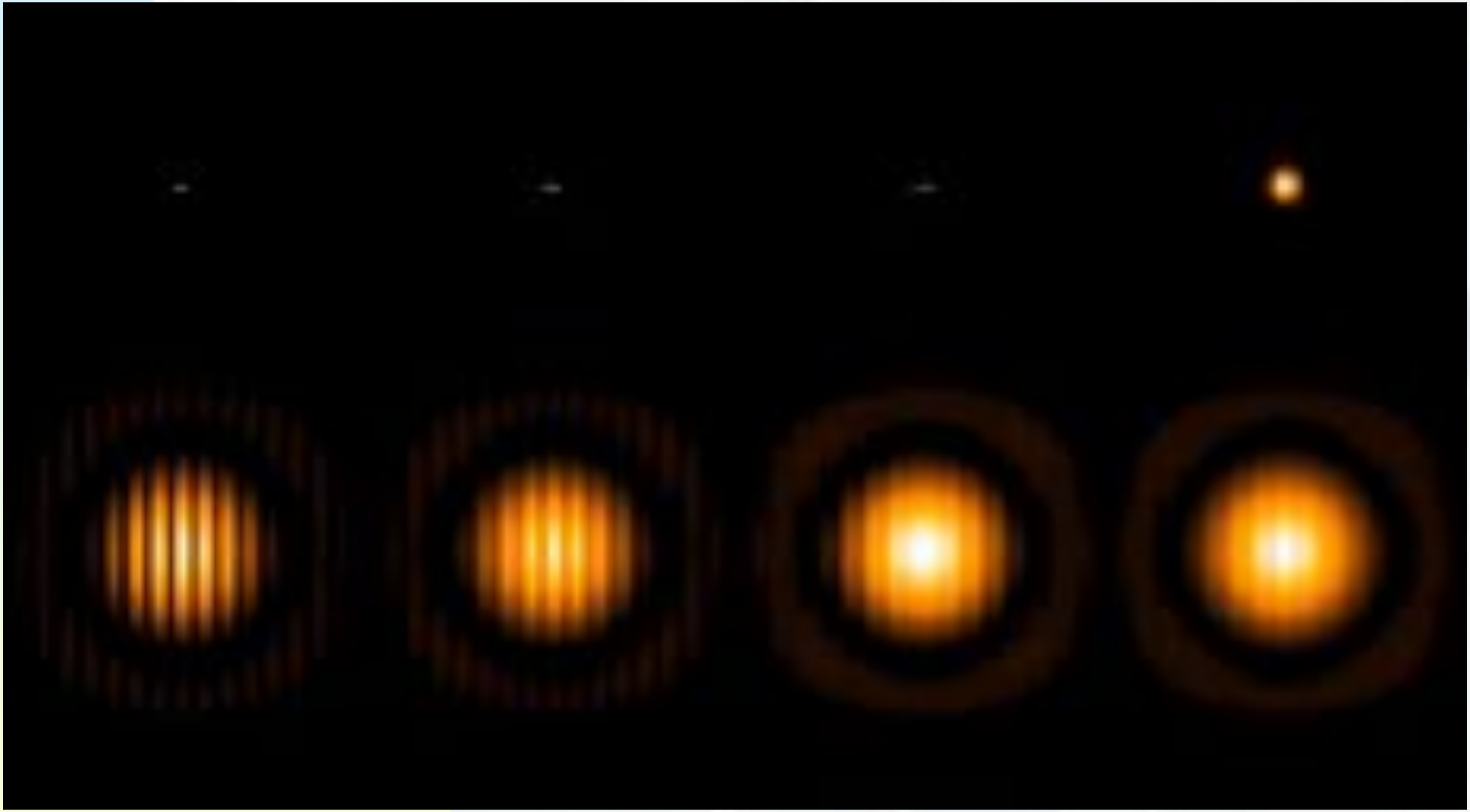
Case of a double point-like source with a flux ratio = 1



Case of a double point-like source with a flux ratio 0.7/0.3

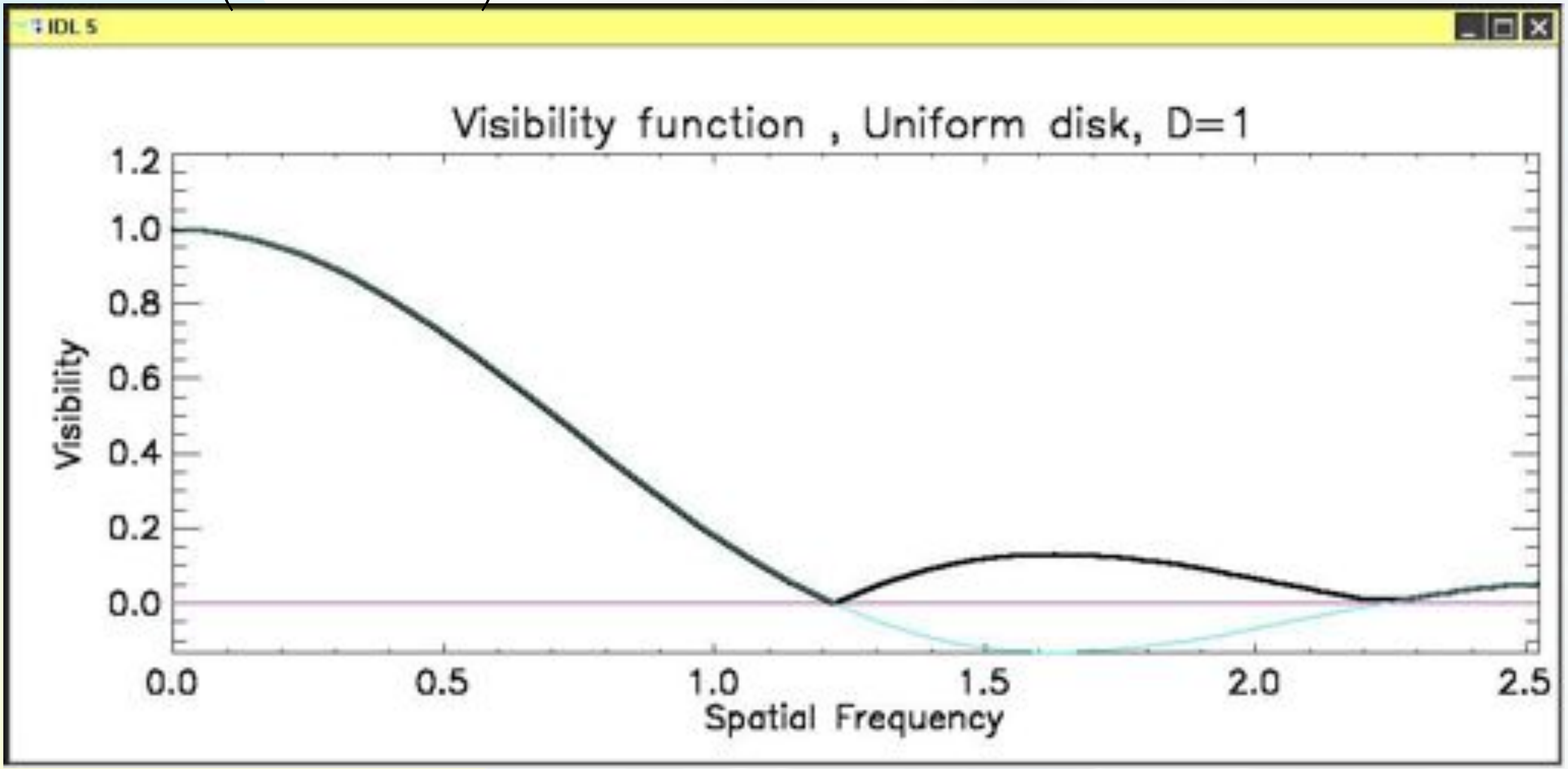


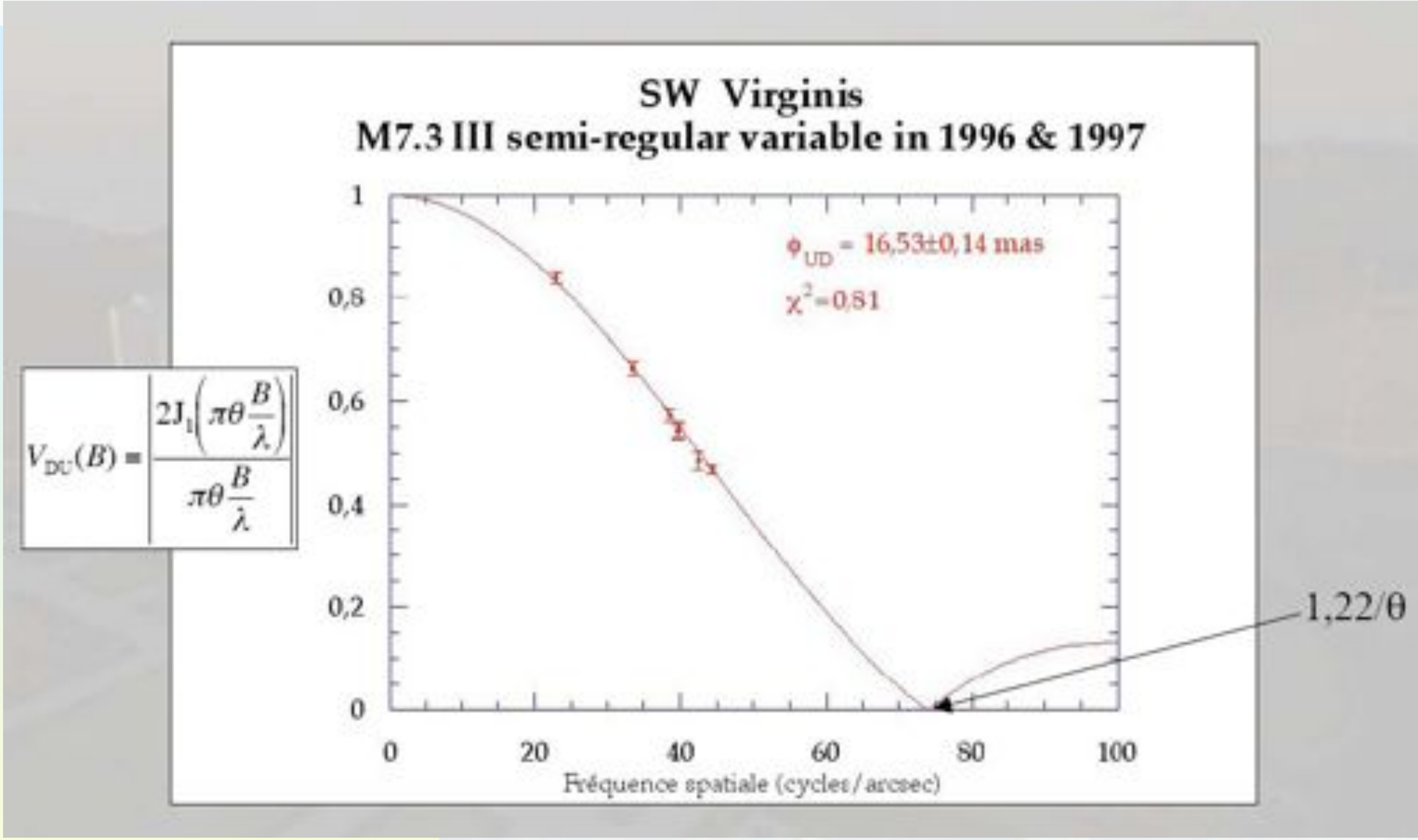
Variation of the fringe contrast as a function of the angular separation between the two stars:

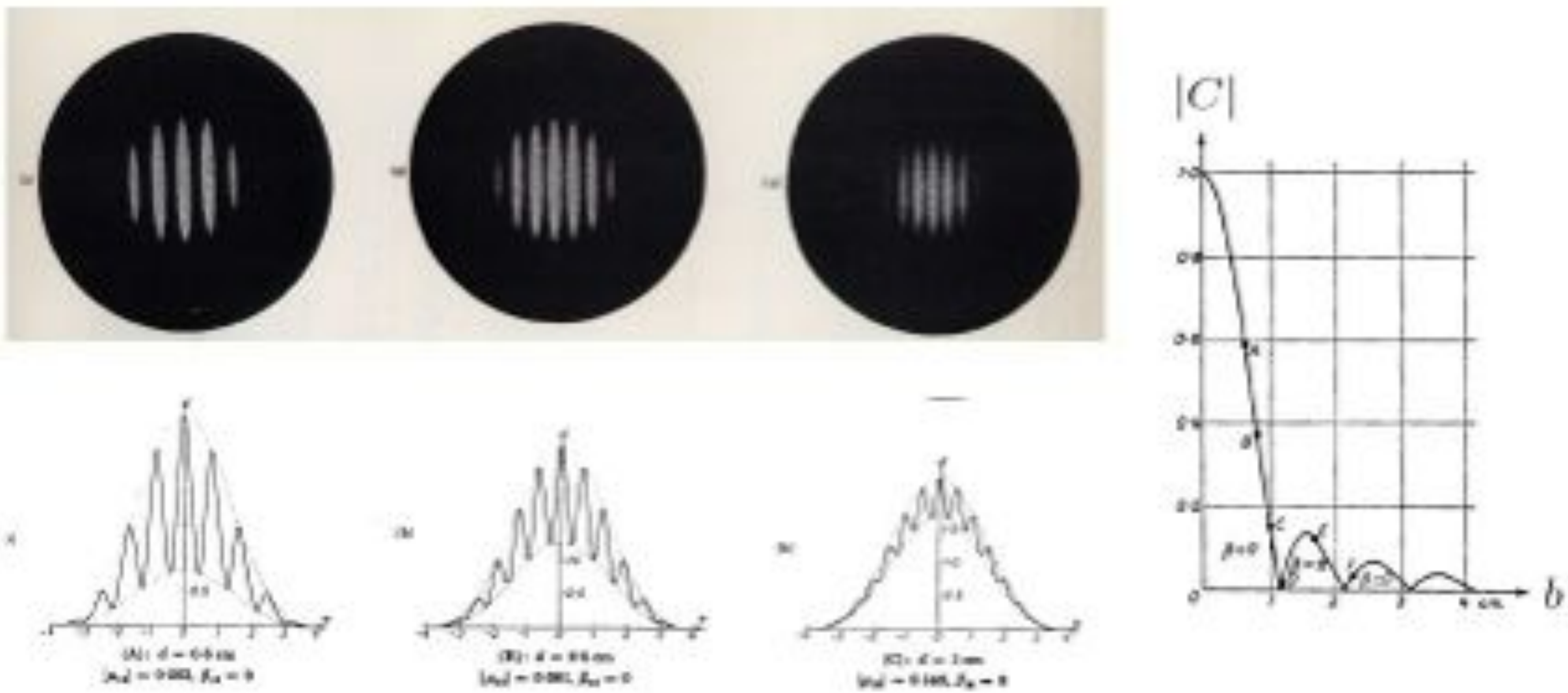


If the source is characterized by a uniform disk light distribution, the corresponding visibility function is given by

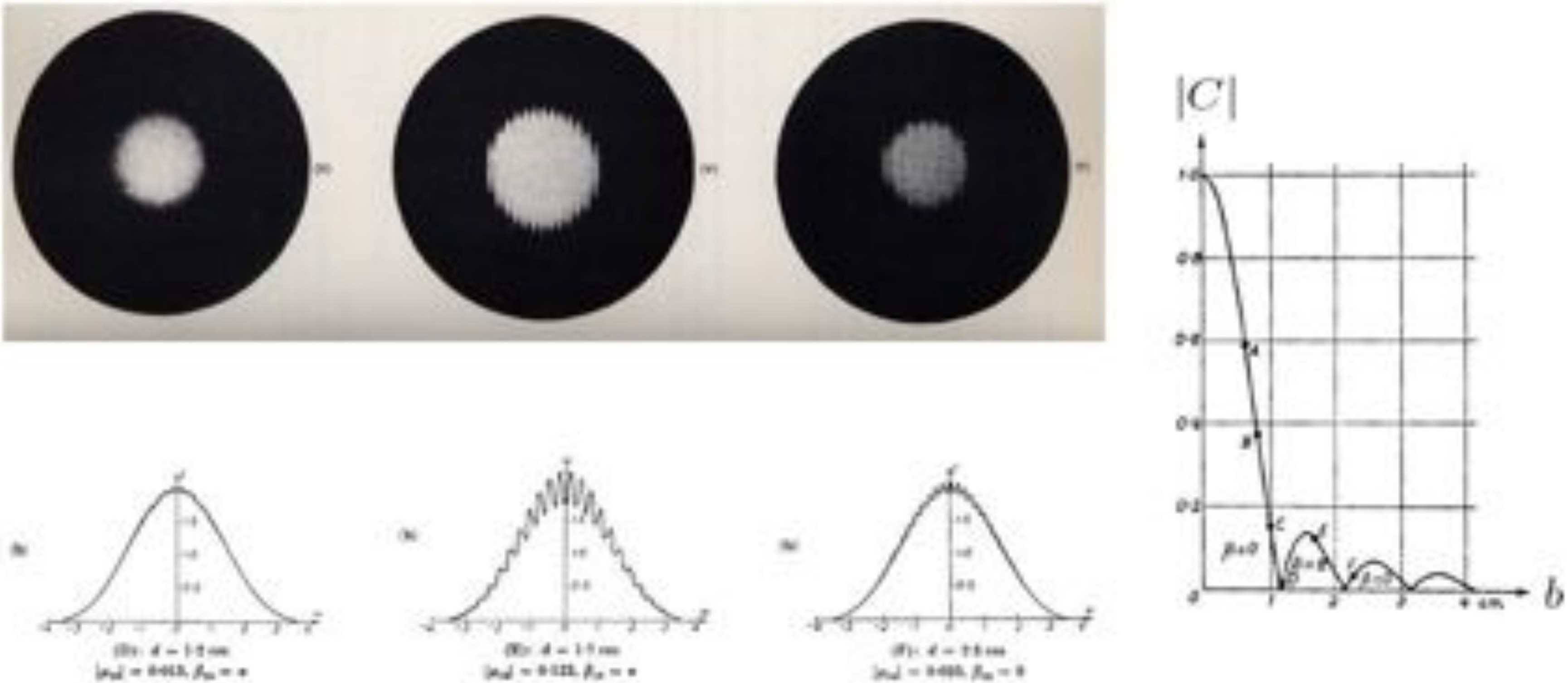
$$\upsilon = \left(\frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}} \right) = \left| \gamma_{12}(0) \right| = TF(I) = \frac{2J_1(\pi\theta_{UD} B / \lambda)}{\pi\theta_{UD} B / \lambda}$$




SW Virginis
M7.3 III semi-regular variable in 1996 & 1997
ϕ_{UD} = 16,53±0,14 mas
χ^2=0,81
$V_{DU}(B) = \left| \frac{2J_1\left(\pi\theta\frac{B}{\lambda}\right)}{\pi\theta\frac{B}{\lambda}} \right|$
1,22/θ
1
0,8
0,6
0,4
0,2
0
0
20
40
60
80
100
Fréquence spatiale (cycles/arcsec)



$$I(x) = I_1 + I_2 + 2\sqrt{I_1 I_2}|C| \cos\left(\frac{bx}{\lambda} + \phi_C\right) \text{ with } |C| = \frac{I_{max} - I_{min}}{I_{max} + I_{min}}$$



$$I(x) = I_1 + I_2 + 2\sqrt{I_1 I_2}|C|\cos\left(\frac{bx}{\lambda} + \phi_C\right) \text{ with } |C| = \frac{I_{\text{max}} - I_{\text{min}}}{I_{\text{max}} + I_{\text{min}}}$$

For the case of the Sun:

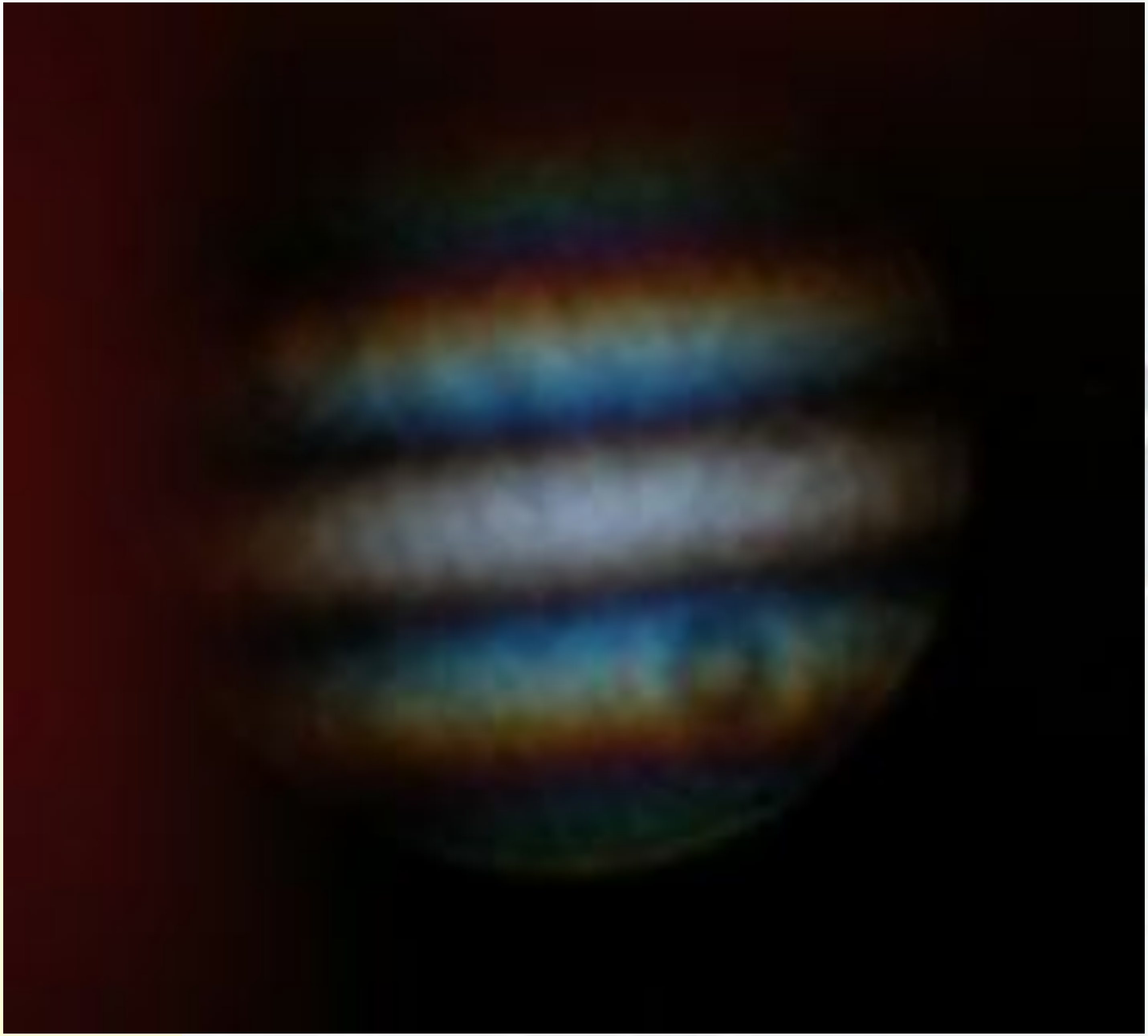
$\vartheta_{UD} = 1.22\lambda / B = 1.22\ 0.55 / B(\mu) = 30' \times 60'' / 206265$

$B(\mu) = 206265 \times 1.22 \times 0.55 / (30 \times 60) = 76.9\ \mu$

$d(\mu) = 7.2$ or $14.4\ \mu$ ➔ $\sigma = 2.44\ \lambda / d = 7.8°$ or $3.9°$

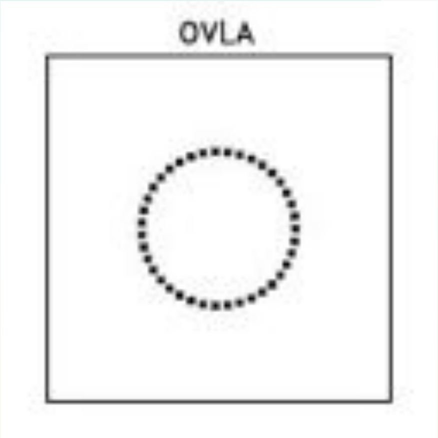
See the masks!

First fringes on the Sun: 9/4/2010



$B = 29.4\mu$
$d = 11.8\mu$

OVLA PSF

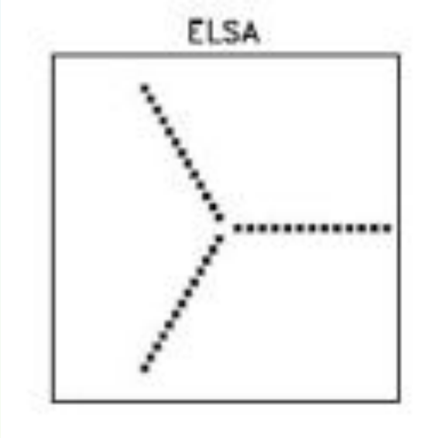


↔ 50μ

. 14μ

OVLA_Sun_2

ELSA PSF

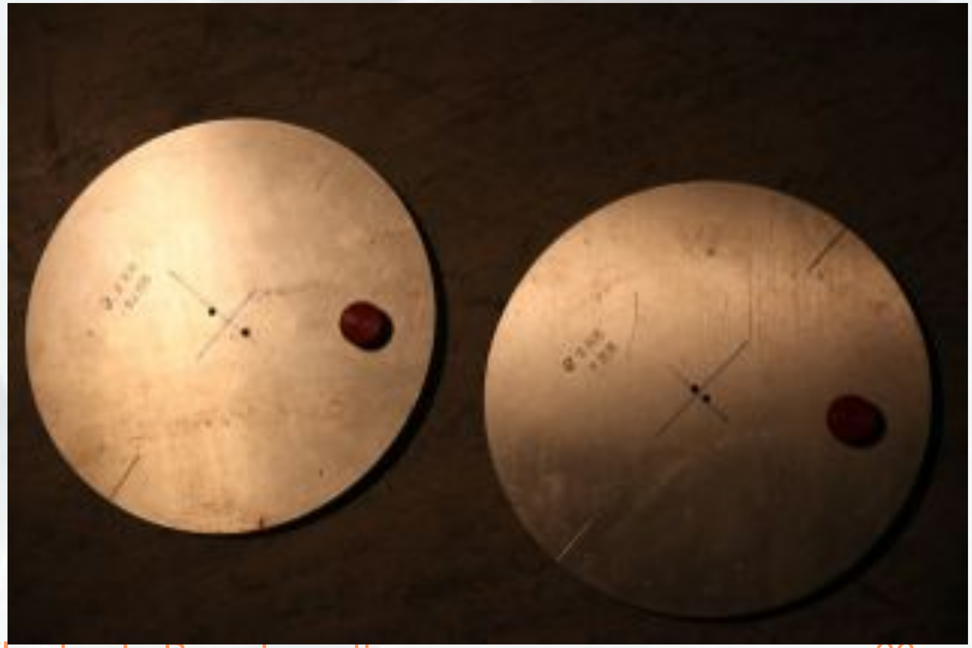


↔ 50μ

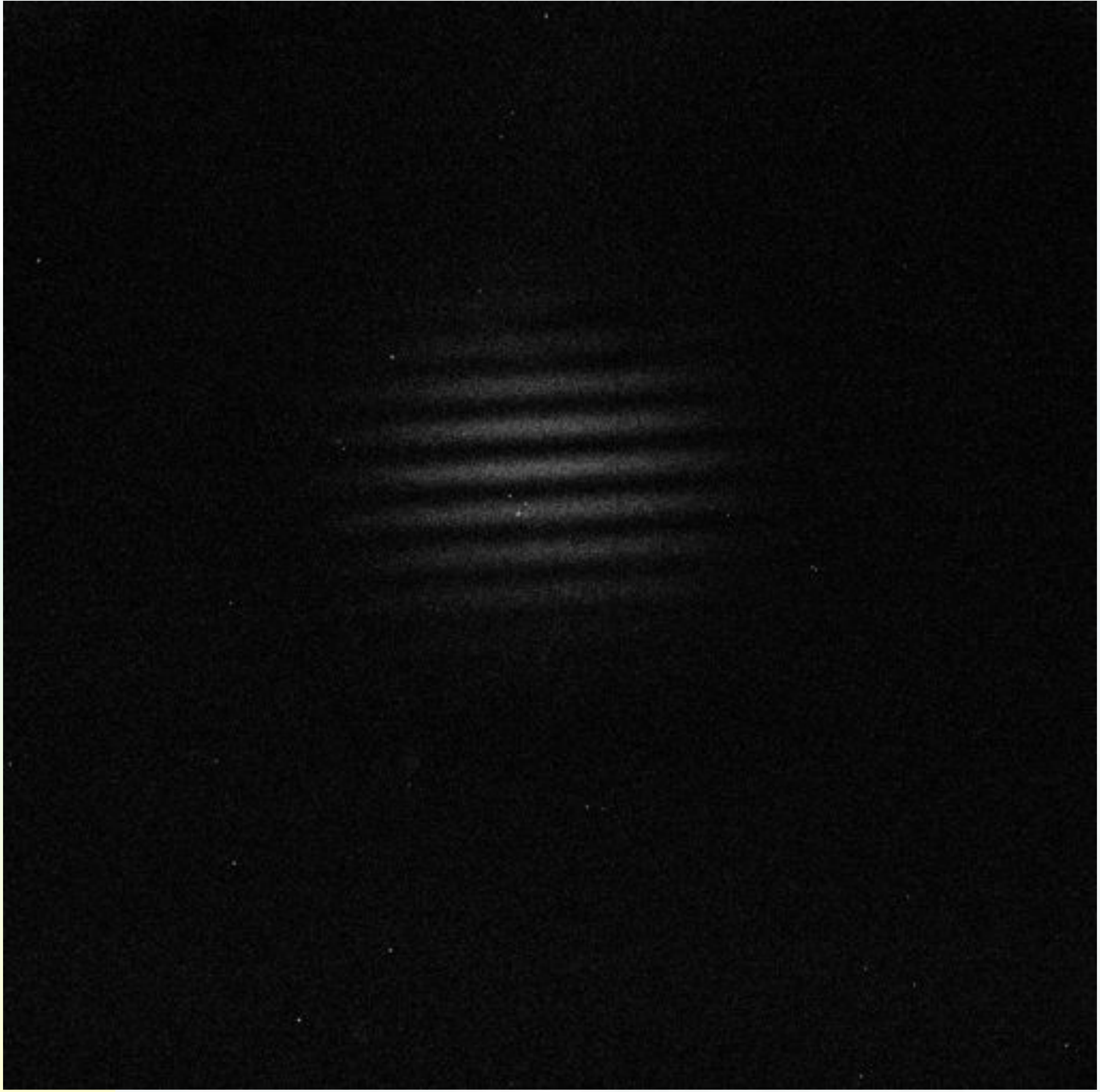
. 14μ

ELSA_Sun_24

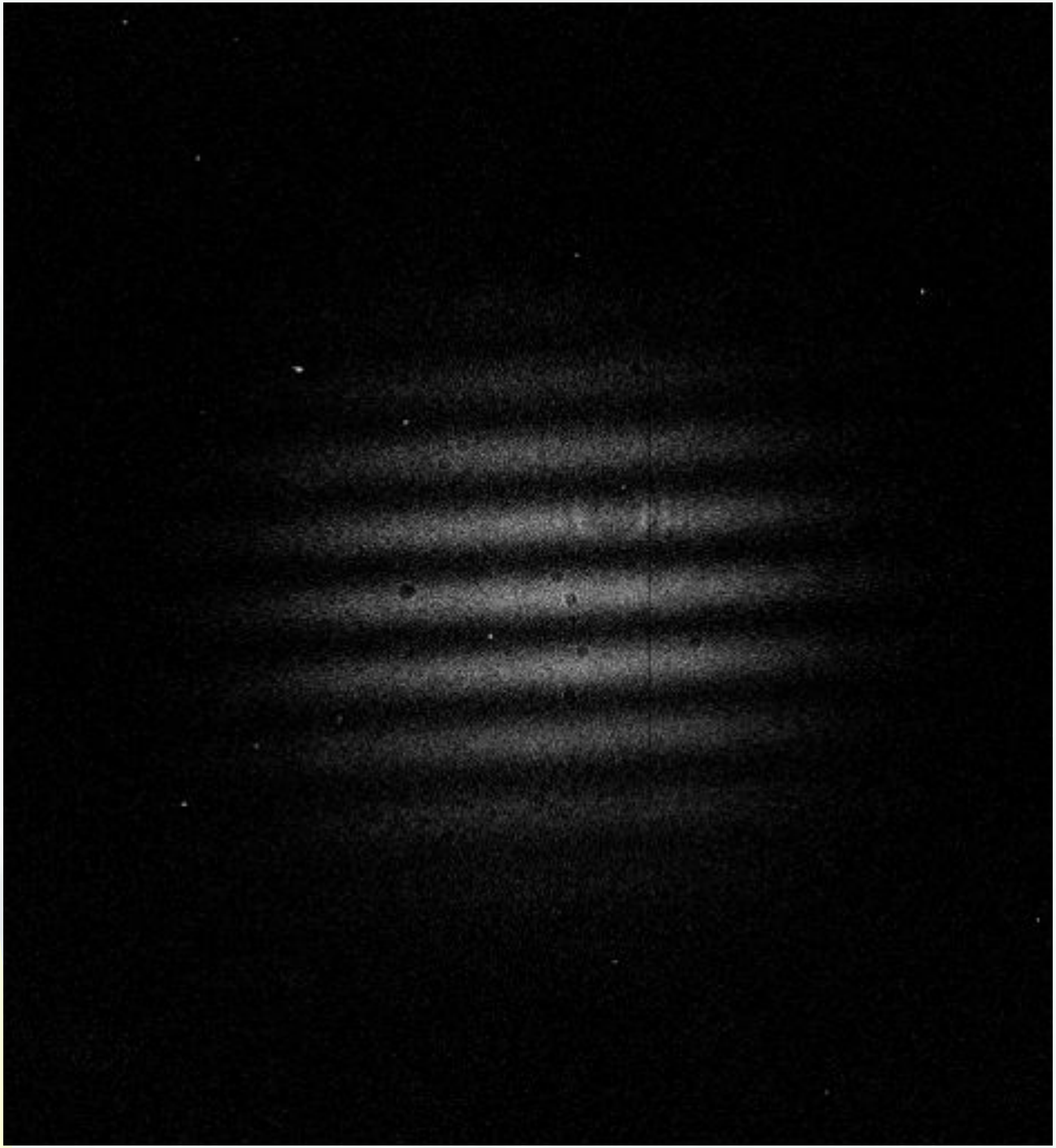
Interferometric observations on 10/4/2010 of Procyon, Mars and Saturn, using the 80cm telescope at Haute-Provence Observatory and adequate masks (coll. with Hervé le Coroller) …



Procyon
B = 12 mm
d = 2 mm



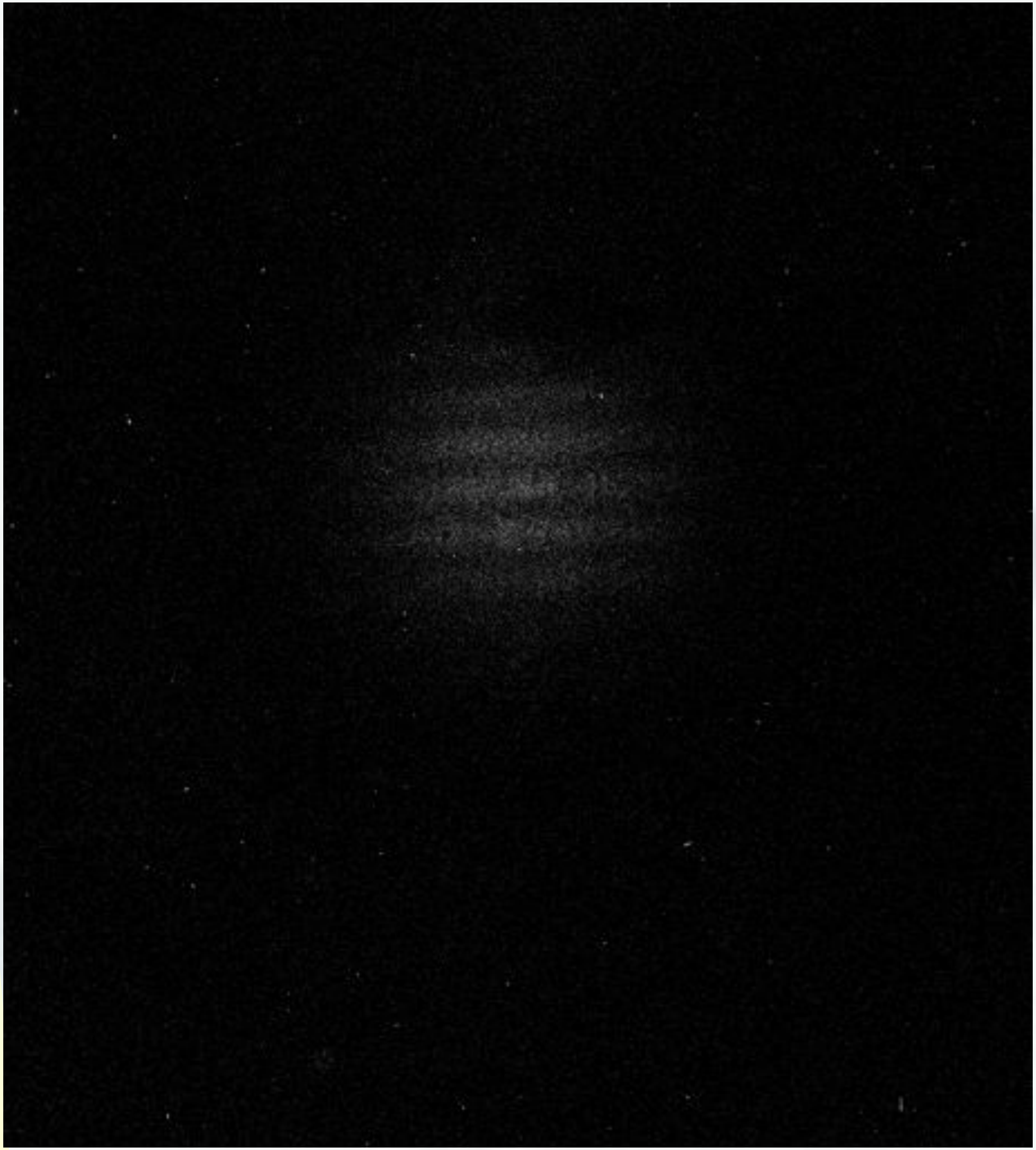
Mars
B = 12 mm
d = 2 mm



Saturn

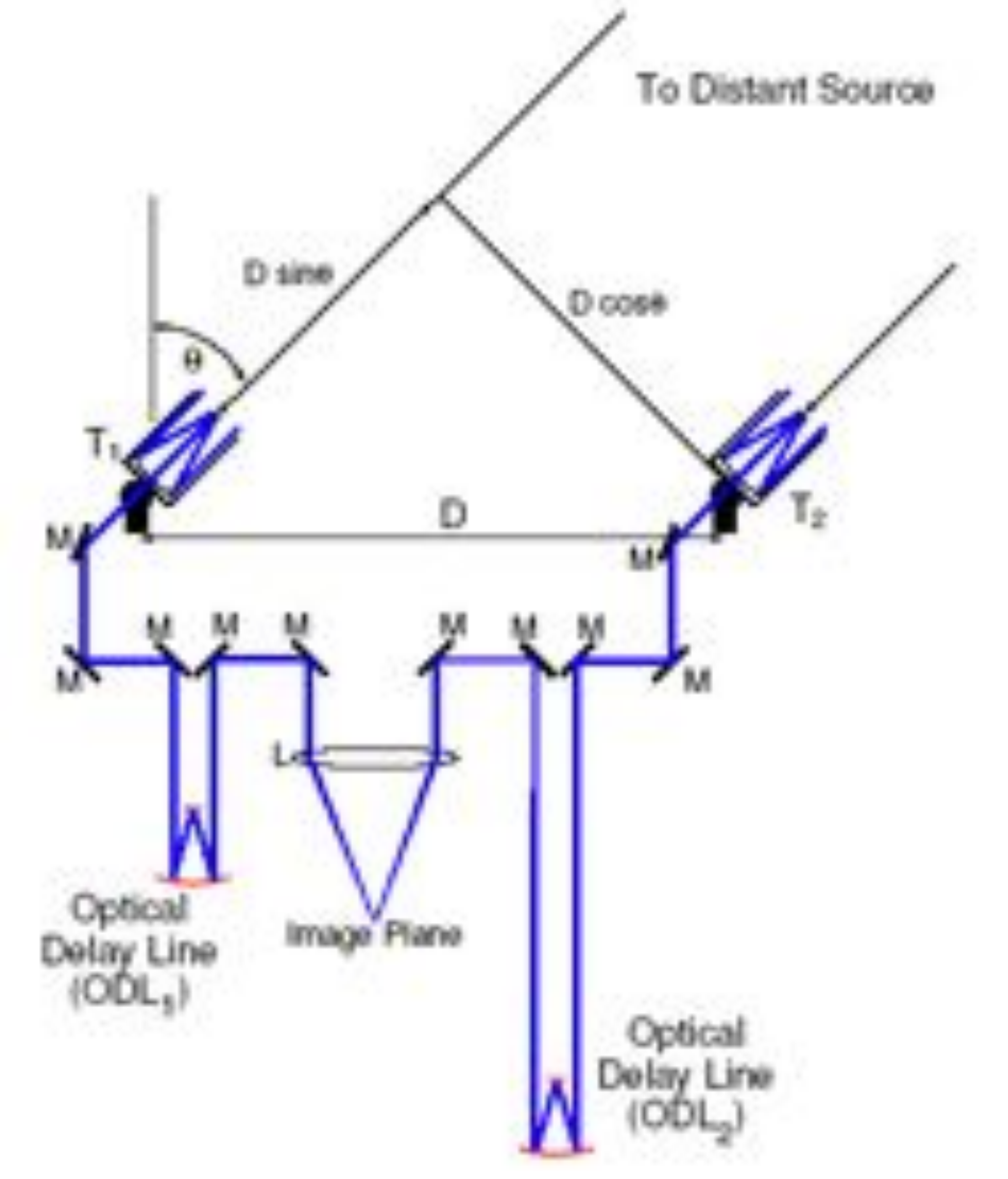
B = 12 mm

d = 2 mm

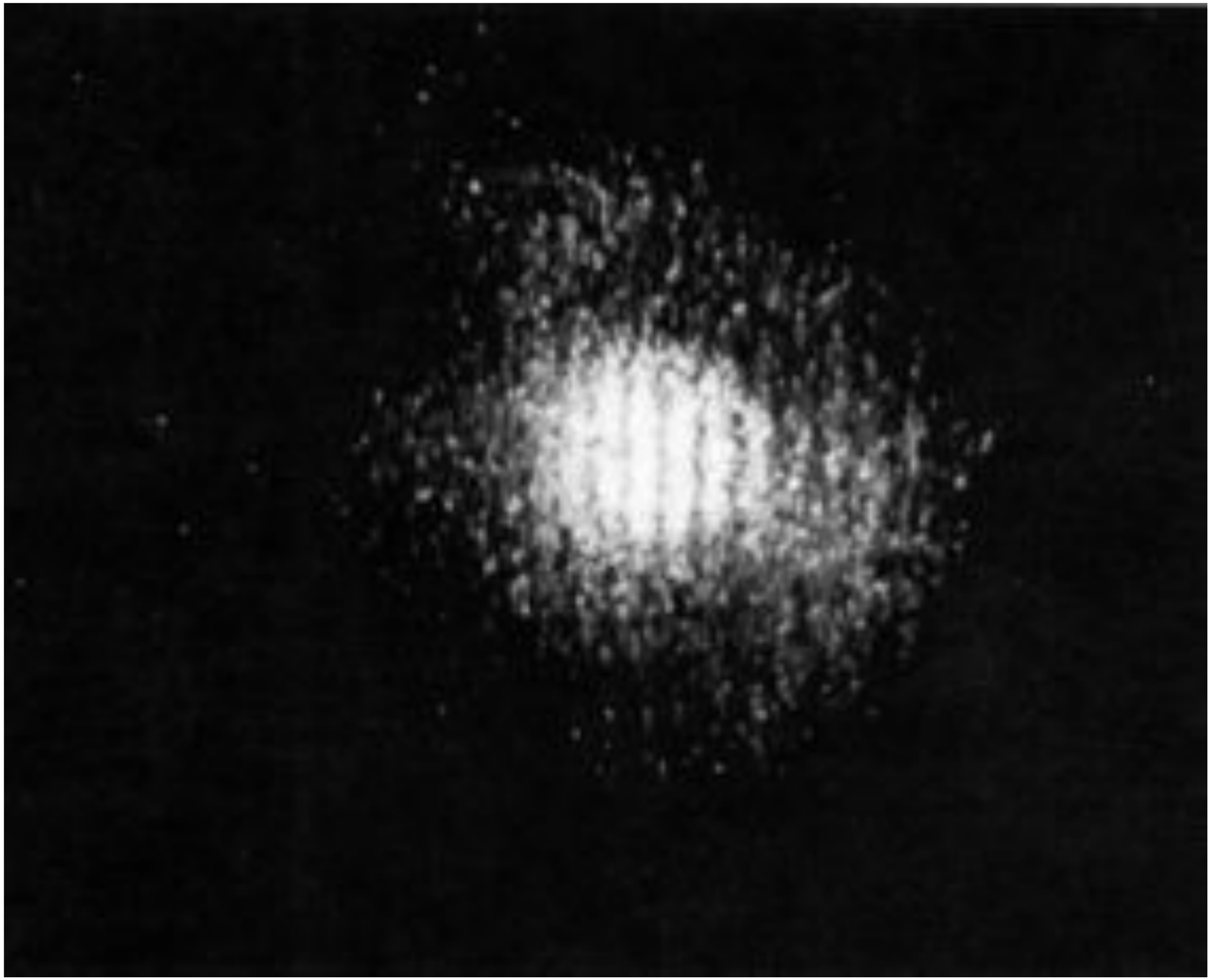


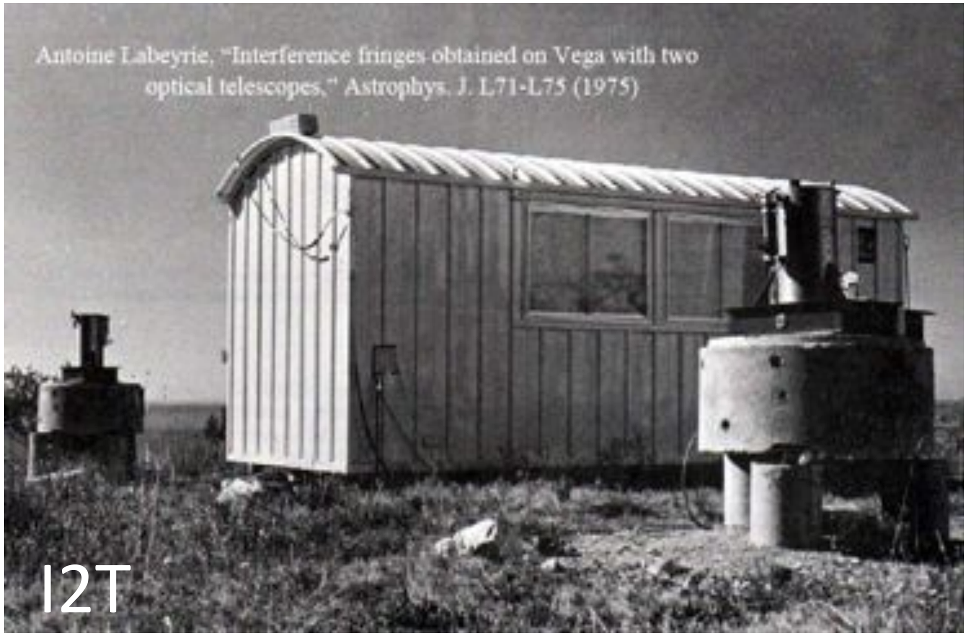
An introduction to optical/IR interferometry

- 6 Some examples of optical interferometers


To Distant Source
D sinθ
D cosθ
θ
T1
T2
D
M
L
Optical
Delay Line
(ODL1)
Image Plane
Optical
Delay Line
(ODL2)

First fringes with I2T




Antoine Labeyrie, "Interference fringes obtained on Vega with two optical telescopes," Astrophys. J. L71-L75 (1975)
I2T


GI2T

An introduction to optical/IR interferometry

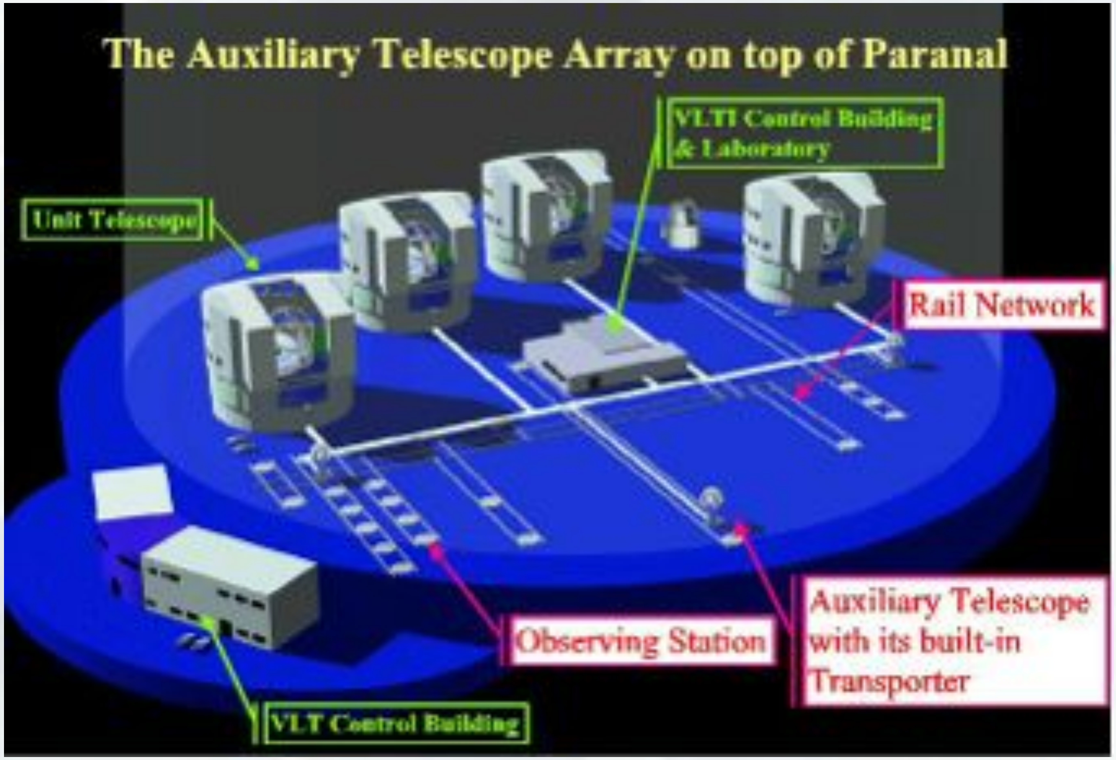
- 7 Some results

Star	Spectral type	Luminosity class	Angular diameter $\times 10^{-3}$ seconds of arc
α Boo	K2	Giant	20
α Tau	K5	Giant	20
α Sco	M1–M2	Super-giant	40
β Peg	M2	Giant	21
o Cet	M6e	Giant	47
α Ori	M1–M2	Super-giant variable	34→47

Table 2.1. Stars measured with Michelson's interferometer. From Pease (1931).

An introduction to optical/IR interferometry

- 6 Some examples of optical interferometers



http://www.aeos.ulg.ac.be/HARI/

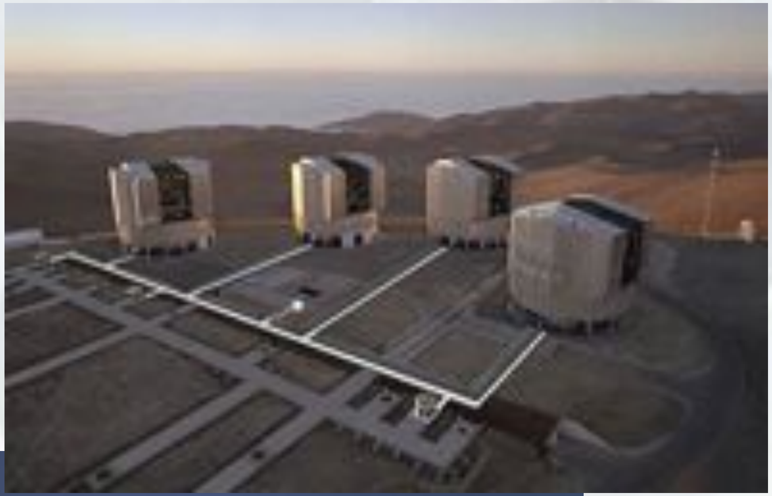
An introduction to optical/IR interferometry

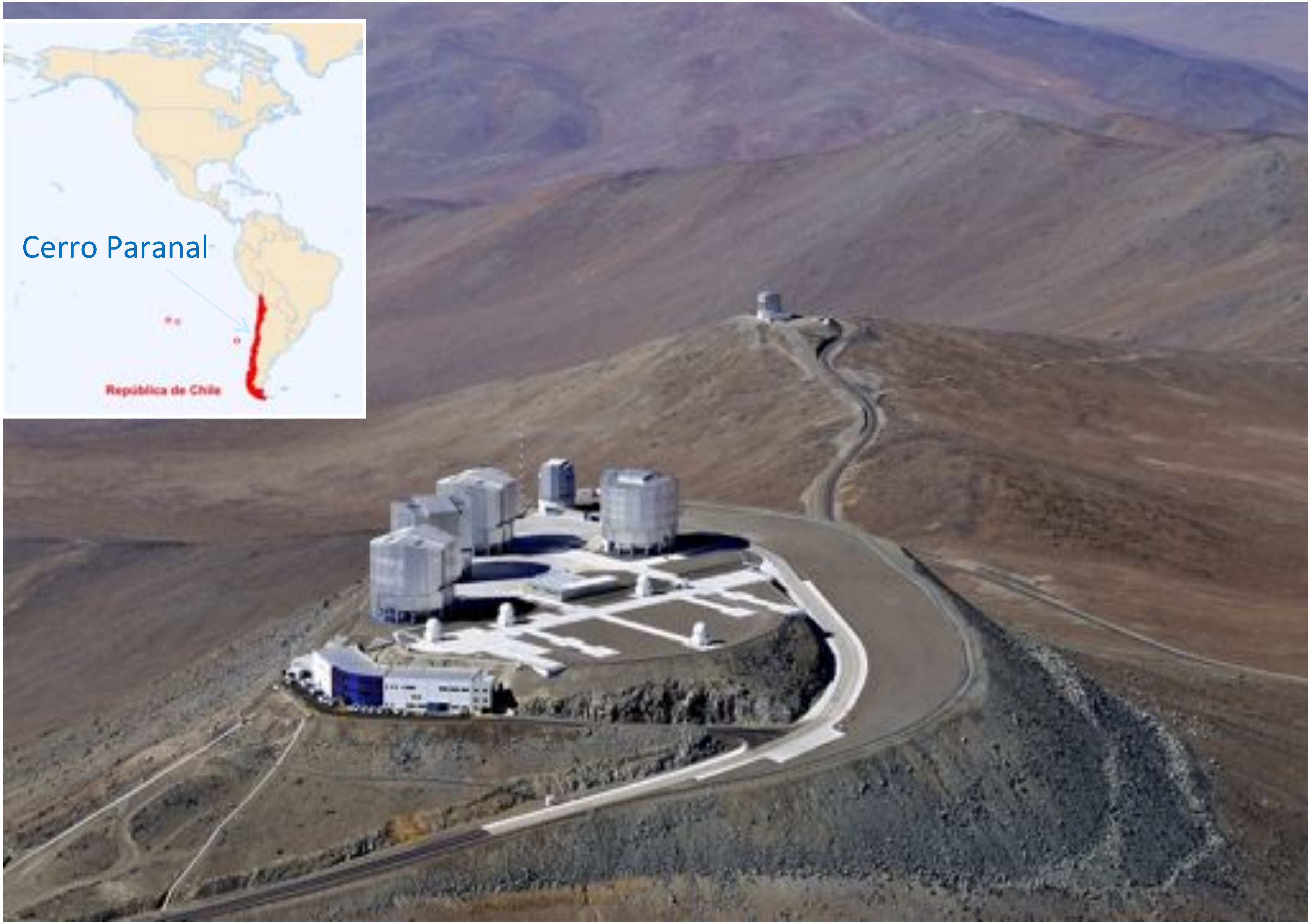
6 Some examples of optical interferometers

Interferometry to-day is:

Very Large Telescope Interferometer (VLTI)

- 4 x 8.2m UTs
- 4 x 1.8m ATs
- Max. Base: 200m




Cerro Paranal
República de Chile

An introduction to optical/IR interferometry

- 6 Some examples of optical interferometers

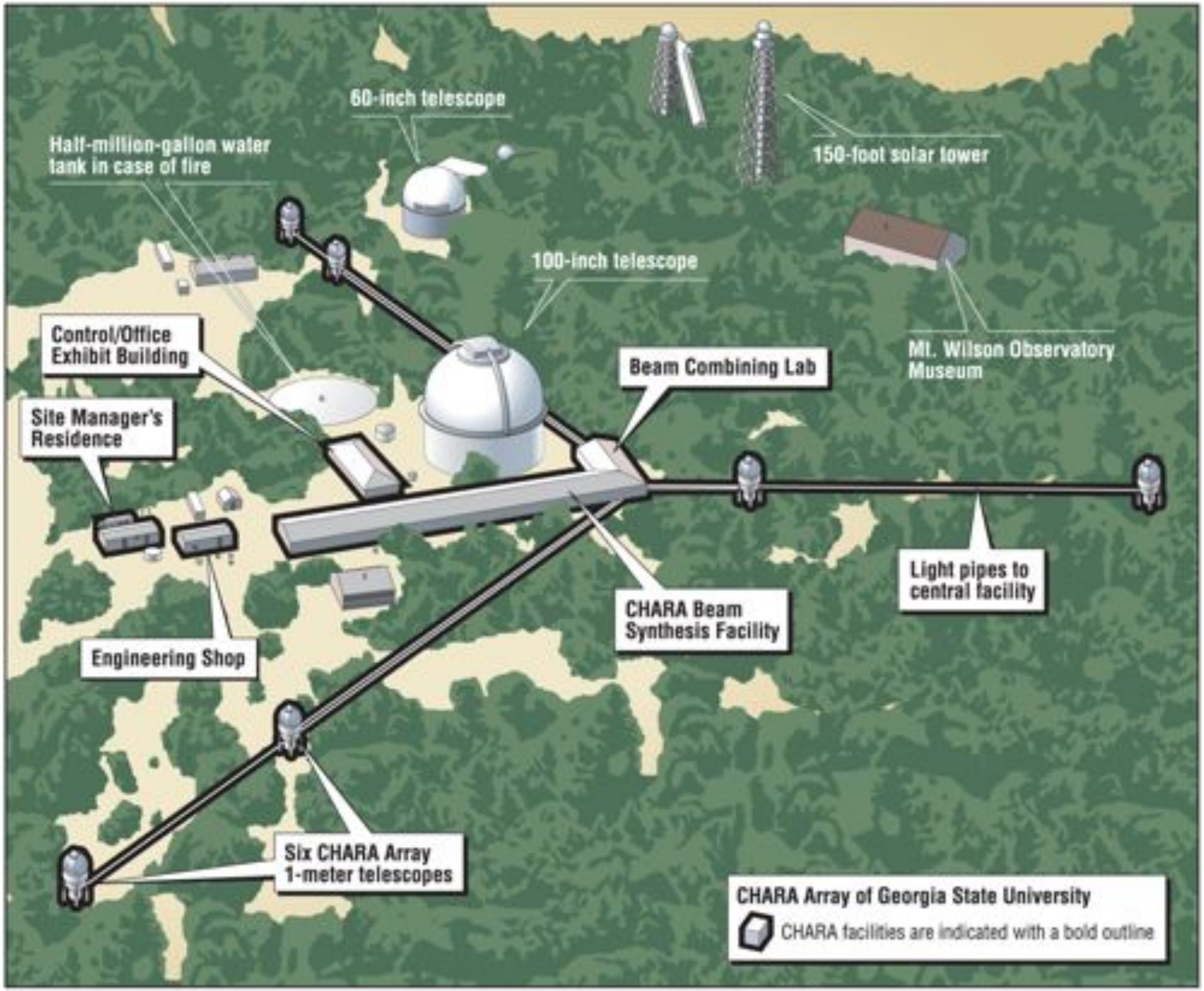
An introduction to optical/IR interferometry

- 6 Some examples of optical interferometers

Interferometry to-day is also:

The CHARA interferometer

• 6 x 1m telescopes
• Max. Base: 330m


60-inch telescope
Half-million-gallon water tank in case of fire
150-foot solar tower
100-inch telescope
Control/Office Exhibit Building
Beam Combining Lab
Mt. Wilson Observatory Museum
Site Manager's Residence
Light pipes to central facility
CHARA Beam Synthesis Facility
Engineering Shop
Six CHARA Array 1-meter telescopes
CHARA Array of Georgia State University
CHARA facilities are indicated with a bold outline

An introduction to optical/IR interferometry

- 6 Some examples of optical interferometers

Interferometry to-day

is also:

Keck interferometer

• 2 x 10m telescopes
• Base: 85m

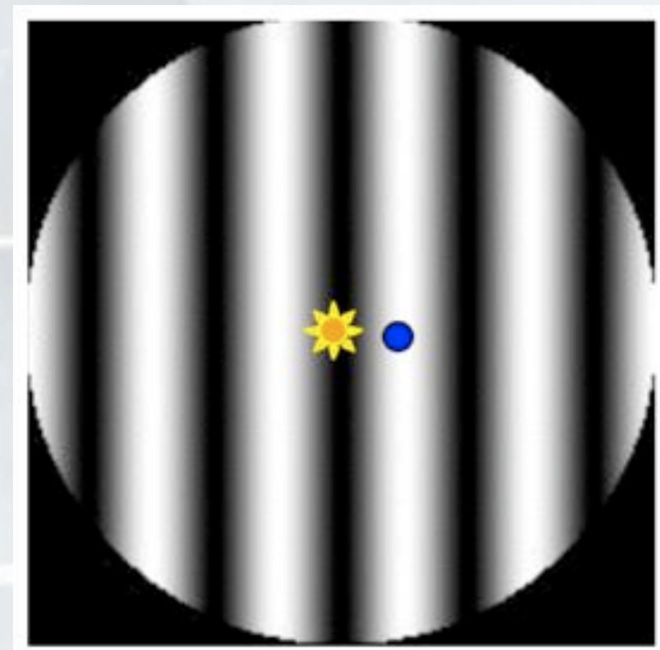
An introduction to optical/IR interferometry

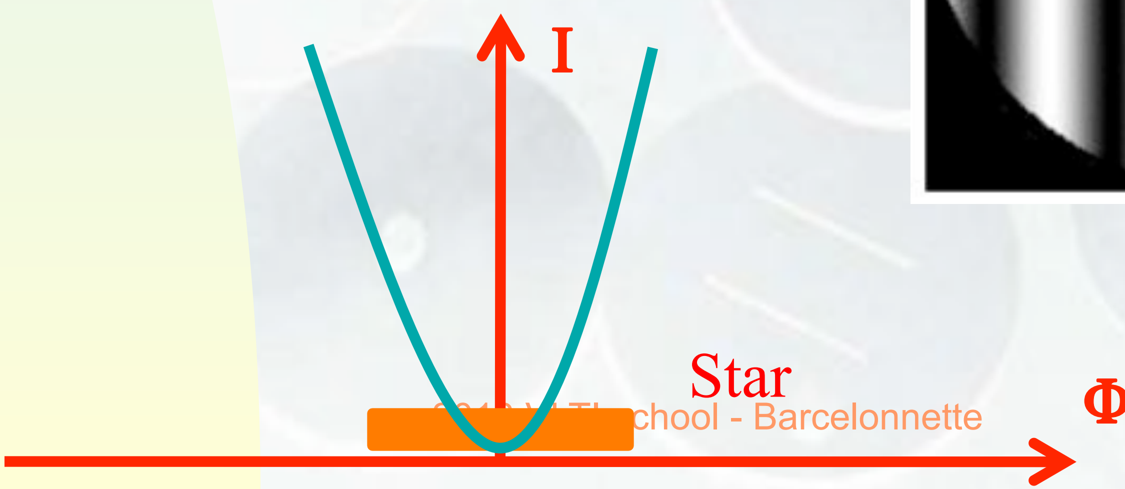
- 6 Some examples of optical interferometers

Interferometry to-day is also:

Nullin interferometry

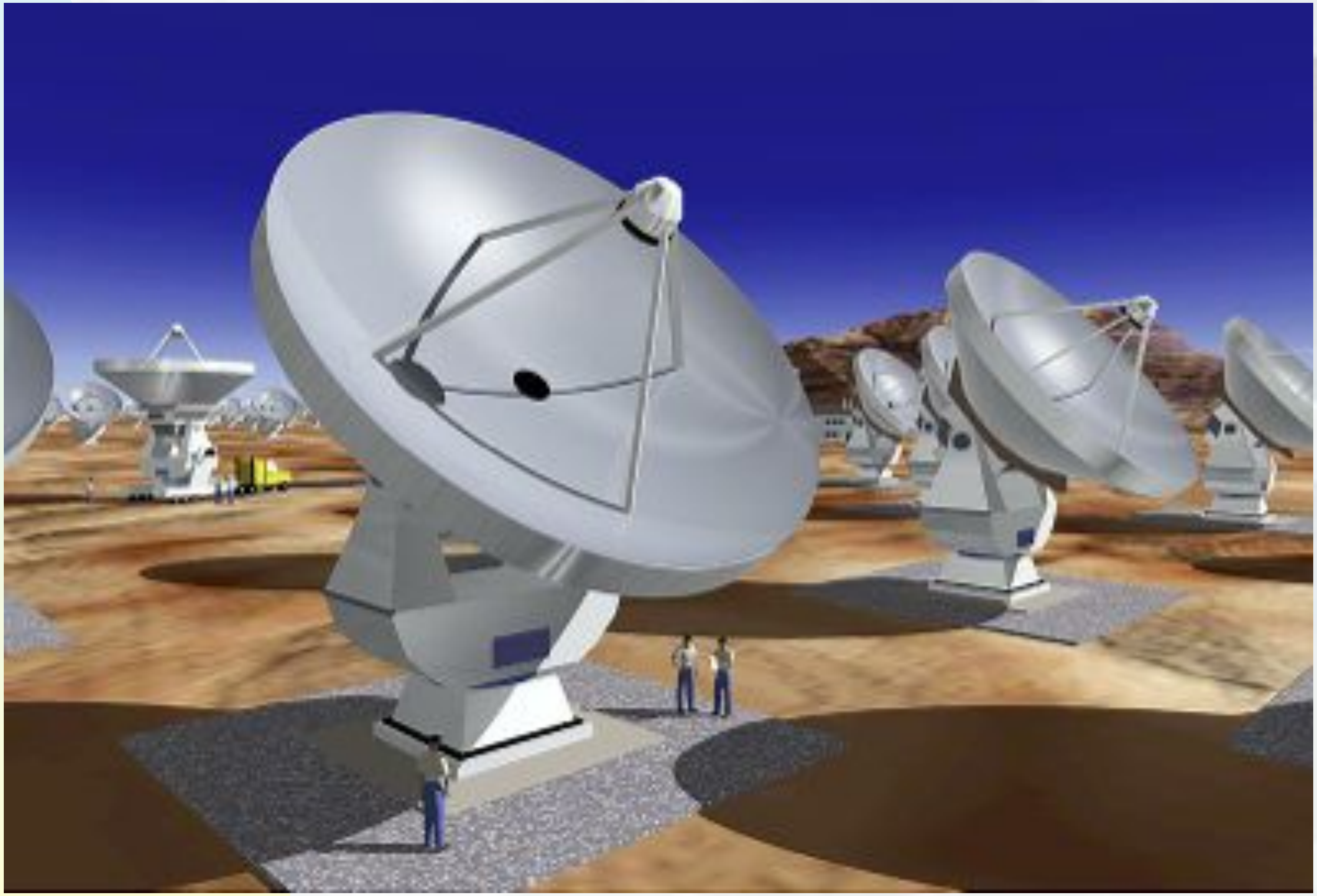
- Measurement of « stellar leakage »
- Allow to resolve stars with a a small size interferometer





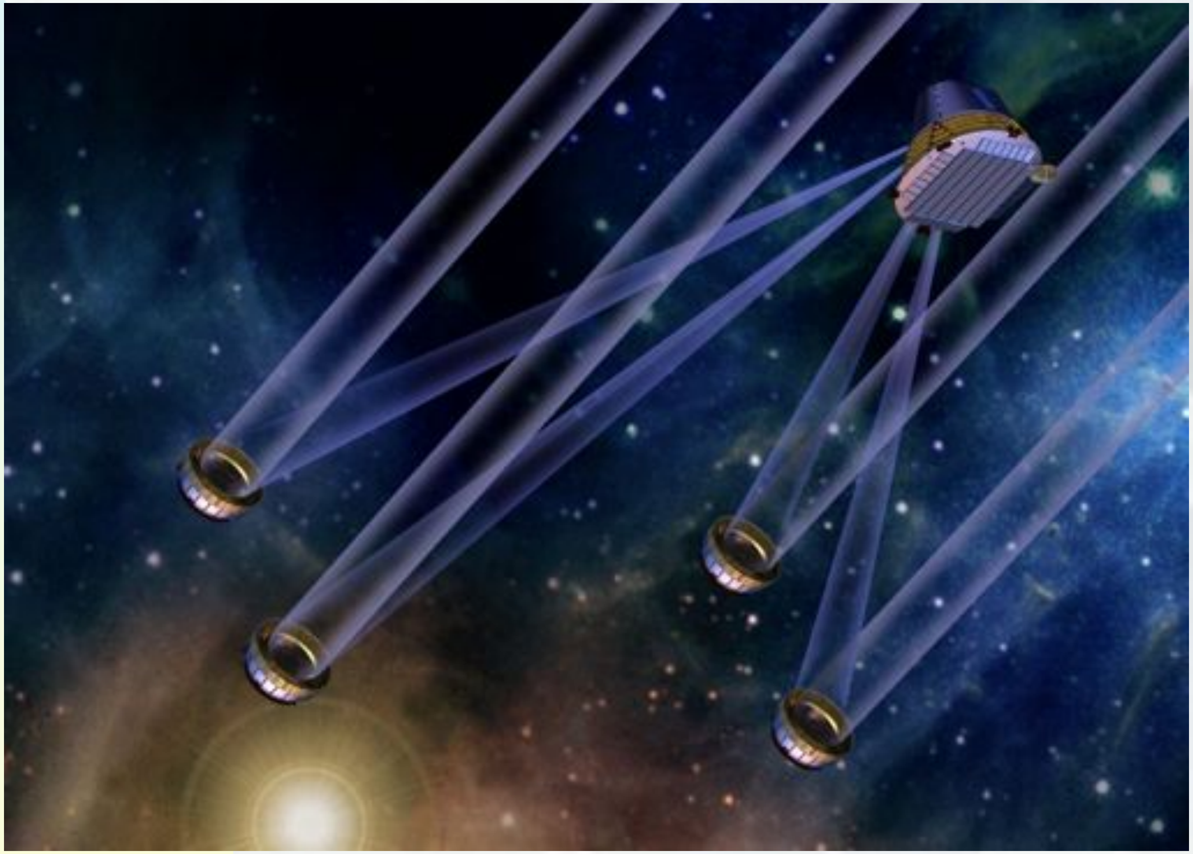
An introduction to optical/IR interferometry

- 6 Other examples of interferometers: ALMA



An introduction to optical/IR interferometry

- 6 Other examples of interferometers: DARWIN



An introduction to optical/IR interferometry

8 Three important theorems … and some applications

8.1 The fundamental theorem

8.2 The convolution theorem

8.3 The Wiener-Khintchin theorem

Réf.: P. Léna; Astrophysique: méthodes physiques de l’observation (Savoirs Actuels / CNRS Editions)

An introduction to optical/IR interferometry

8.1 The fundamental theorem

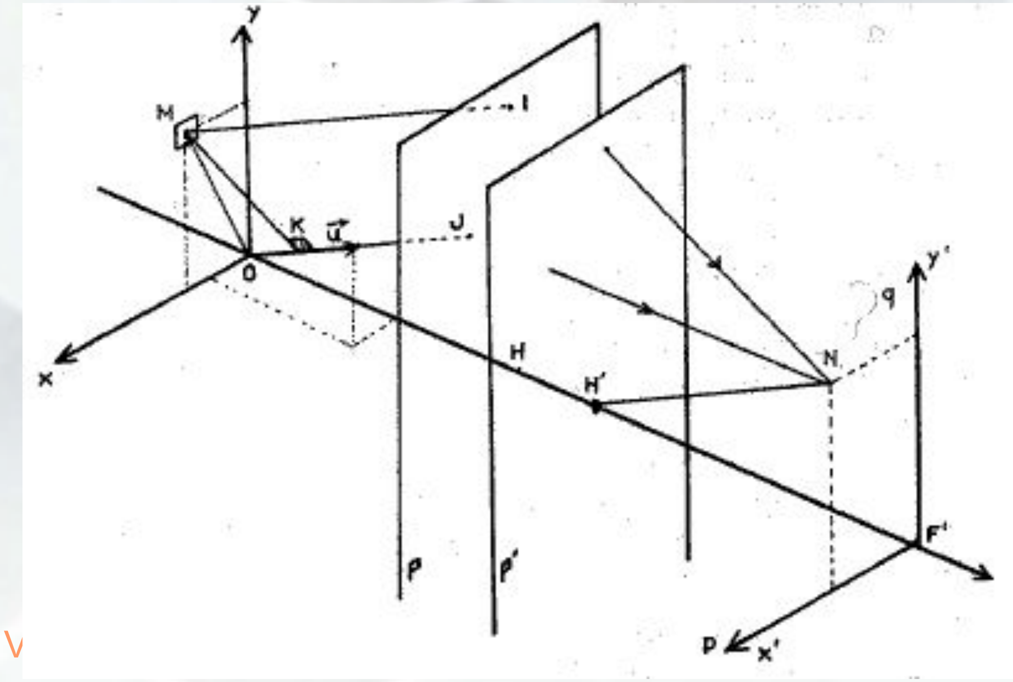
a(p,q) = TF_(A(x,y))(p,q),

$$a(p,q) = \int_{R^2} A(x,y) \exp\left[-i2\pi\left(px+qy\right)\right] dxdy,$$

with

p = x' /(λ f)

q = y' /(λ f)



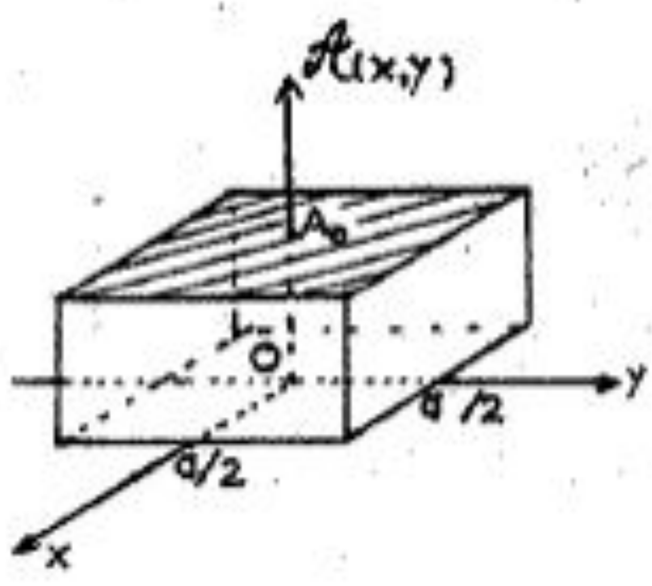
An introduction to optical/IR interferometry

8.1 The fundamental theorem

The distribution of the complex amplitude a(p,q) in the focal plane is given by the Fourier transform of the distribution of the complex amplitude A(x,y) in the entrance pupil plane.

8.1 The fundamental theorem

Application: Point Spread Function determination



$$A(x,y) = A_0 \, P_0(x,y), \qquad (8.1.1)$$

$$P_0(x,y) = \Pi(x/a)\,\Pi(y/a). \qquad (8.1.2)$$

An introduction to optical/IR interferometry

8.1 The fundamental theorem

$$a(p,q) = TF\_[A(x,y)](p,q) = \int_{-a/2}^{a/2} \int_{-a/2}^{a/2} A_0 \exp[-i2\pi(px+qy)]dxdy \qquad (8.1.3)$$

$$a(p,q) = A_0 \int_{-a/2}^{a/2} \exp[-i2\pi px]dx \int_{-a/2}^{a/2} \exp[-i2\pi qy]dy \qquad (8.1.4)$$

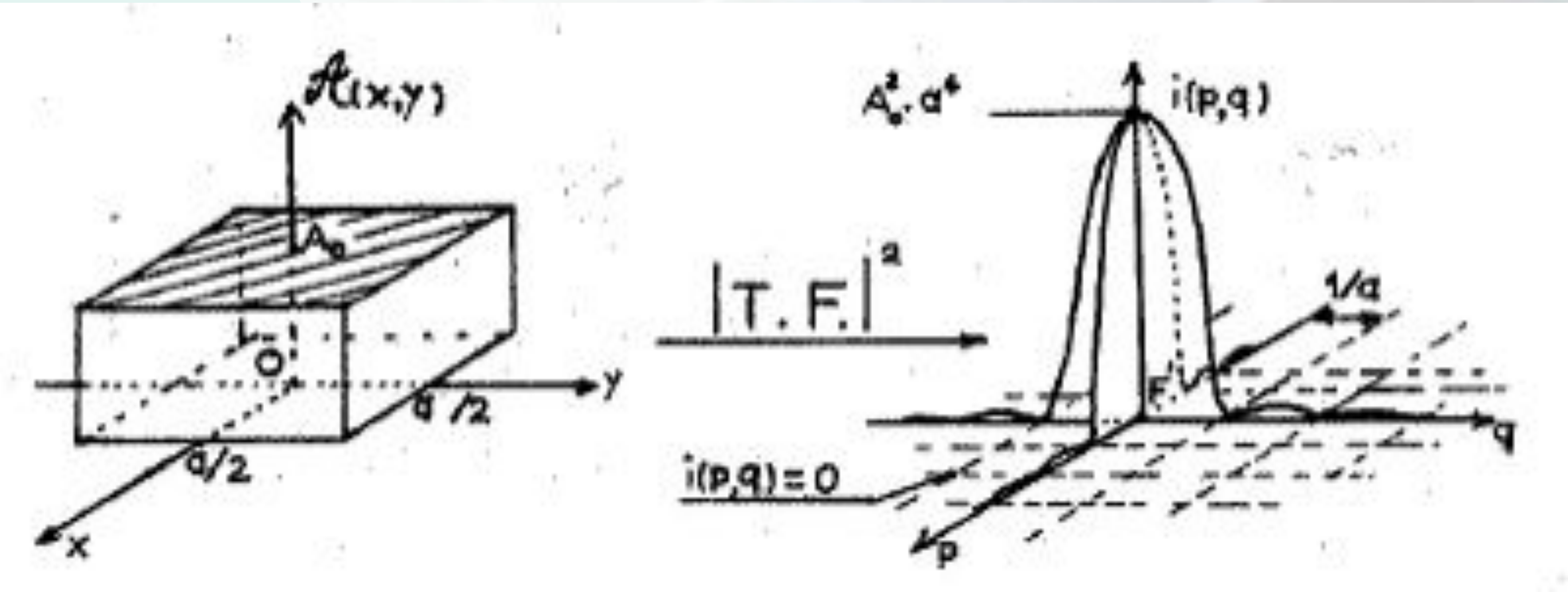
$$a(p,q) = A_0\, a^2\, [\sin(\pi pa) / (\pi pa)]\, [\sin(\pi qa) / (\pi qa)]. \qquad (8.1.5)$$

$$i(p,q) = a(p,q)\, a^*(p,q) = |a(p,q)|^2 = |h(p,q)|^2 =$$
$$= i_0\, a^4\, [\sin(\pi pa) / (\pi pa)]^2\, [\sin(\pi qa) / (\pi qa)]^2. \qquad (8.1.6)$$

An introduction to optical/IR interferometry

8.1 The fundamental theorem

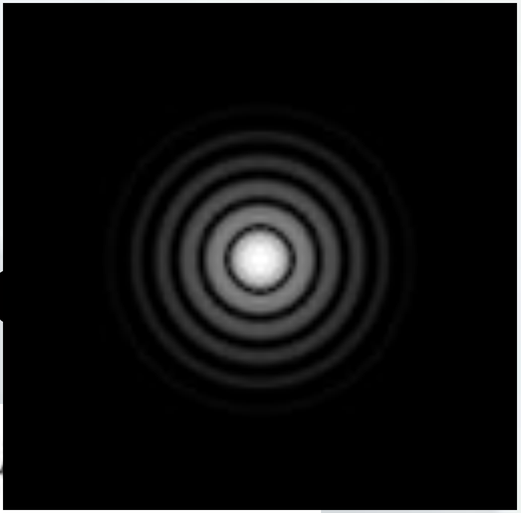
Application: Point Spread Function determination



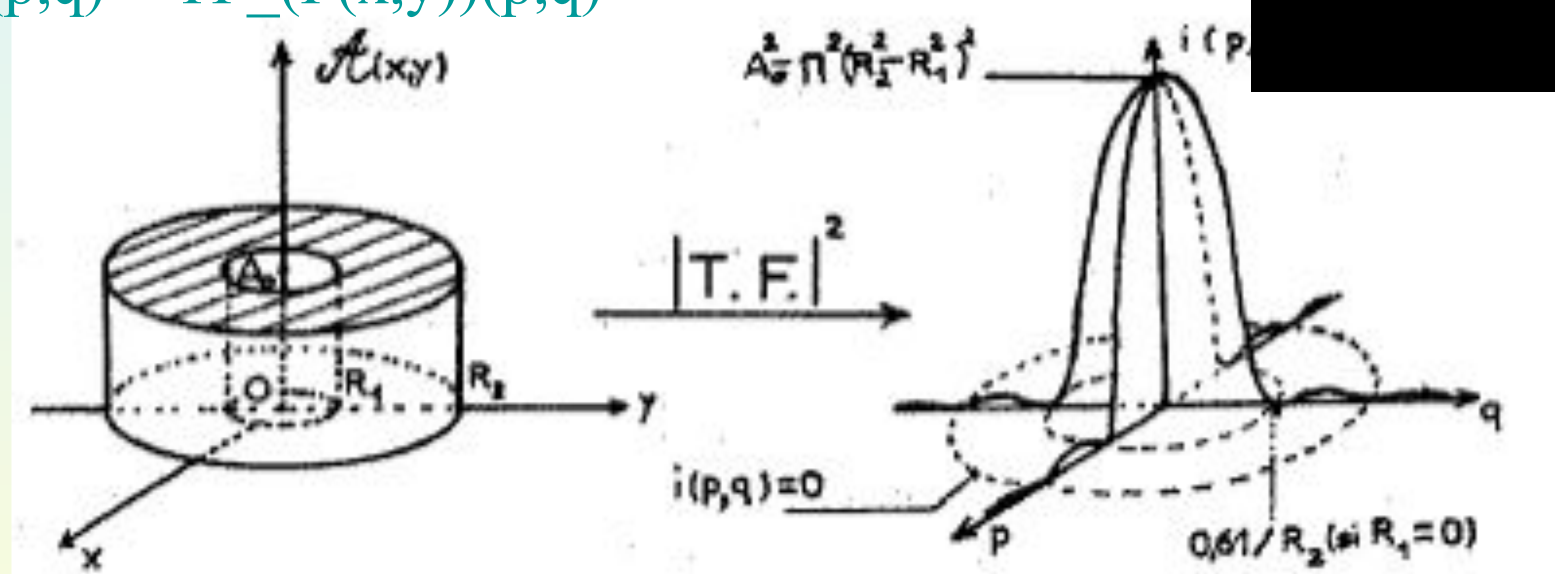
$\Delta p = \Delta x' / (\lambda f); \Delta q = \Delta y' / (\lambda f) = 2/a \rightarrow \Delta\phi_{x'} = \Delta\phi_{y'} = 2\lambda/a$ (8.1.7)

8.1 The fundamental theorem

Application: Point Spread Function det



$h(p,q) = TF\_(P(x,y))(p,q)$

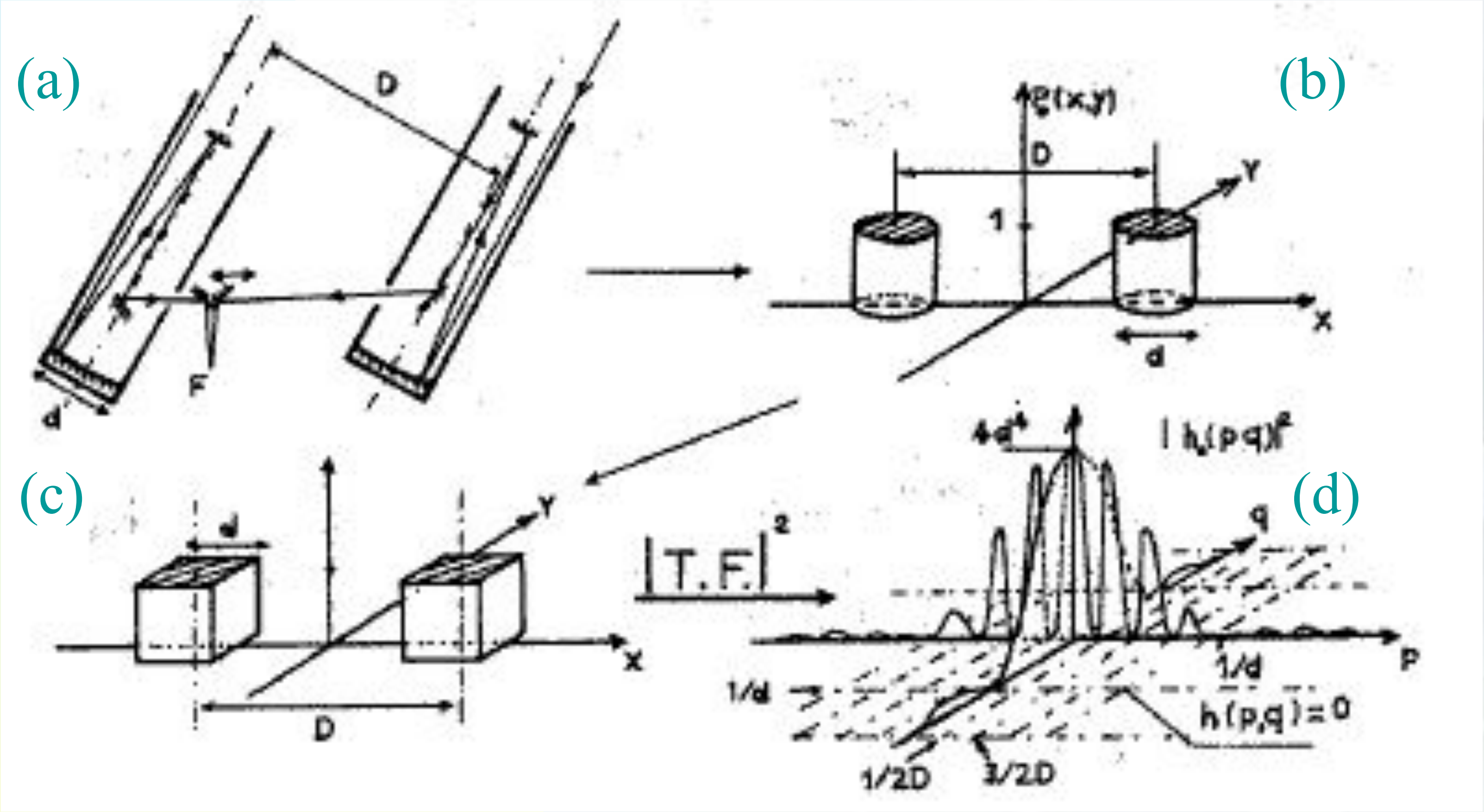


$$i(\rho') = |a(\rho')|^2 = (A_0\,\pi)^2\,[R_2^2\;2\,J_1(Z_2)/Z_2\; - R_1^2\;2\,J_1(Z_1)/Z_1]^2, \quad (8.1.8)$$

with $Z_2 = 2\pi\,R_2\,\rho' / (\lambda f)$ and $Z_1 = 2\pi\,R_1\,\rho' / (\lambda f)$. (8.1.9)

An introduction to optical/IR interferometry

8.1 The fundamental theorem: 2 telescope interferometer



Two coupled optical telescopes: simplified optical scheme (a). Distribution of the complex amplitude for the case of two circular (b) or square (c) apertures and corresponding impulse response (d).

An introduction to optical/IR interferometry

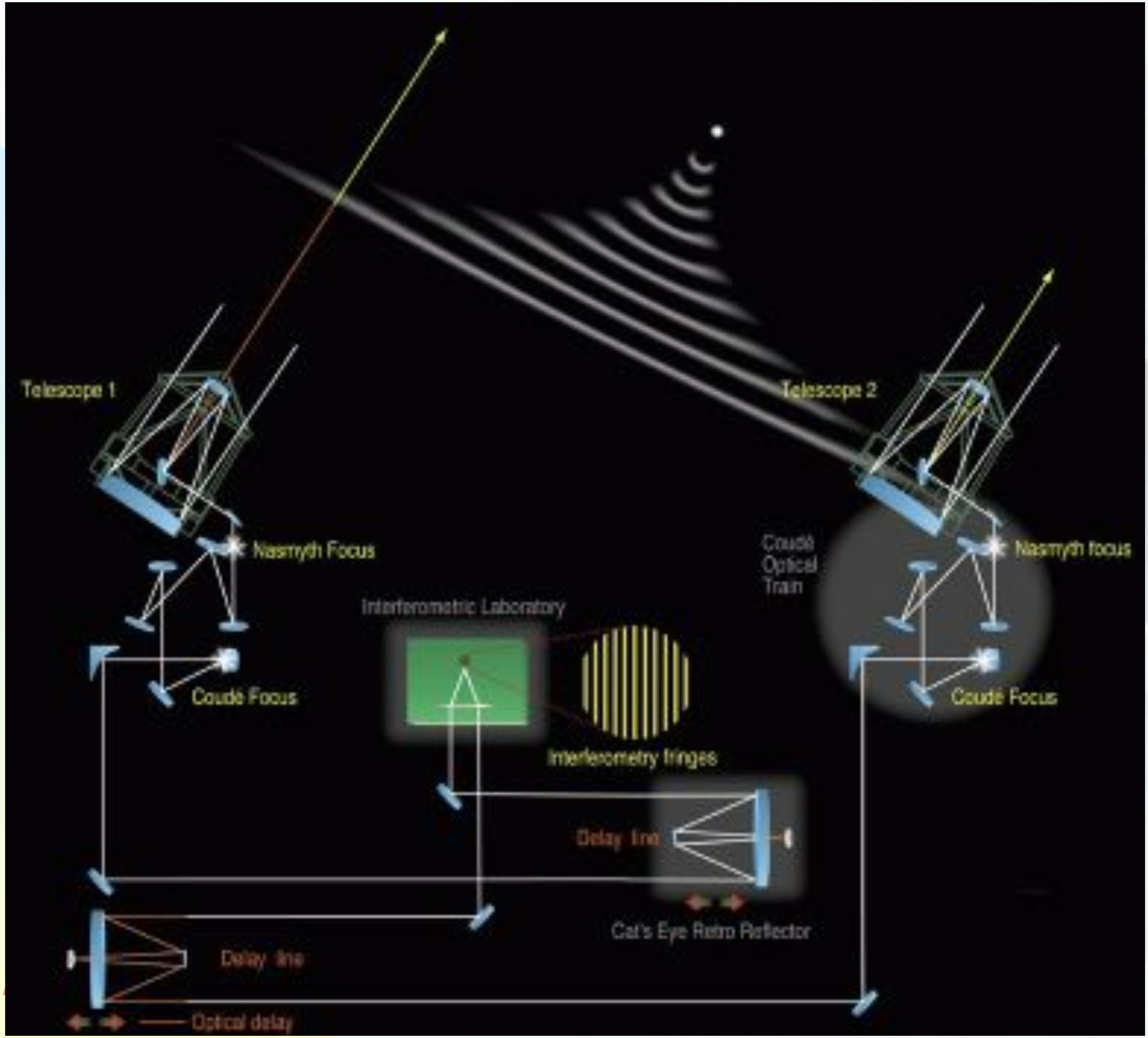
8.1 The fundamental theorem: 2 telescope interferometer

$$h(p,q) = TF(P(x,y)(p,q) = \int_{R^2} P(x,y)\exp\left[-i2\pi(px+qy)\right]dxdy \qquad (8.1.10)$$

$$\begin{aligned}
h(p,q) &= TF(P_0(x+D/2)+P_0(x-D/2))(p,q) = \\
&TF(P_0(x+D/2))(p,q)+TF(P_0(x-D/2))(p,q) = \\
&\exp(i\pi D)\,TF(P_0(x))(p,q)+\exp(-i\pi D)\,TF(P_0(x))(p,q) = \\
&(\exp(i\pi D)+\exp(-i\pi D))\,TF(P_0(x))(p,q) = \\
&2\cos(\pi D)\,TF(P_0(x))(p,q)
\end{aligned} \qquad (8.1.11)$$

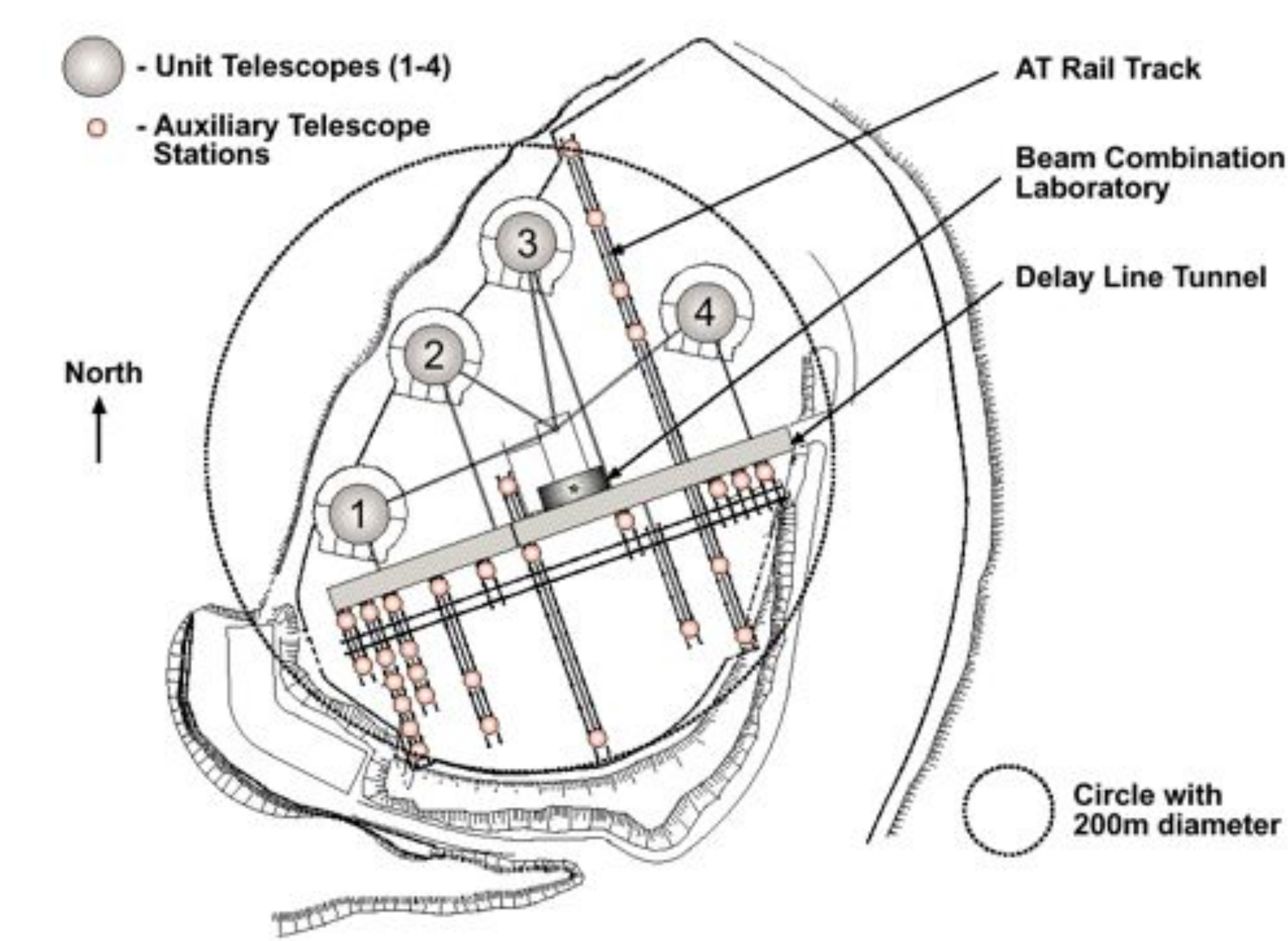
For the particular case of two square apertures:

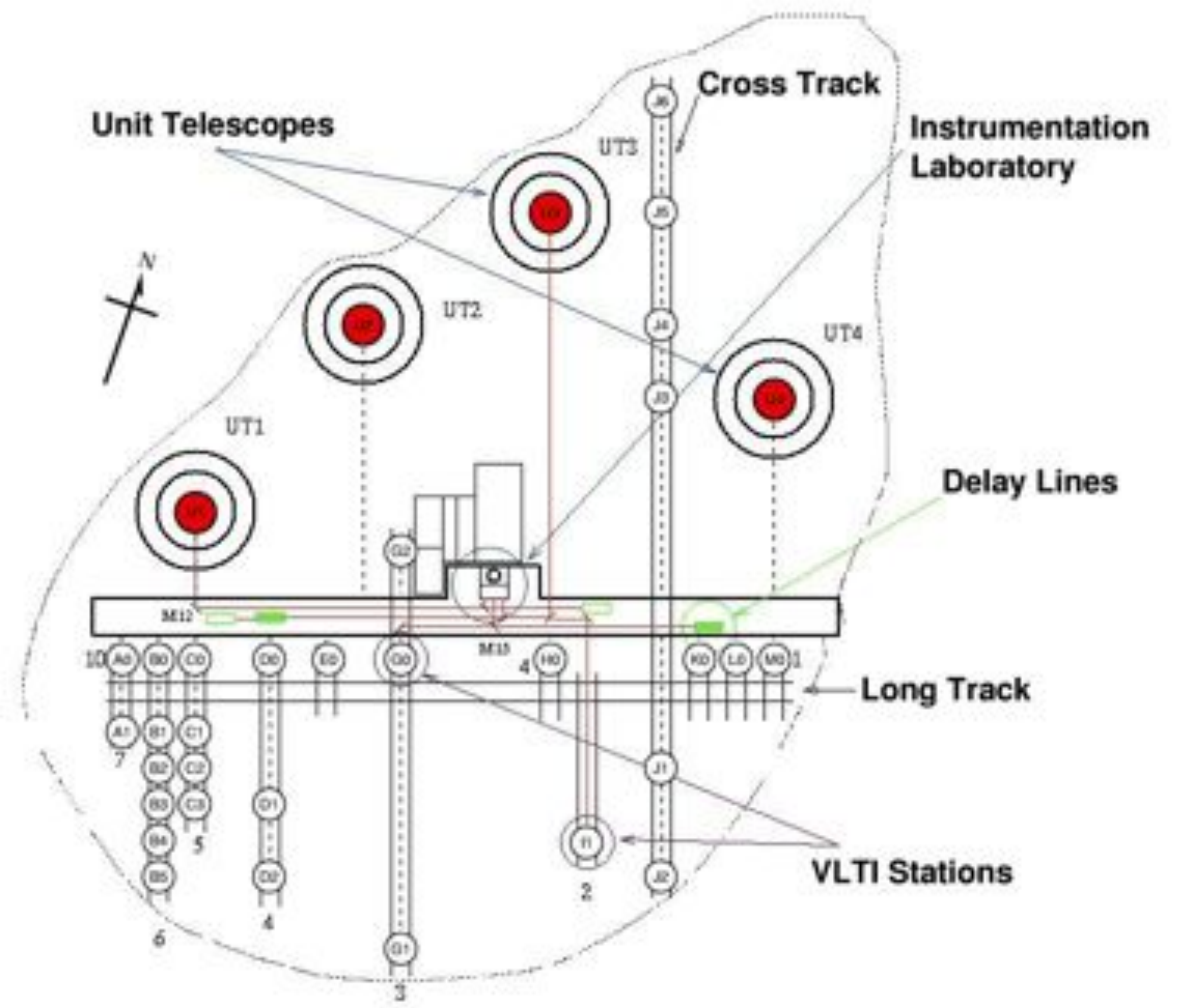
$$i(p,q) = \left|h(p,q)\right|^2 = 4\cos^2(\pi pD)\,d^4\left(\frac{\sin(\pi qd)}{\pi qd}\right)^2\left(\frac{\sin(\pi pd)}{\pi pd}\right)^2 \qquad (8.1.12)$$


Telescope 1
Telescope 2
Nasmyth Focus
Nasmyth focus
Coudé Optical Train
Interferometric Laboratory
Coudé Focus
Coudé Focus
Interferometry fringes
Delay line
Cat's Eye Retro Reflector
Delay line
Optical delay

Delay lines at the VLTI

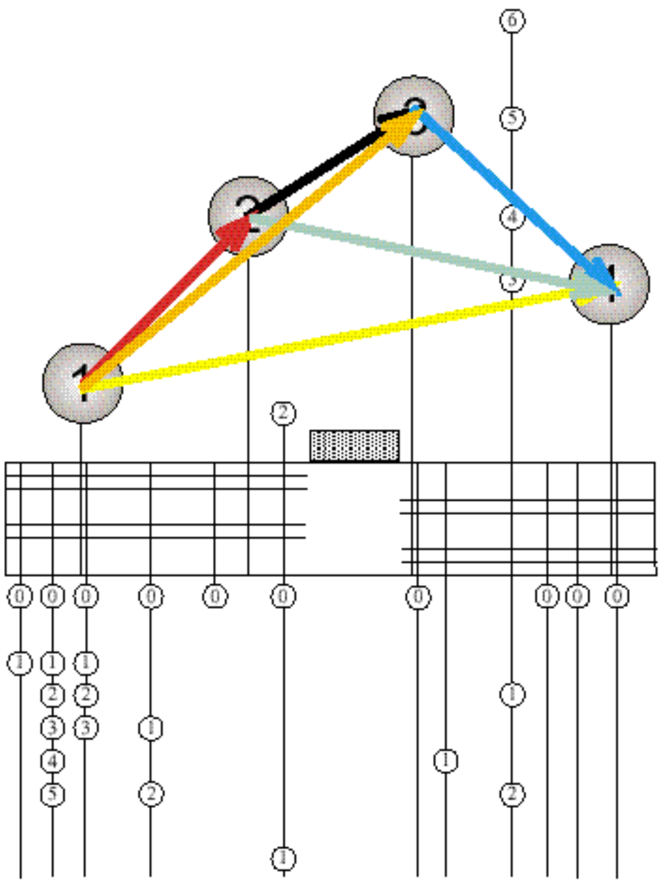
What do the VLTI telescope locations look like?



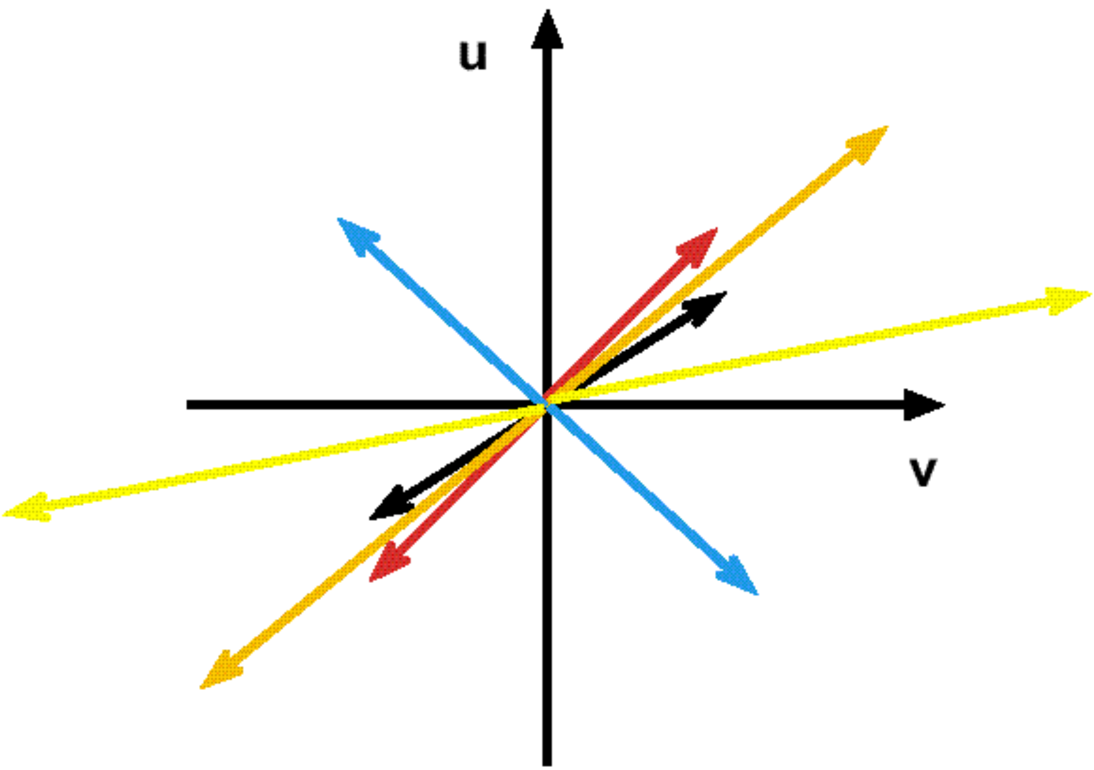


Cross Track
Unit Telescopes
Instrumentation Laboratory
UT3
UT2
UT4
UT1
N
Delay Lines
Long Track
VLTI Stations

How are those locations related to the *uv* coverage?



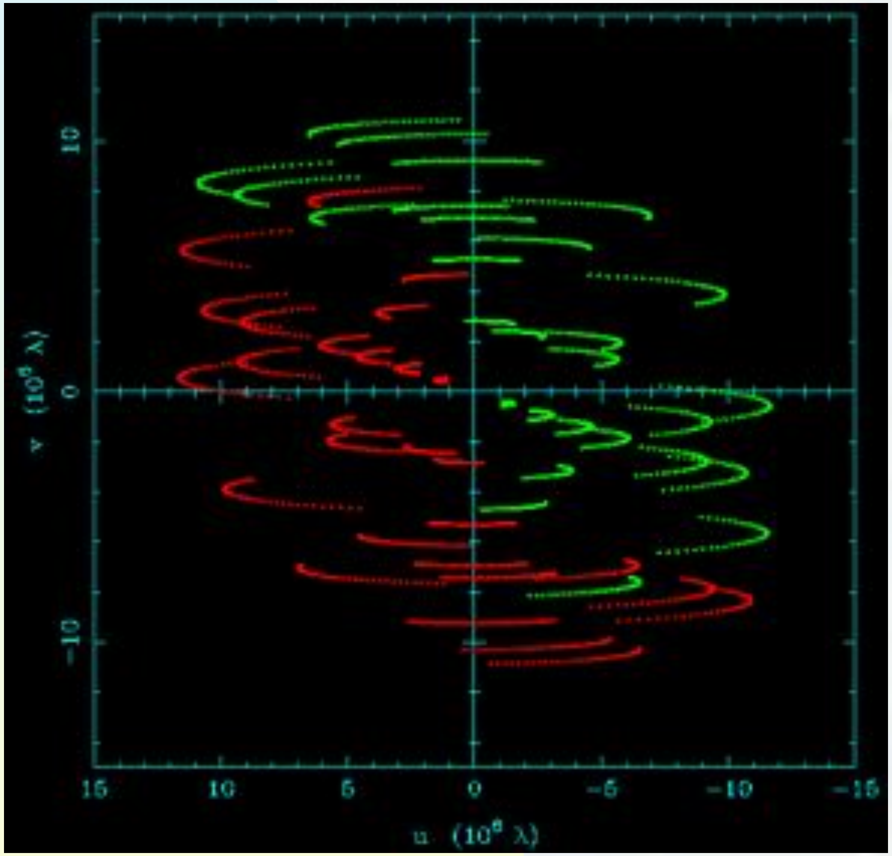
This is the uv-plane:



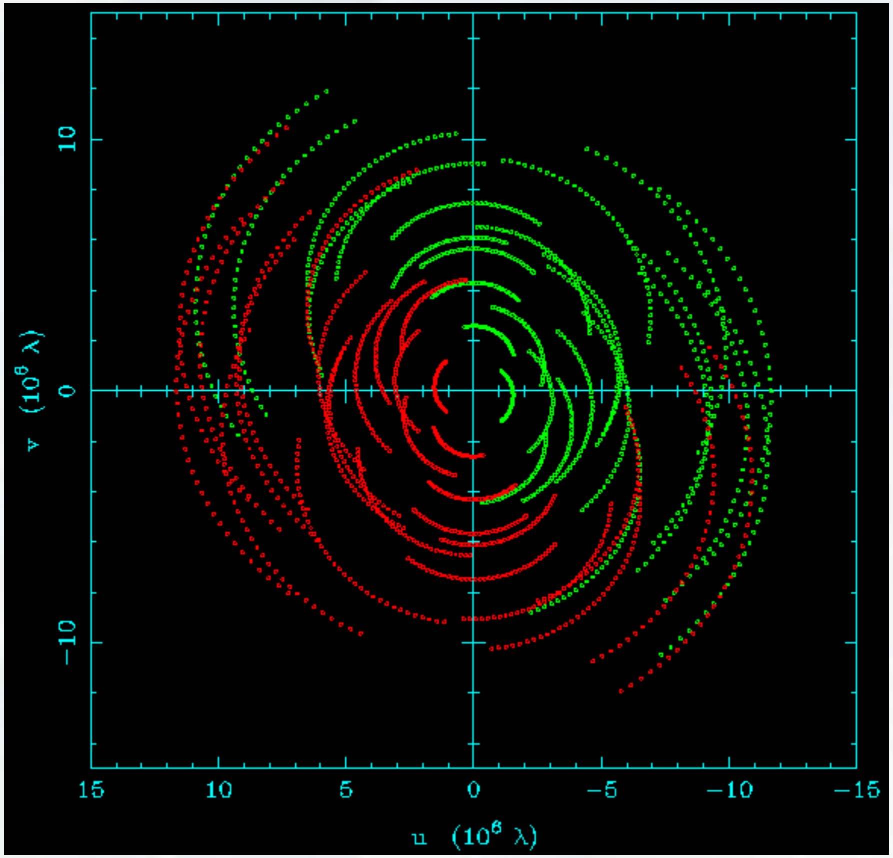
Note: This is the uv-plane for an object at zenith.
In general, the projected baselines have to be used.

Examples of Fourier plane coverage

Dec -15

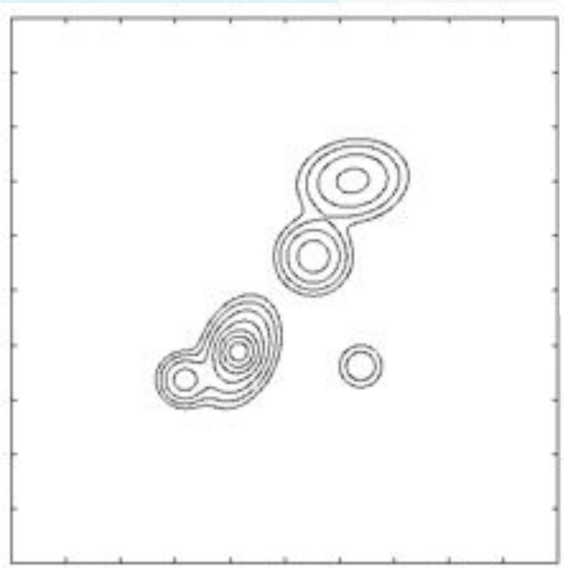


Dec -65

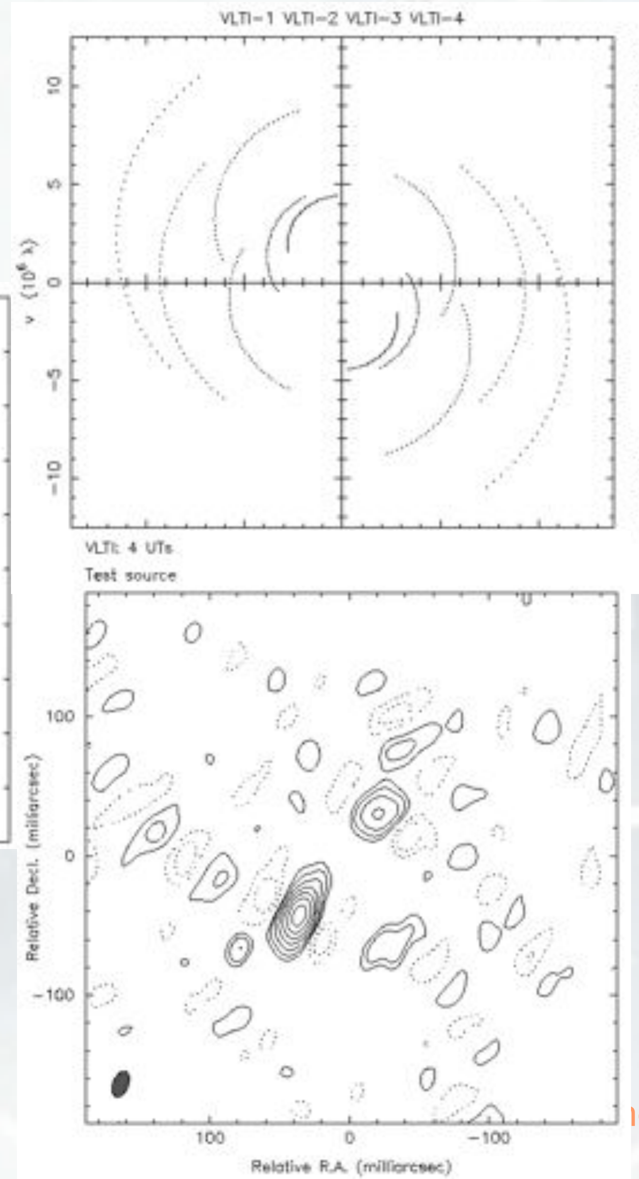


How does the uv plane coverage impact imaging?

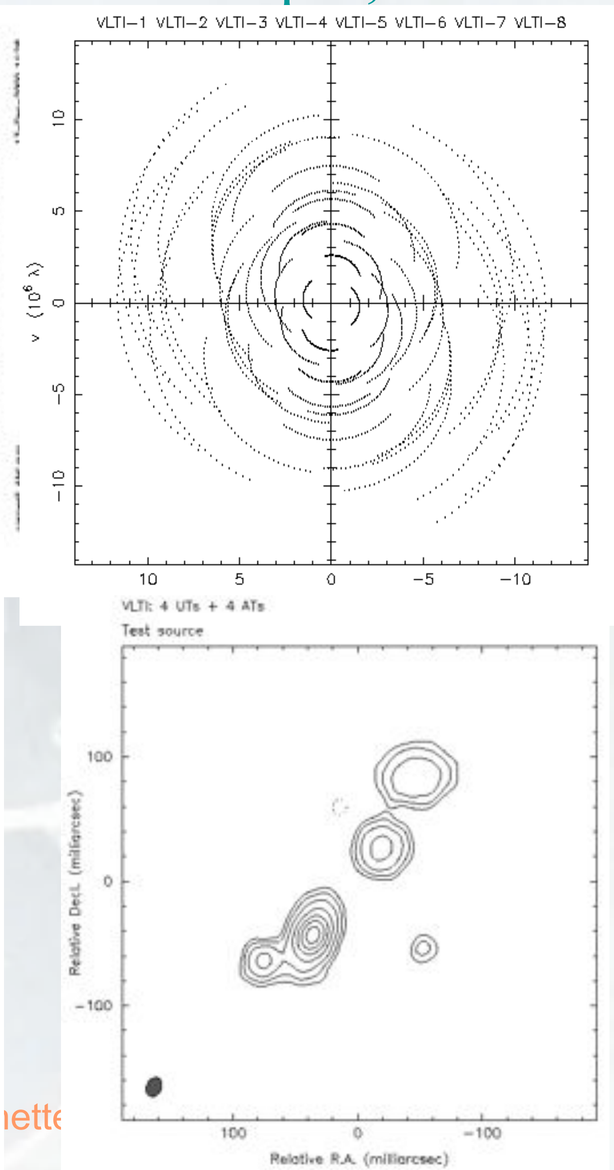
Model



4 telescopes, 6 hours



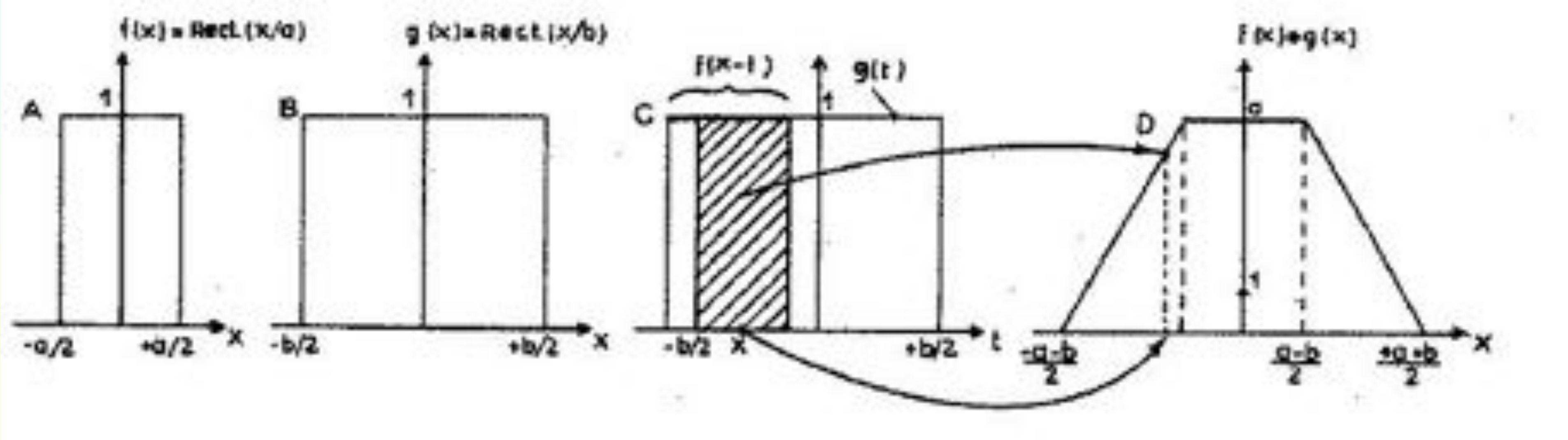
8 telescopes, 6 hours



An introduction to optical/IR interferometry

8.2 The convolution theorem

$$f(x)*g(x)=(f*g)(x)=\int_{R^n} f(x-t)g(t)dt$$

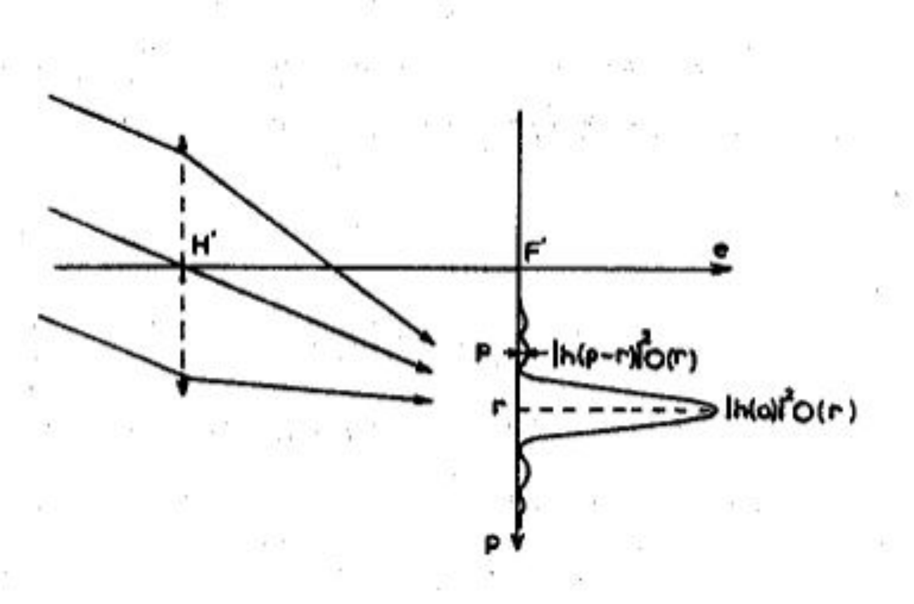


Convolution product of two 1D rectangle functions. A) f(x), B) g(x), C) g(t) and f(x-t); the dashed area represents the integral of the product of f(x-t) and g(t) for the given x offset, D) f(x)*g(x) = (f*g)(x) represents the previous integral as a function of x.

An introduction to optical/IR interferometry

8.2 The convolution theorem

$e(p,q) = O(p,q) * |h(p,q)|^2$,



$$e(p,q) = \int_{R^2} O(r,s) \left| h(p-r, q-s) \right|^2 drds$$

8.2 The convolution theorem

For the case of a point-like source:

$O(p,q) = E\,\delta(p,q)$, (8.2.1)

[$\delta(x) = 0$ if $x \neq 0$, $\delta(x) = \infty$ if $x = 0$] and (8.2.2)

$e(p,q) = O(p,q) * |h(p,q)|^2 = E\,\delta(p,q) * |h(p,q)|^2 = E\,|h(p,q)|^2$ (8.2.3)

An introduction to optical/IR interferometry

8.2 The convolution theorem

More generally, since
TF_(f * g) = TF_(f) TF_(g). (8.2.4)

We find, because
$e(p,q) = O(p,q) * |h(p,q)|^2$ (8.2.5)

that:
$TF\_(e(p,q)) = TF\_(O(p,q))\ TF\_(|h(p,q)|^2)$, (8.2.6)

and, finally,
$O(p,q) = TF^{-1}\ [TF\_(e(p,q)) / TF\_(|h(p,q)|^2)]$. (8.2.7)

An introduction to optical/IR interferometry

8.2 The convolution theorem

$O(p,q) = (\lambda^2 E / \phi^2)\, \Pi(p \lambda / \phi)\, \Pi(q \lambda / \phi).$ (8.2.8)

$e(p,q) = O(p,q) * |h_0(p,q)|^2.$

$e(p) = O(p) * |h_0(p)|^2,$ (8.2.9)

$$e(p) = 2d^2 (\lambda/\phi)\sqrt{E}\int_{p-\phi/2\lambda}^{p+\phi/2\lambda}\left(\frac{\sin(\pi r d)}{\pi r d}\right)^2 \cos^2(\pi r D)\,dr \qquad (8.2.10)$$

$\left(\frac{\sin(\pi r d)}{\pi r d}\right)^2 \approx$ Cte sur [p-$\phi/2\lambda$, p+$\phi/2\lambda$], et (8.2.11)

$$e(p) = 2d^2 (\lambda/\phi)\sqrt{E}\left(\frac{\sin(\pi p d)}{\pi p d}\right)^2 \int_{p-\phi/2\lambda}^{p+\phi/2\lambda} \cos^2(\pi r D)\,dr. \qquad (8.2.12)$$

An introduction to optical/IR interferometry

8.2 The convolution theorem

$$e(p) = 2d^2\left(\frac{\sin(\pi pd)}{\pi pd}\right)^2\left[O(p) * \cos^2(\pi pD)\right], \qquad (8.2.13)$$

$$e(p) = 2d^2\left(\frac{\sin(\pi pd)}{\pi pd}\right)^2\left[\frac{1}{2}\int_R O(p)dp + \frac{1}{2}O(p) * \cos(2\pi pD)\right] \qquad (8.2.14)$$

$$e(p) = A\left[B + \frac{1}{2}\mathrm{Re}\left(O(p) * \exp(i2\pi pD)\right)\right], \qquad (8.2.15)$$

$$A = 2d^2\left(\frac{\sin(\pi pd)}{\pi pd}\right)^2 \quad et \quad B = \frac{1}{2}\int_R O(p)dp, \qquad (8.2.16)$$

An introduction to optical/IR interferometry

8.2 The convolution theorem

$$e(p) = A\left[B + \frac{1}{2}\mathrm{Re}\left(\int_R O(r)\exp\left(i2\pi(p-r)D\right)dr\right)\right], \qquad (8.2.17)$$

$$e(p) = A\left[B + \frac{1}{2}\cos(2\pi pD)\,TF\_(O(r))(D)\right], \qquad (8.2.18)$$

$$\gamma(D) = (e_{max} - e_{min}) / (e_{max} + e_{min}), \qquad (8.2.19)$$

$$\gamma(D) = TF\_(O(r))(D) / (2B) = TF\_(O(r))(D) / \int O(p)\,dp. \qquad (8.2.20)$$

An introduction to optical/IR interferometry

8.3 The Wiener-Khintchin theorem

The **Wiener–Khinchin theorem** states that the power spectral density of a wide-sense-stationary random process is the Fourier transform of the corresponding autocorrelation function. **In our case, this theorem merely states that the Fourier tranform of the PSF (see Eq. (8.2.7)) is the auto-correlation function of the distribution of the complex amplitude in the pupil plane**:

$$TF\left(\left|h(p,q)\right|^2\right) = \iint A^*(x,y)\, A\,(x+p, y+q)\,dx\,dy$$