Interferometric Data Reduction

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Contents

- Data reduction in context
- The forward problem
  - atmospheric piston & pupil distortion
  - spectral decoherence
  - bias and noise sources
- The inverse problem
  - debias & flatfielding
  - coherent flux extraction (Fourier, ABCD, P2VM)
  - integration (coherent & incoherent)
- Calibration
  - photometric calibration
  - transfer function calibration

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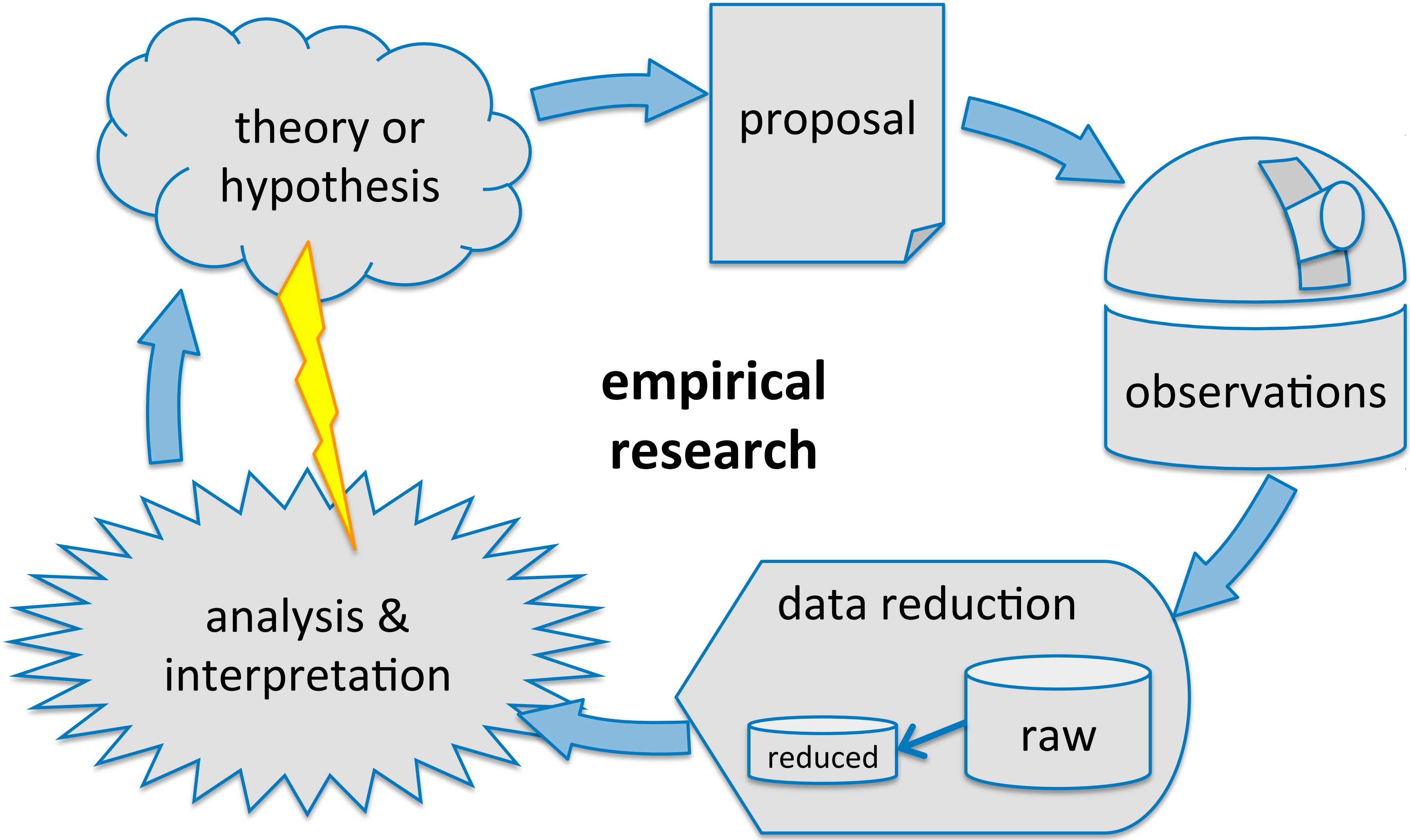
Data reduction in context



Is there a problem?

Simple expression for the fringes:

$$I(\delta) = I_0\left[1 + \mathrm{Re}\left(\mathcal{V} \cdot \mathrm{e}^{-ik\delta}\right)\right]$$

with the complex visibility

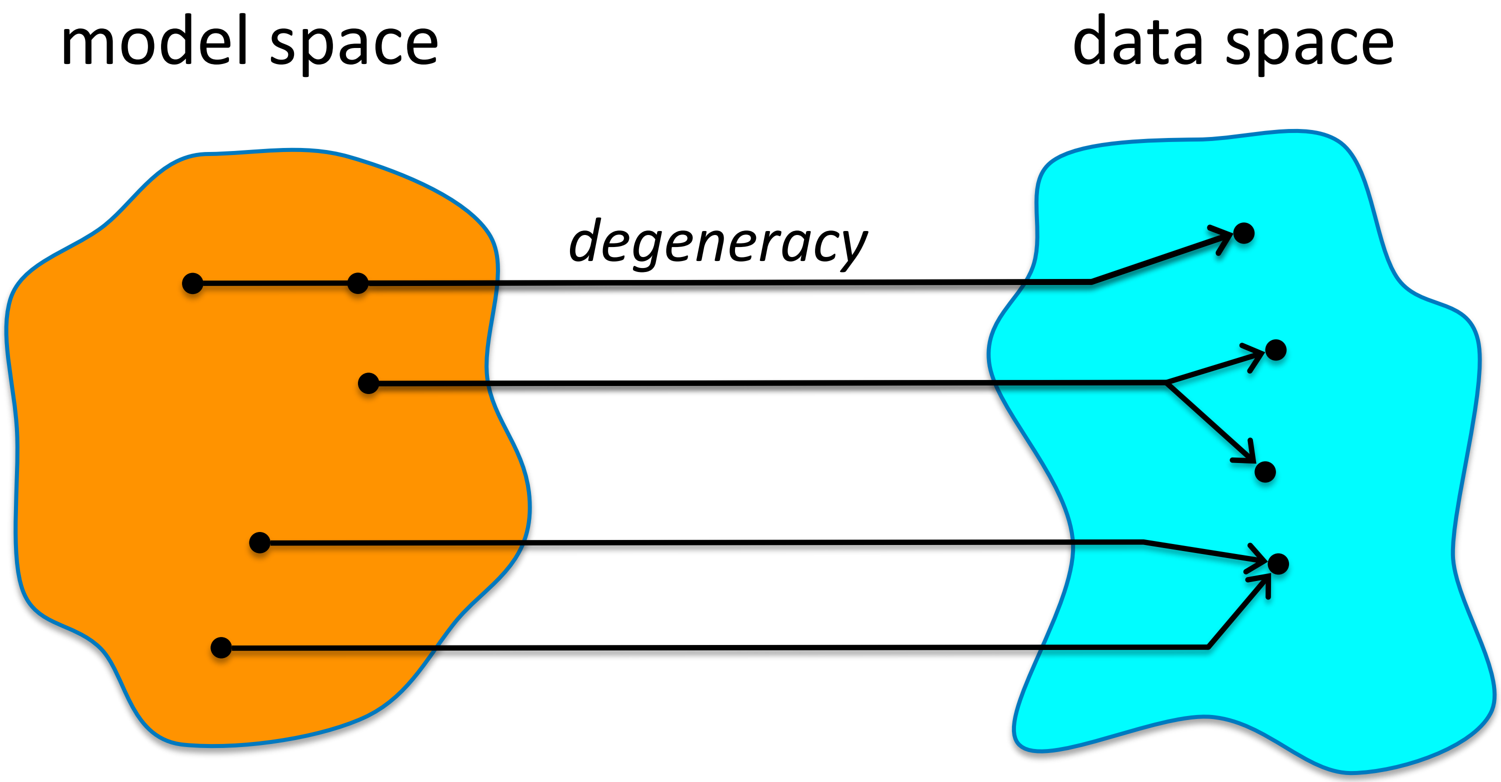
$$\mathcal{V} = V \cdot \mathrm{e}^{i\varphi} = \Re(\mathcal{V}) + i\Im(\mathcal{V}).$$

⇒ The visibility can be estimated in an easy form:

$$\Re(\mathcal{V}) = I(0)/I_0 - 1$$

$$\Im(\mathcal{V}) = I(\tfrac{\lambda}{4})/I_0 - 1$$

Scientific inference



one set of model parameters $\{\pi_i\}$

one set of measurements $\{\mu_i\}$

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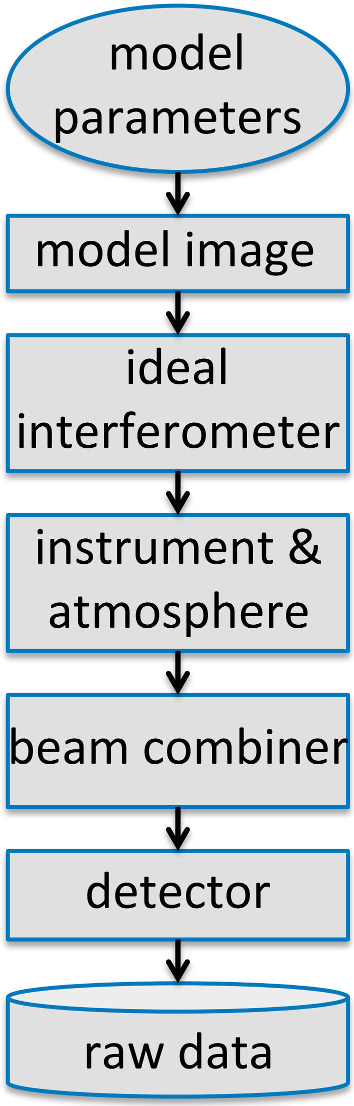
Forward Problem



$\{\pi_i\}$, e.g. $\{\pi_i\} = \{d_1, d_2, s, b\}$

$I(\sigma)$

$F(\boldsymbol{u}_{ij})$

$f_{ij} = \gamma_{ij} F(\boldsymbol{u}_{ij})$

$\{I_p\}$, e.g. $I_p = 0.5\,(f_{11} + f_{22}) + \mathrm{Re}\{f_{12} e^{2isp}\}$

$r_p = g_p(I_p + \delta n_p) + b_p$

Forward problem

$$I(\delta) = I_0\left[1 + \mathrm{Re}\left(\mathcal{V} \cdot \mathrm{e}^{-ik\delta}\right)\right]$$

Using the identity $\mathrm{e}^{i\vartheta} = \cos\vartheta + i\sin\vartheta$:

$$I(\delta) = \left[1 + \mathrm{Re}\left(V\left(\cos\left(\varphi - k\delta\right) - i\sin\left(\varphi - k\delta\right)\right)\right)\right]$$

$$= I_0\left[1 + V\cos\left(\varphi - k\delta\right)\right]$$

Forward Problem

- Idealised formula: $I(\delta_p) = I_0\left[1 + V \cdot \cos\left(\varphi - k\delta_p\right)\right]$
- More realistic raw data:

$$r(\delta) = I_{\mathrm{src}}\left[\frac{\eta_i(t)+\eta_j(t)}{2} + \sqrt{\eta_i(t)\eta_j(t)} \cdot \right.$$

$$e^{-\sigma^2_{\mathrm{jit}}(t)} \cdot \mathrm{sinc}\left(\frac{\Delta k}{2}\left(\delta_{\mathrm{ins}}(t)+\delta_{\mathrm{atm}}(t)\right)\right) \cdot$$

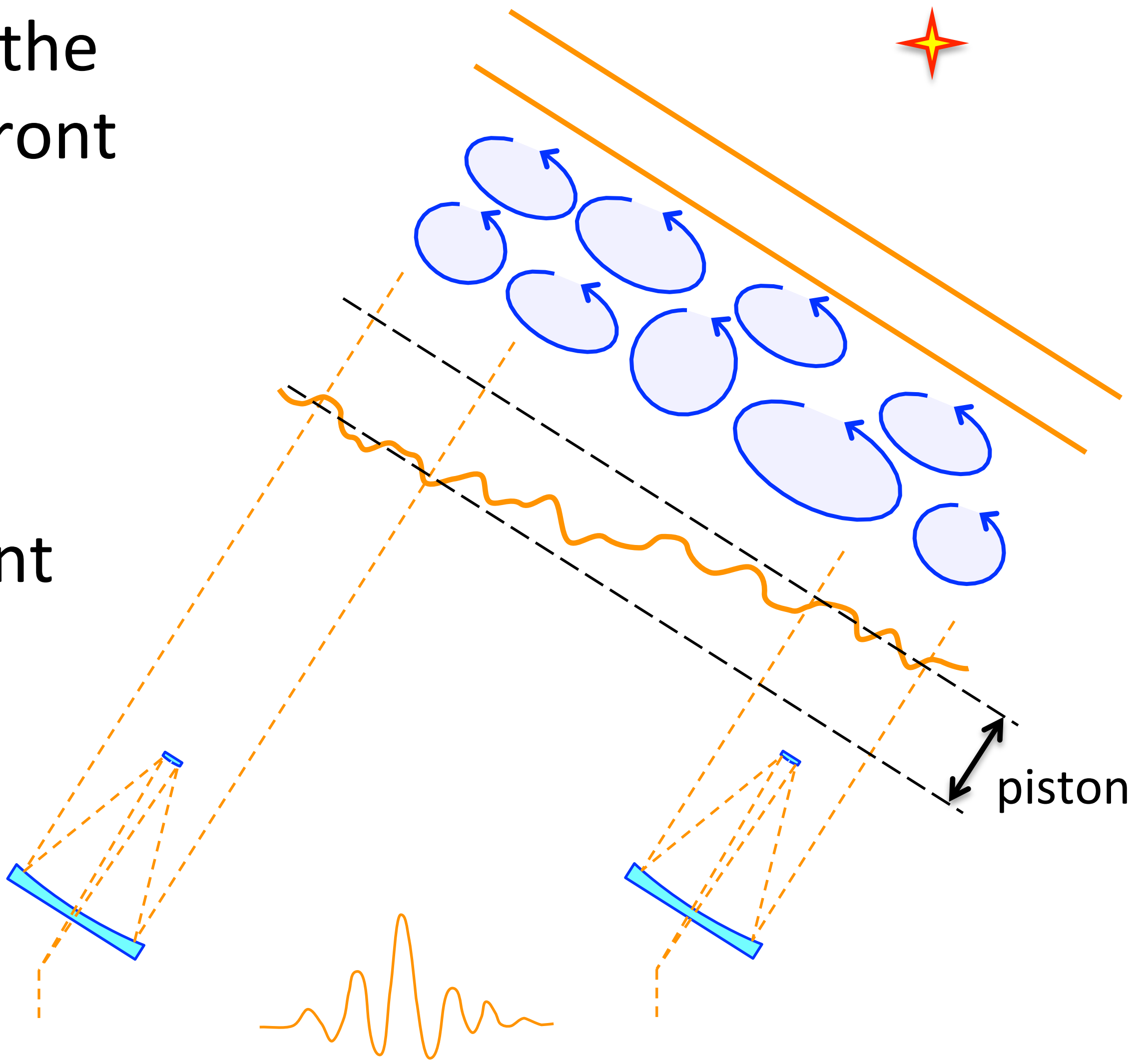
$$\left. V(\vec{u}_{ij}) \cdot \cos\left(\varphi(\vec{u}_{ij}) - k\left(\delta_{\mathrm{ins}}(t)+\delta_{\mathrm{atm}}(t)\right)\right)\right] \cdot$$

$$g(t) + n(t) + b(t)$$

Noise – the atmosphere

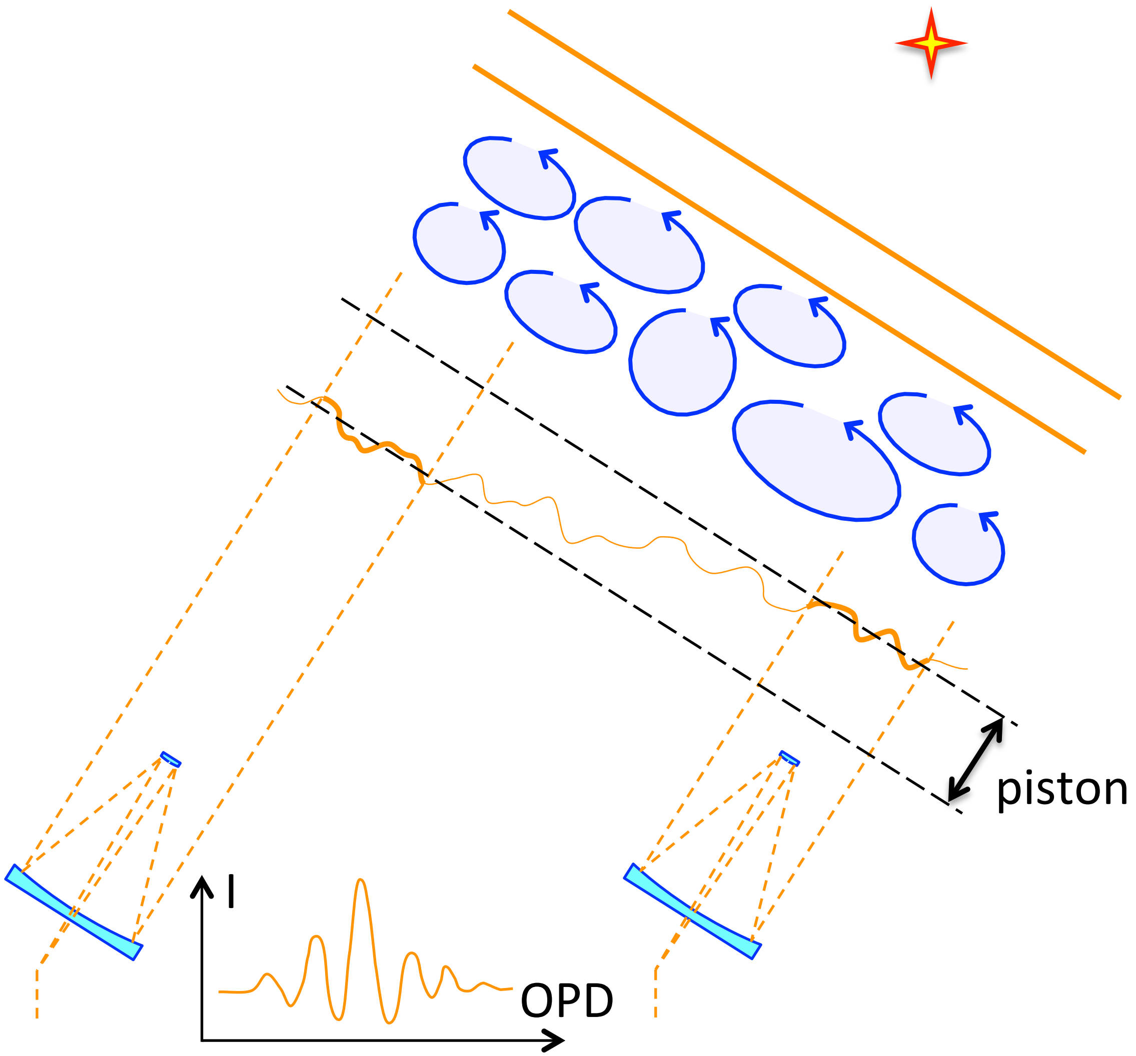
Turbulence distorts the the incoming wavefront

- 1) piston → OPD
- 2) pupil wavefront distortion



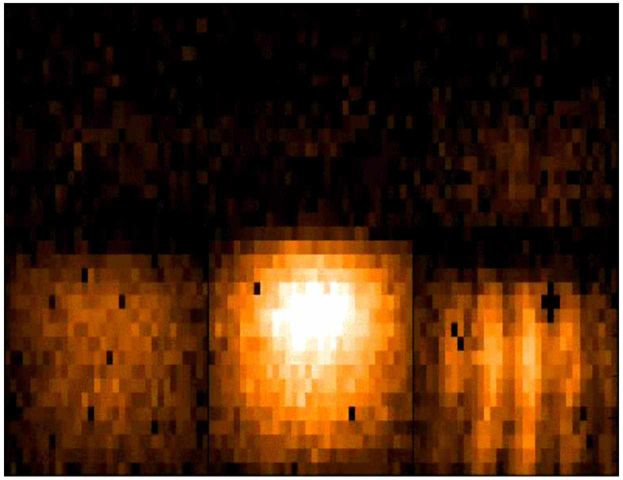
Noise – piston

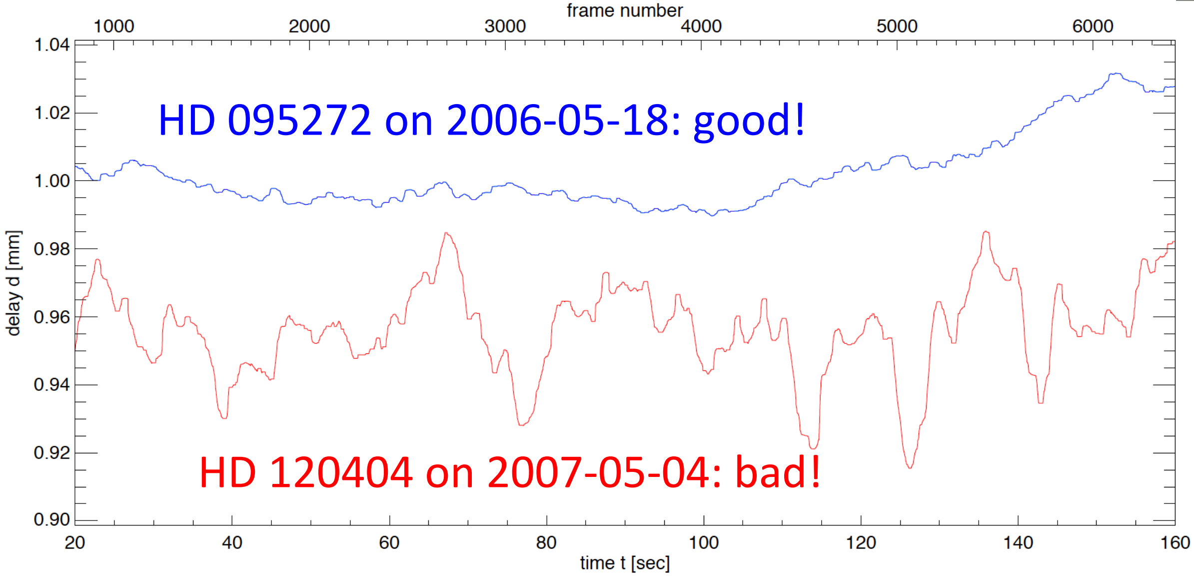
Piston leads to a movement of the fringe packet in OPD space.



Noise – piston

H and K band fringes with AMBER of HD 048433





Group delay measured with MIDI.

Noise – piston

The piston has two effects:

- Time dependent phase shift
  - ↳ fringe motion phase lost
- Fringe blurring
  - ↳ fringe amplitude lost

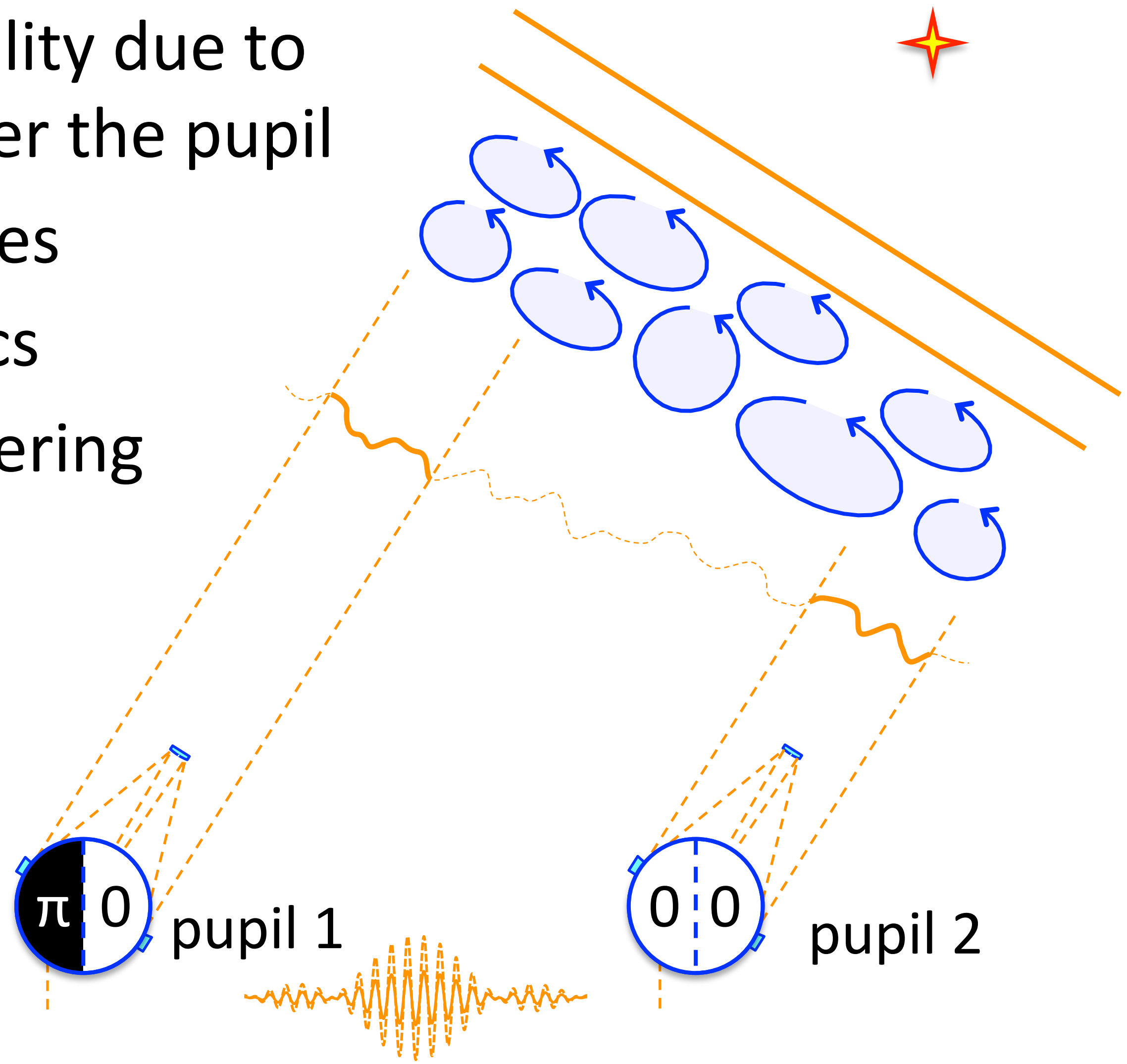
blurring

phase shifts

$$I(\delta_p) = I_{\mathrm{src}}\left[e^{-\sigma^2_{\mathrm{jit}}(t)} \cdot V(\vec{u}_{ij}) \cdot \cos\left(\varphi(\vec{u}_{ij}) - k\left(\delta_{\mathrm{ins}}(t) + \delta_{\mathrm{atm}}(t)\right)\right)\right]$$

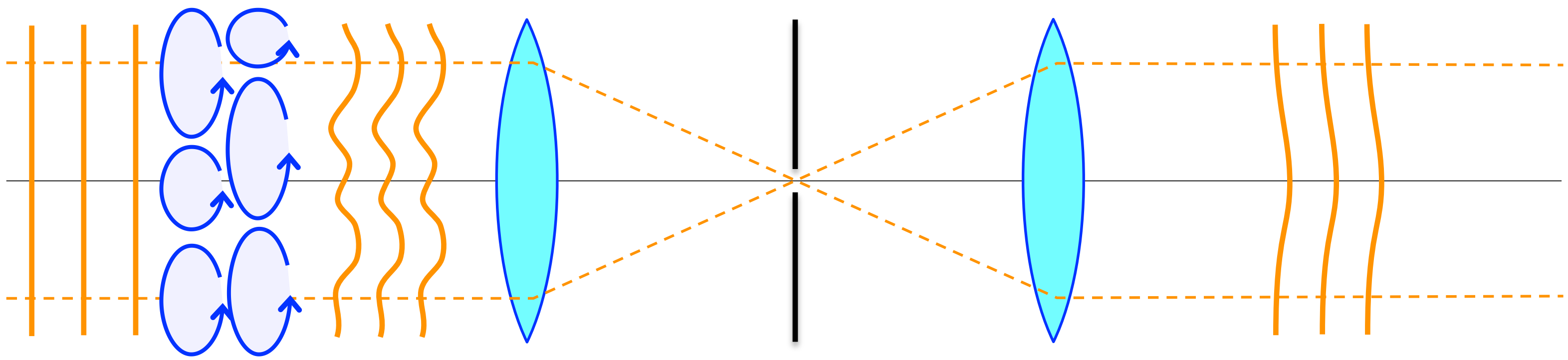
Noise – pupil distortion

- Reduction of visibility due to phase variance over the pupil
  - ↳ small telescopes
  - ↳ adaptive optics
  - ↳ wavefront filtering



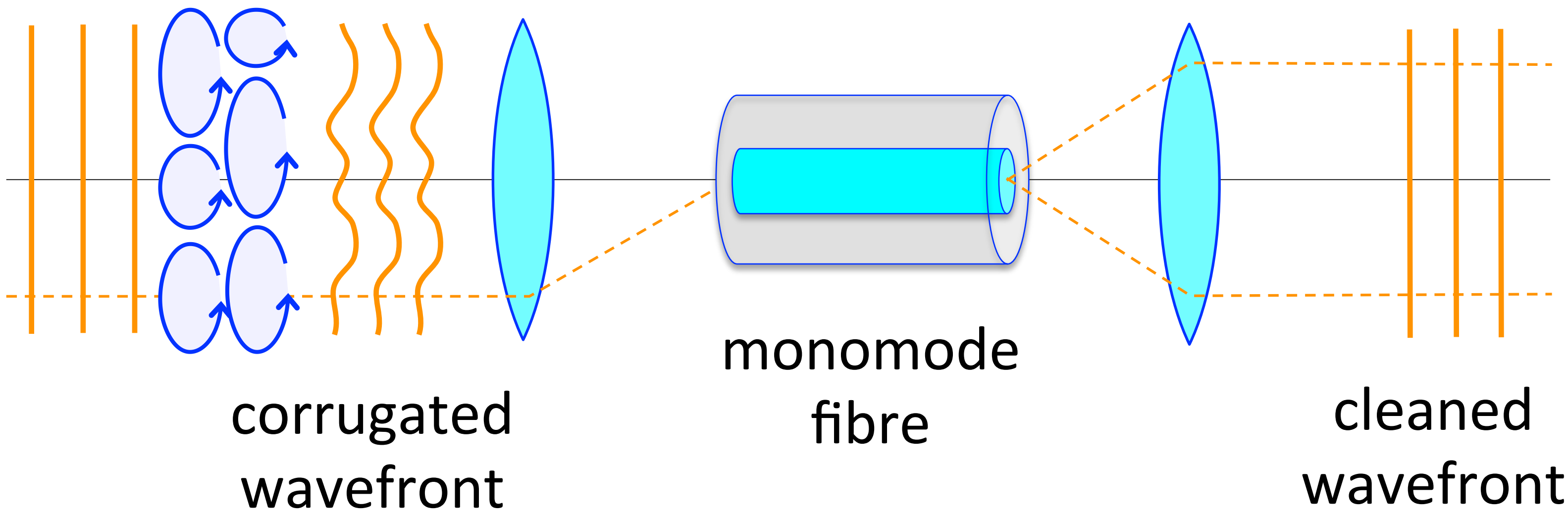
Noise – pupil distortion

Spatial filtering:



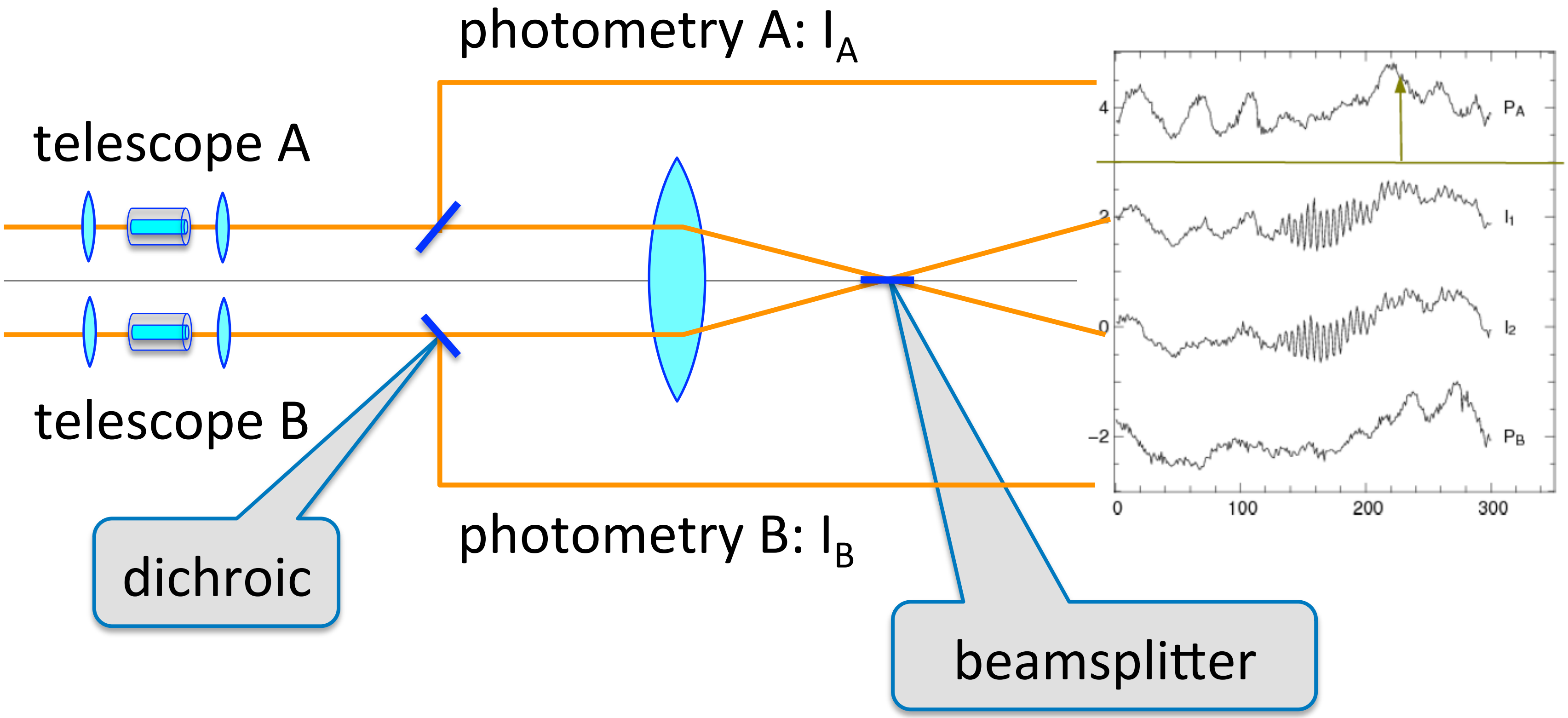
Noise – pupil distortion

Modal filtering:



Noise – pupil distortion

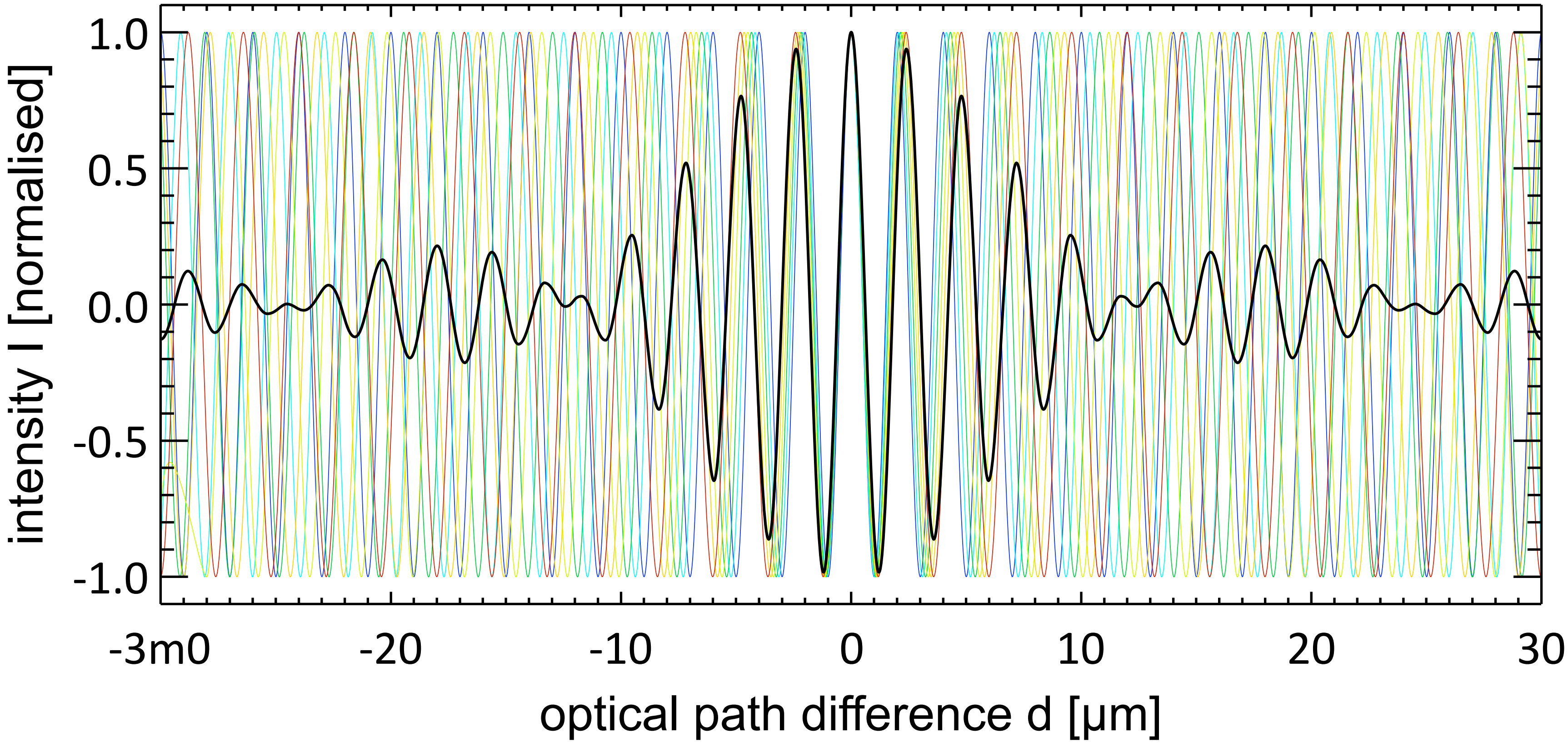
Photometric fluctuations can be monitored:



Spectral decoherence

Typically not only a single wavelength is observed:

Idealised example of the K band: $\lambda = 2.2\mu m$, $\Delta\lambda = 0.4\ \mu m$



Spectral decoherence

Remember? $$I(\delta_p) = I_0\left[1 + V \cdot \cos\left(\varphi - k\delta_p\right)\right]$$

Actually: $$I(k,\delta_p) = t(k) \cdot I_0(k)\left[1 + V(k) \cdot \cos\left(\varphi(k) - k\delta_p\right)\right]$$

$$\Rightarrow I(\delta_p) = \int t(k) \cdot I_0(k)\left[1 + V(k) \cdot \cos\left(\varphi(k) - k\delta_p\right)\right] \mathrm{d}k$$

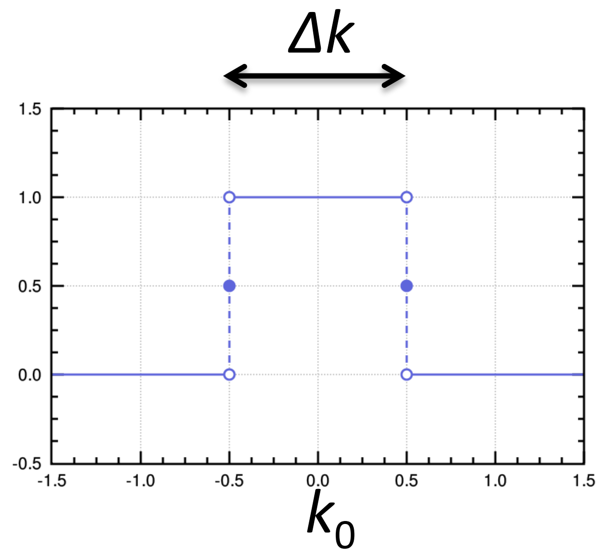
Limited band pass t centred on k_0 and I_0, V, φ constant:

$$\Rightarrow I(\delta_p) = I_0\left[1 + V \cdot \underbrace{\hat{t}\left(\tfrac{\delta}{2\pi}, k_0\right)}_{\text{Fourier transform of the band pass}} \cdot \cos\left(\varphi - k_0\delta_p\right)\right]$$

Spectral decoherence

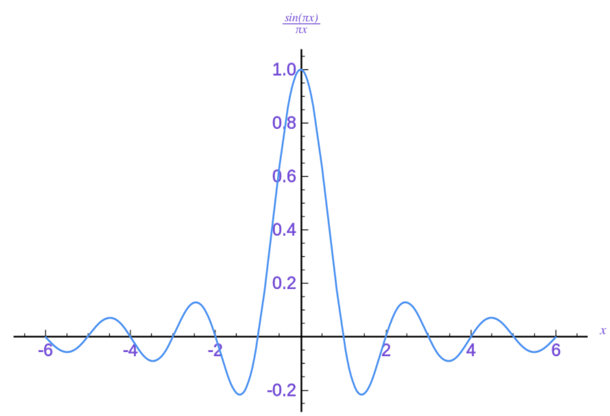
Example: top hat function

$$t(k) = \mathrm{rect}(k) = \begin{cases} 0 & \text{if } \left|k - k_0\right| > \frac{\Delta k}{2} \\ \frac{1}{2} & \text{if } \left|k - k_0\right| = \frac{\Delta k}{2} \\ 1 & \text{if } \left|k - k_0\right| < \frac{\Delta k}{2} \end{cases}$$



$$\int_{-\infty}^{\infty} rect(k) \cdot e^{-2\pi ikx} \; \mathrm{d}k = \frac{\sin(\pi \cdot x)}{\pi \cdot x}$$

$$= \mathrm{sinc}(\pi \cdot x)$$



Biases & noise

- Bias: additive value with non-zero mean, e.g.
  - detector bias
  - thermal background
  - EM detector perturbations

  ⇘ Estimate and subtract it!

- Noise: additive value with zero mean, e.g.
  - photon noise from the source / backgound
  - readout noise

  ⇘ Average it away!

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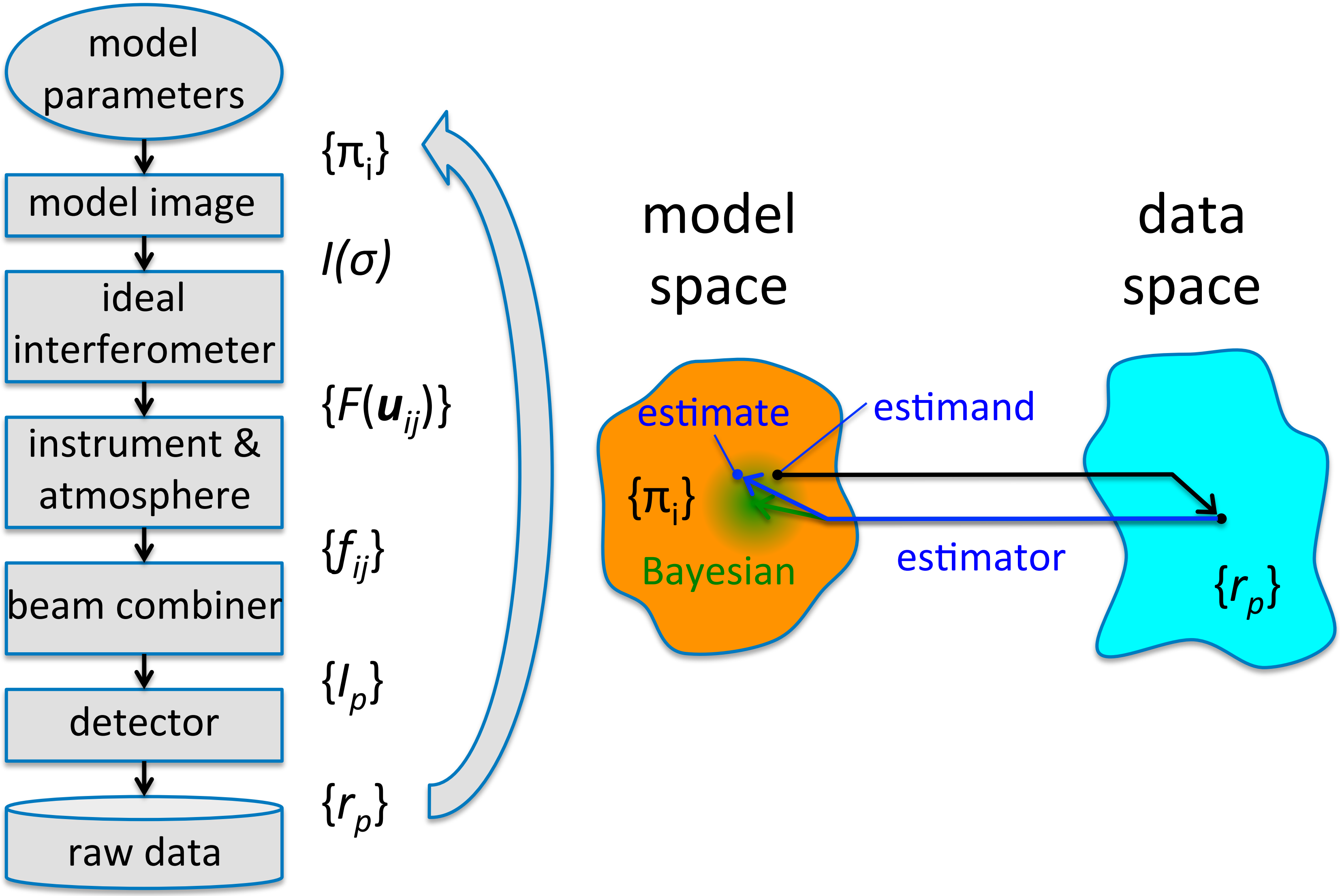
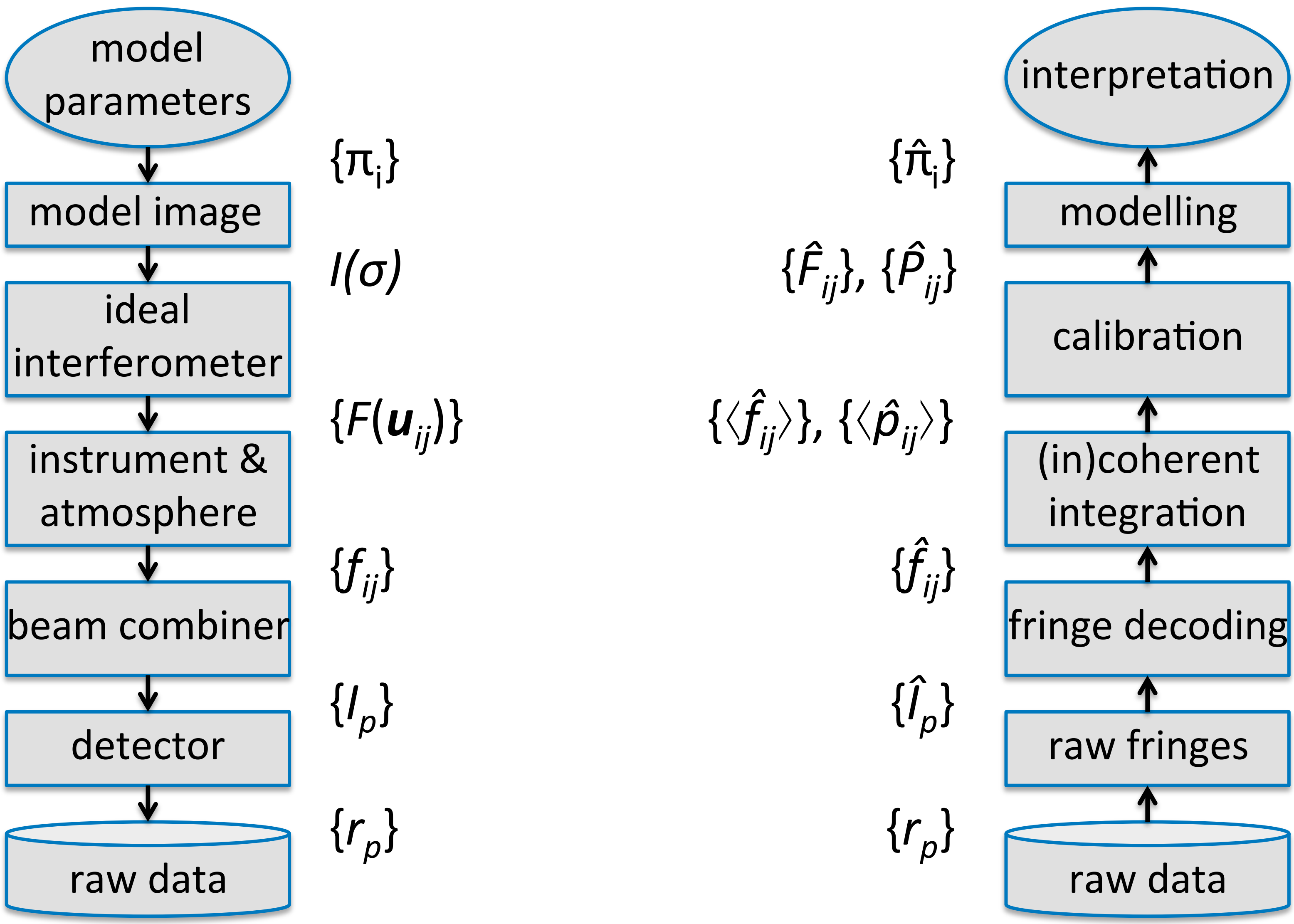
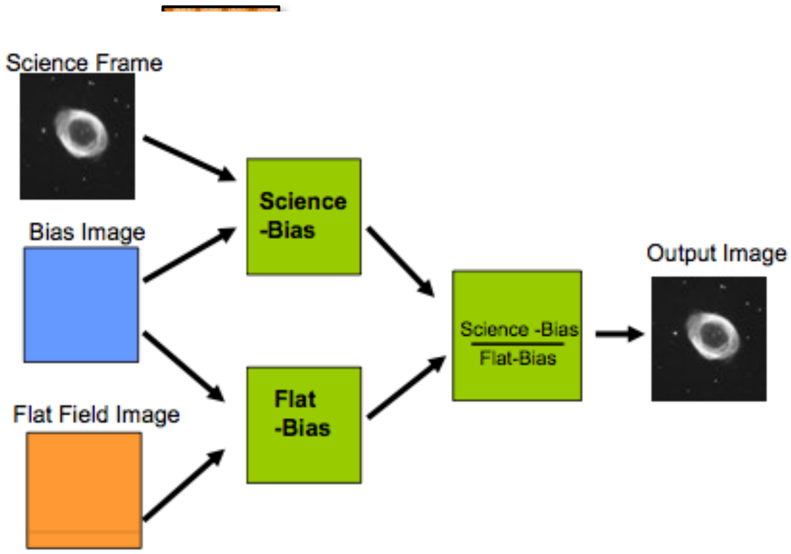
Inverse problem



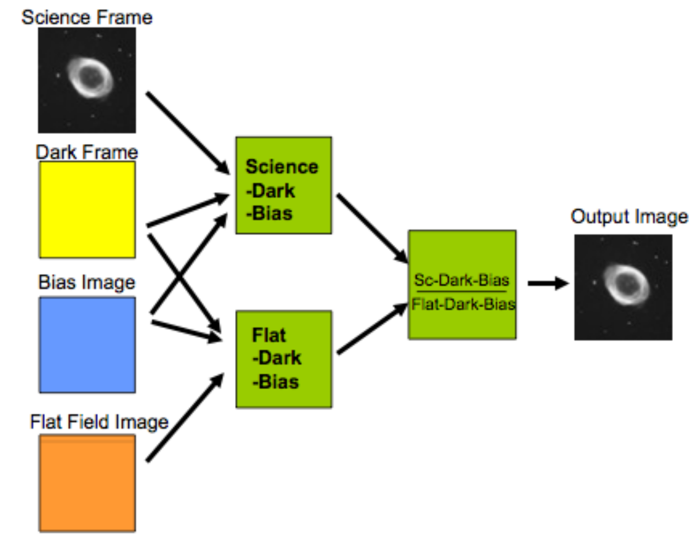
Inverse problem



(1) Debias & flat fielding



If there is significant dark current present:



$$\hat{I}_p = \frac{r_p - \hat{b}_p}{\hat{g}_p}$$

(2) Extraction of the coherent flux

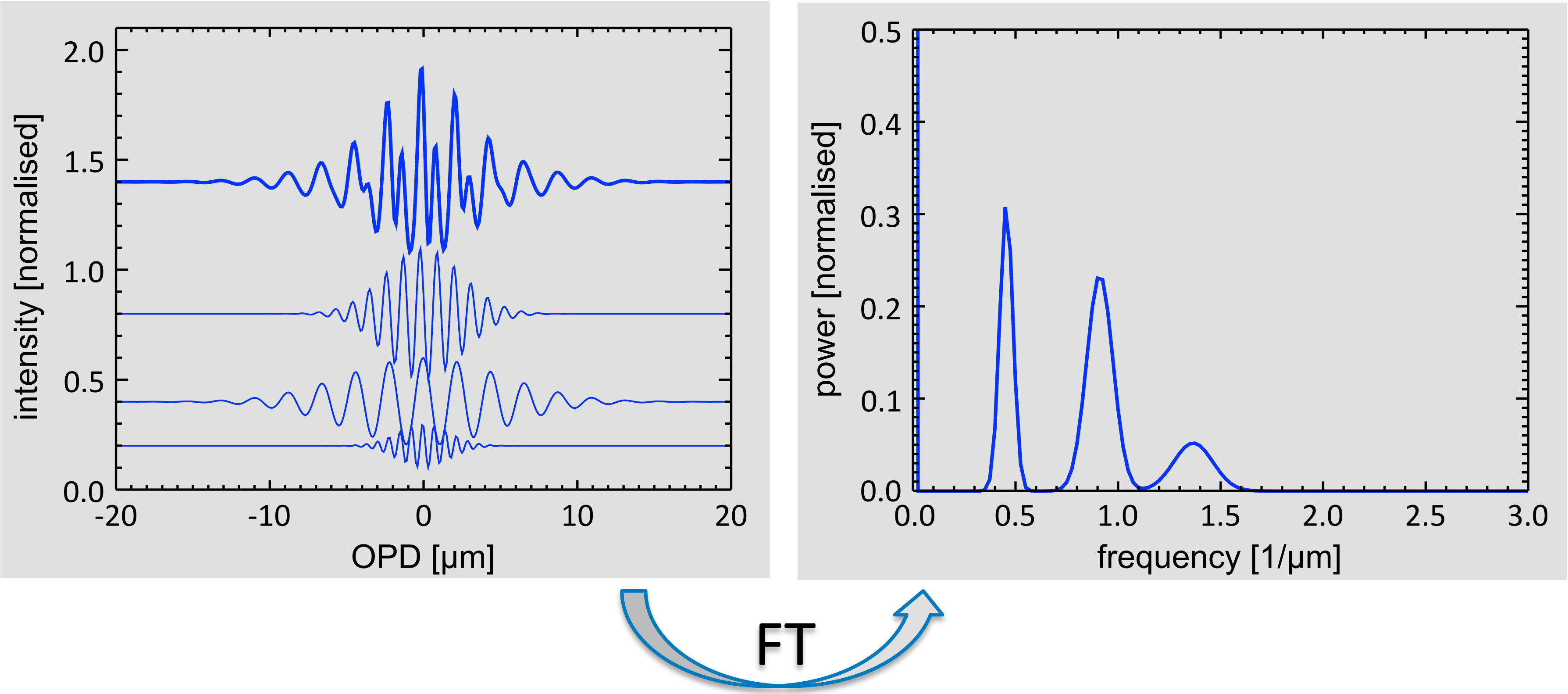
Three different methods:

- Fourier and ABCD (e.g. MIDI and PIONIER):
  take a Fourier transform to extract the oscillating part

- P2VM: pixel to visibility matrix (e.g. AMBER):
  least squares fit of the fringes in the image plane

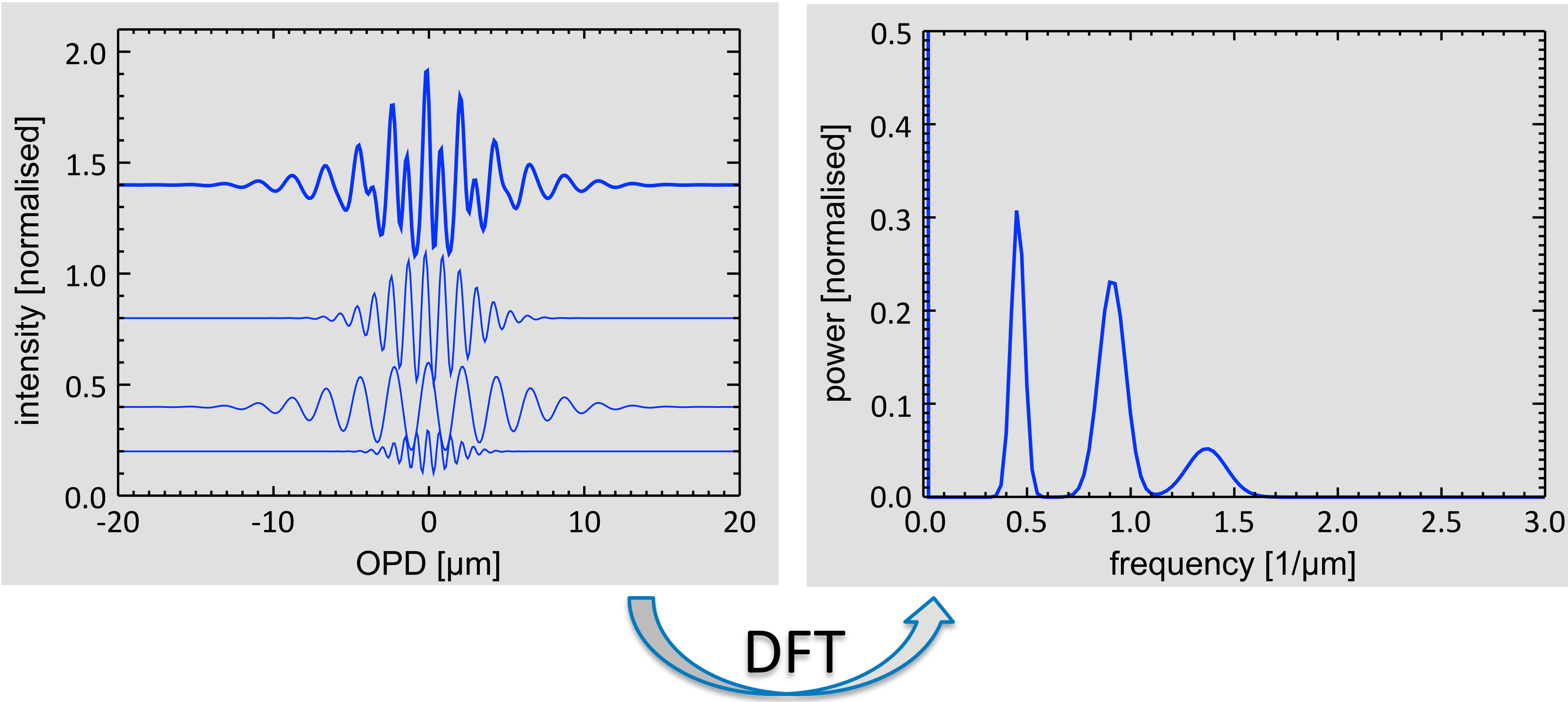
- Coherent integration (e.g. MIDI):
  determine and remove the group delay and then integrate

(2a) Fourier method



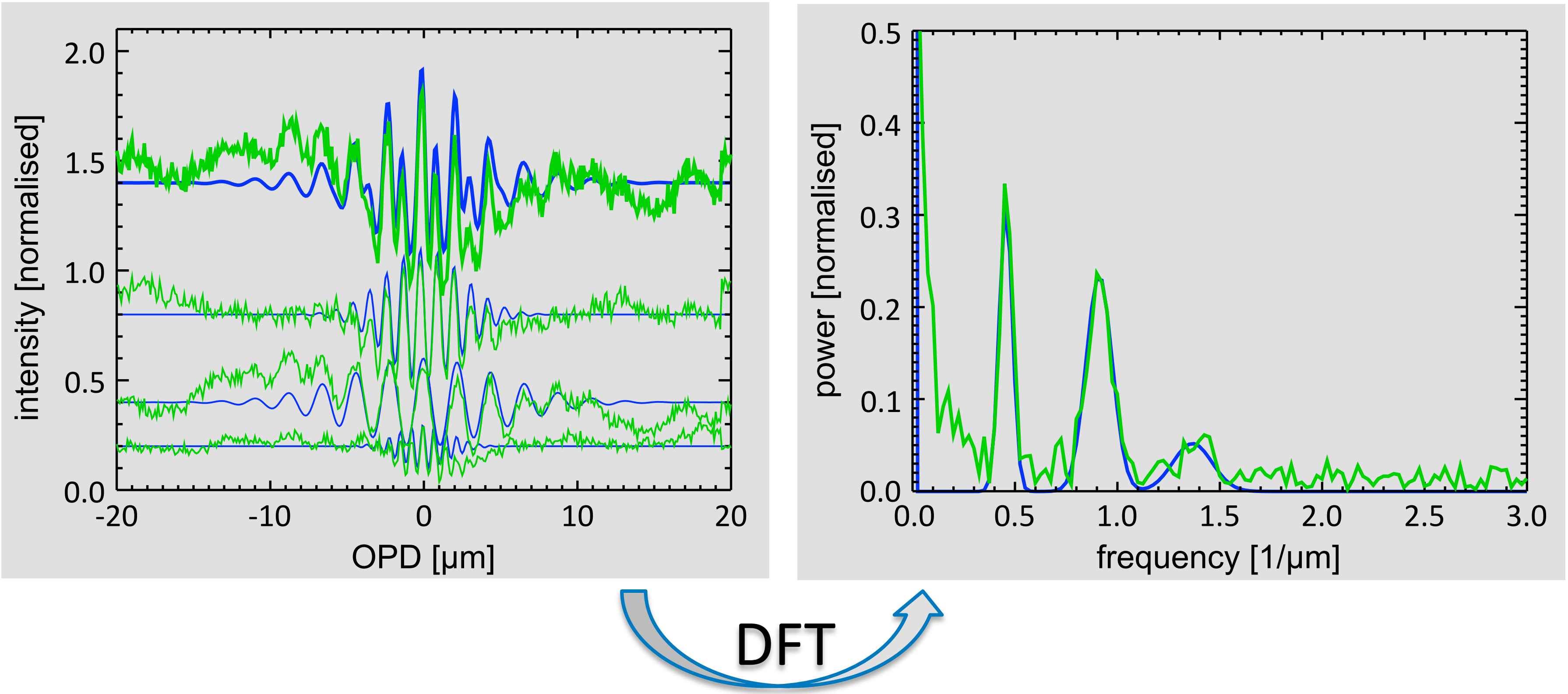
$$f(k) = \int_{-\infty}^{+\infty} I(\delta) \cdot e^{-2\pi i k \delta}$$

(2a) Fourier method



$$f_k = \sum_{p=0}^{N-1} \hat{I}_p \cdot e^{-2\pi i k \frac{n}{N}}, \text{ with } k = 0, \dots, N$$

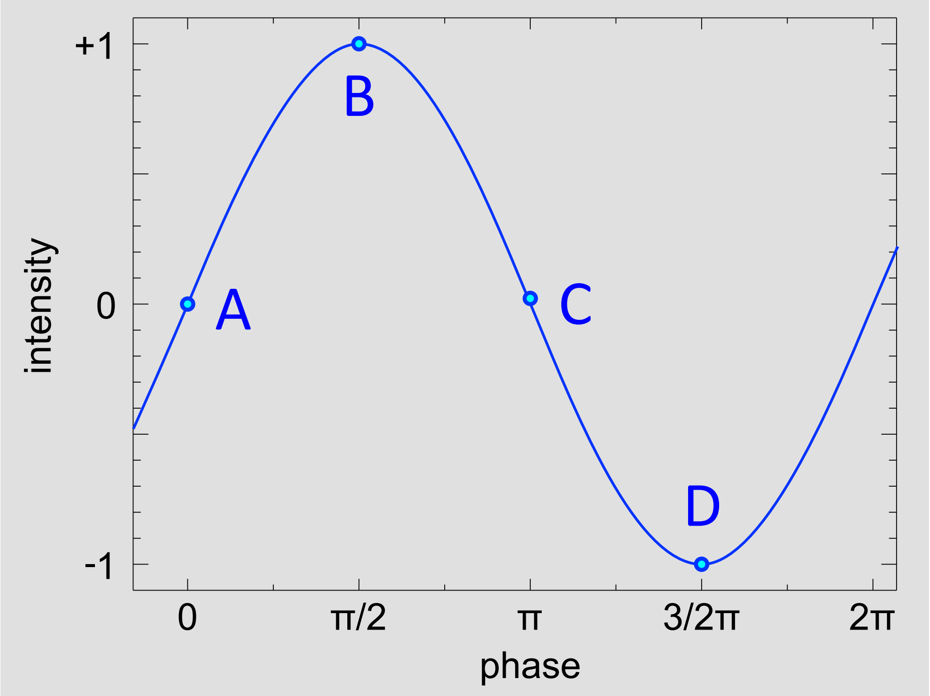
(2a) Fourier method



$$f_k = \sum_{p=0}^{N-1} \hat{I}_p \cdot e^{-2\pi i k \frac{n}{N}}, \text{ with } k = 0, \ldots, N$$

(2b) ABCD method

Optimised Fourier Method – ABCD:



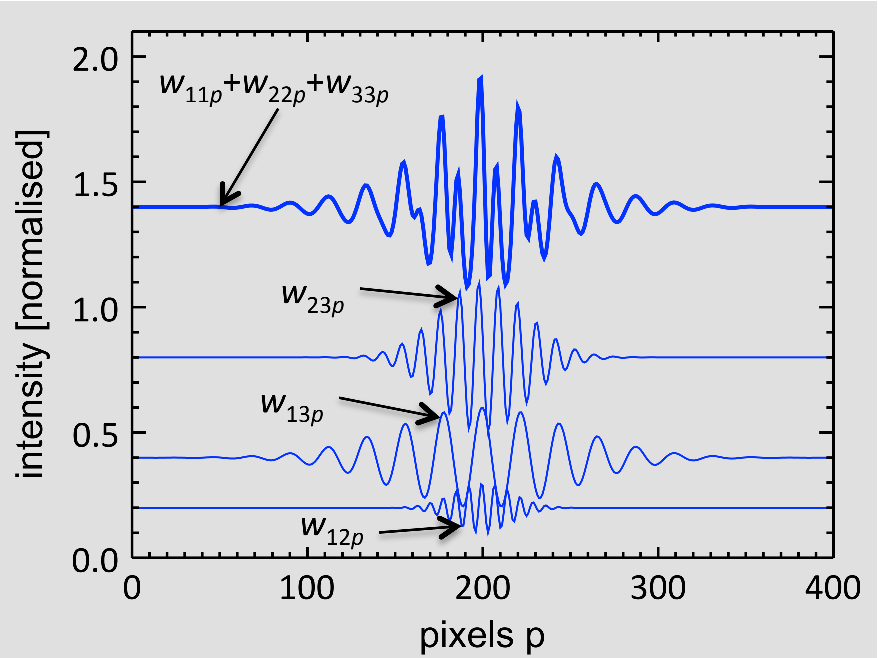
$$\hat{f}_{ij} = \sum_{p=0}^{3} \hat{I}_p \cdot e^{ip\pi/2}$$

$$= A - C + i\left(B - D\right)$$

for $\hat{I}_0 = A,\ \hat{I}_1 = B,\ \hat{I}_2 = C$ and $\hat{I}_3 = D$

$$\Rightarrow \left|\hat{f}_{ij}\right| = \sqrt{(A-C)^2 + (B-D)^2}$$

(2c) P2VM method



$$I_p = \sum \mathrm{Re}\left(f_{ij} w_{ijp} \right)$$

pixel intensity — corr. flux — carrier waveform

Rewrite this as a matrix equation:

$$I = M\, f$$

vector of pixel intensities — V2PM — vector of coherent fluxes

(2c) P2VM method

Forward matrix equation:

$$I = M f$$

Backward matrix equation:

$$\hat{f} = H \hat{I}$$

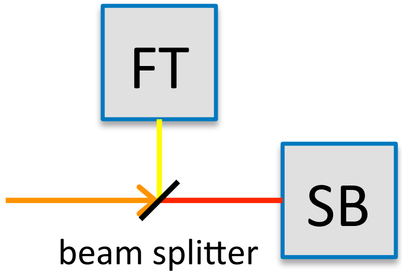
estimate of coherent fluxes ($\hat{f}$) — P2VM (H) — estimate of pixel intensities ($\hat{I}$)

Only pseudoinverse matrix exists ⟶ least squares fit:

$$\text{minimise } \chi^2 = \left| \hat{I} - MH\hat{I} \right|^2$$

Coherent integration

Coherent flux estimate very noisy.

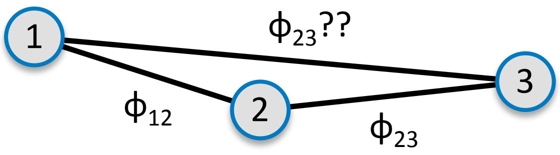


↳ Need to know the fringe motion:

1) External fringe tracker

2) Different channel

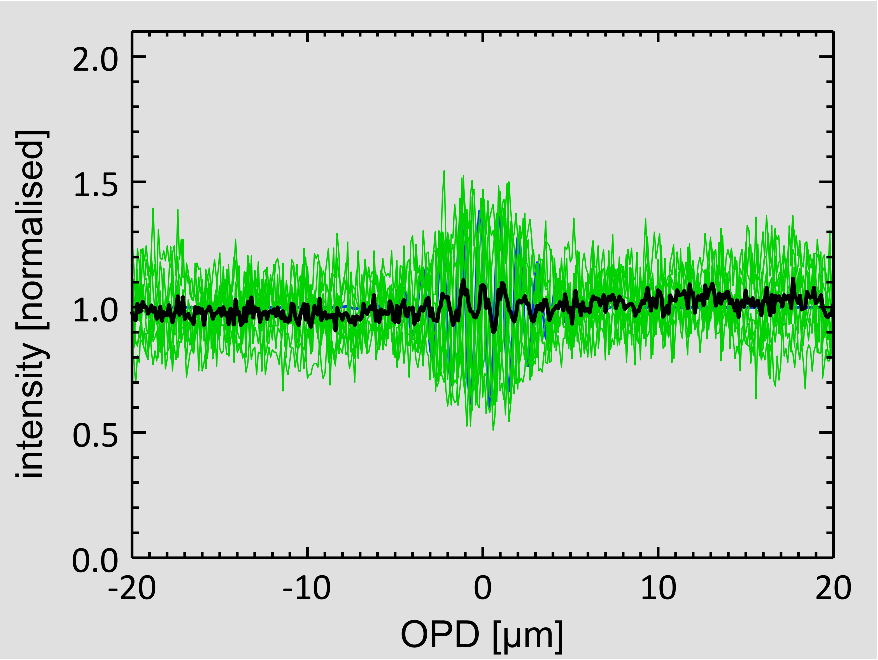
3) Baseline bootstrapping



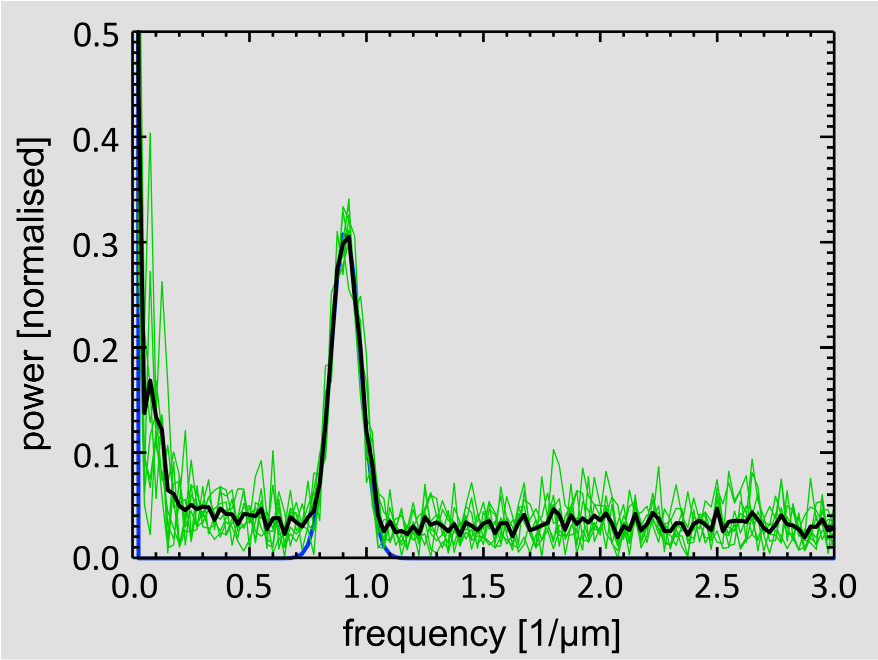
4) Group delay & phase delay

↳ "phase rotation" ⟶ coherent integration

(In-)Coherent integration



coherent integration



incoherent integration

Contents

Calibration

$$f_{ij} = \gamma_{ij} F(\boldsymbol{u}_{ij})$$

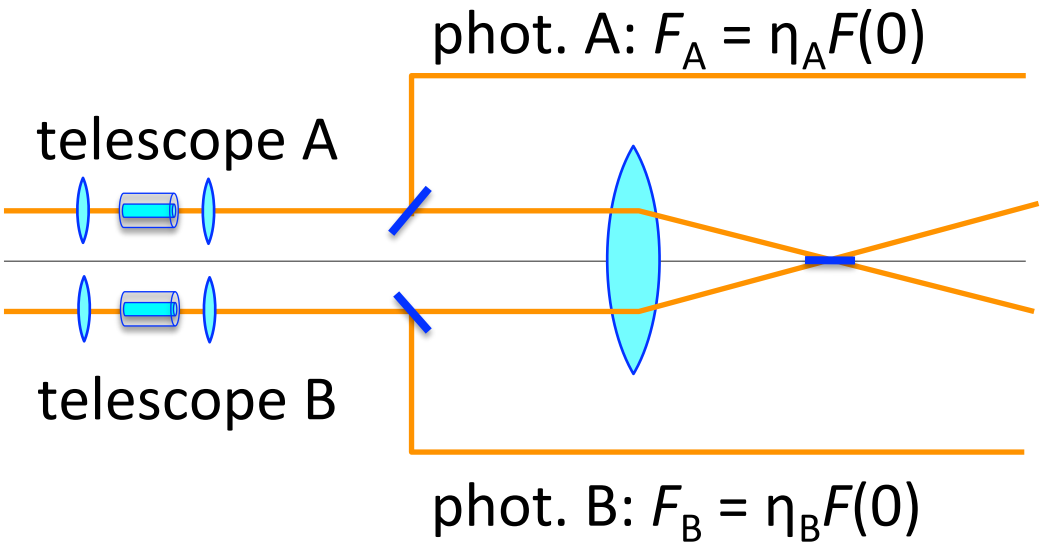
↖ fringe degradation factor

- Antenna-based gains: $\gamma_{ij} = \eta_i \eta_j$
  - pupil misalignment
  - spatial / modal filtering

  ↳ photometric calibration
- Transfer function calibration
  - fringe corruption
  - imperfect optics

  ↳ observations of calibrator stars

Calibration – photometric calibration



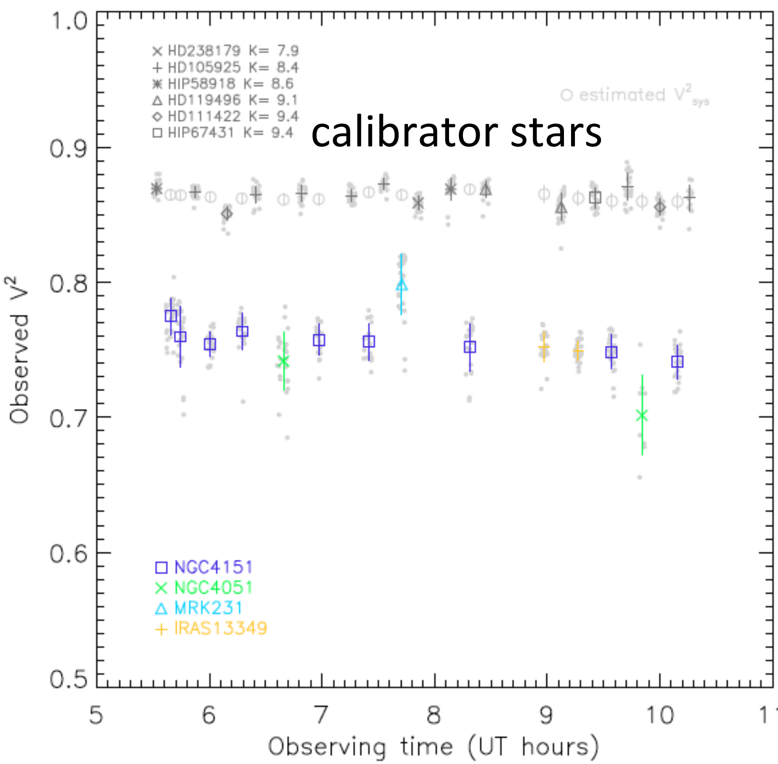
e.g. power spectrum:

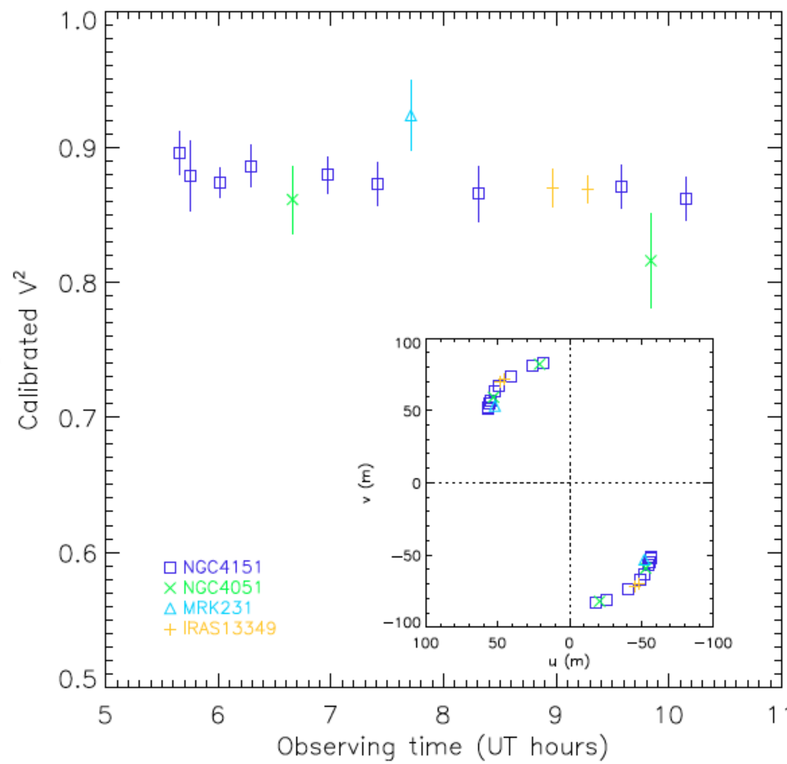
$\hat{p}_{ij} = \eta_i \eta_j \, |F(\boldsymbol{u}_{ij})|^2$

$$\langle \hat{V}_{ij} \rangle = \sqrt{\frac{\langle \hat{p}_{ij} \rangle}{\langle \hat{F}_i \, \hat{F}_j \rangle}} \approx \sqrt{\frac{\langle |\eta_i \eta_j| \rangle |F(\boldsymbol{u}_{ij})|^2}{\langle \eta_i F(0) \, \eta_j F(0) \rangle}} = |V(\boldsymbol{u}_{ij})|$$

Calibration – transfer function

Data from the Keck interferometer:



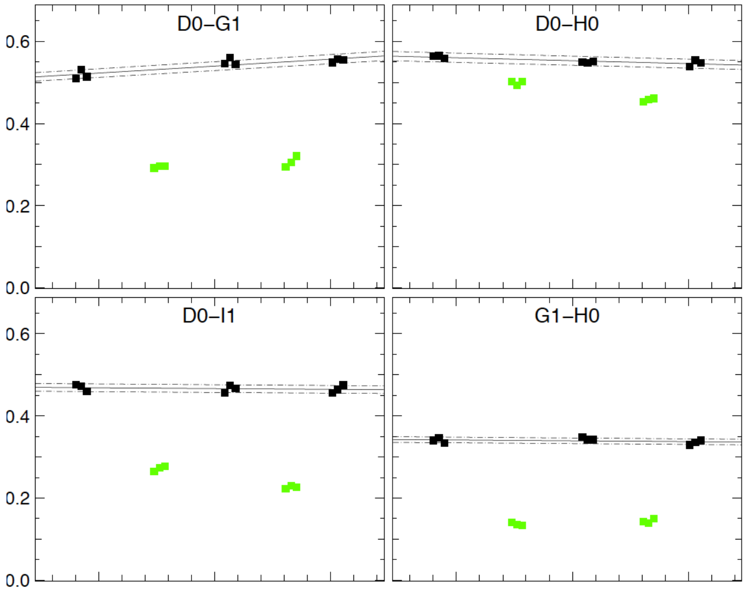


Kishimoto et al. 2009

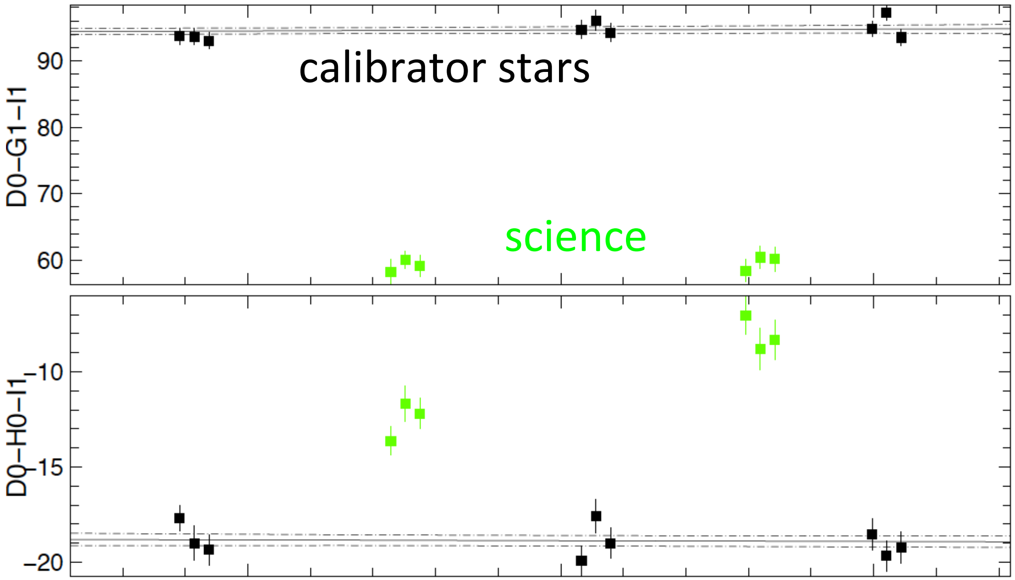
Calibration – transfer function

PIONIER data:

Visibilities:



Closure phases:



Summary

Interferometric data reduction
Is a well chosen sequence of:

- Calibrations (additive & multiplicative)
- Averaging of data
- Fourier Transforming
- Data fitting

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