formulae cheat-sheet for simple models

1 Generic properties of Fourier Transform

- **linearity (addition):** $FT[f+g] = FT[f] + FT[g]$,
- **translation (shift):** $FT[f(x-x_0, y-y_0)] = FT[f](u,v) \times e^{2i\pi,(ux_0+vy_0)}$,
- **similarity (zoom and shrink):** $FT[f(\alpha x, \beta y)] = \frac{1}{\alpha\beta} FT[f](\frac{u}{\alpha}, \frac{v}{\beta})$,
- **convolution ("blurring"):** $FT[f \otimes g] = FT[f] \times FT[g]$,
- ∞ **limit ("small" details):** $FT[f] \stackrel{\infty}{\longmapsto} 0$,
- 0 **limit ("large" details):** $FT[f] \stackrel{0}{\longmapsto} 1$.
- **(u,v) symmetry:** if f is a real function, the modulus of $FT[f]$ is an even function and the phase of $FT[f]$ is an odd function.

2 Generic models

Shape	Brightness distribution	Visibility
Point source	$\delta(\overrightarrow{x})$	1
Background	I_0	$\begin{cases} 1 & \text{if } \rho = 0 \\ 0 & \text{otherwise} \end{cases}$
Binary star	$I_0\left[\delta(\overrightarrow{x}) + R\delta(\overrightarrow{x} - \overrightarrow{x_0})\right]$	$\sqrt{\frac{1+R^2+2R\cos\left(\frac{\overrightarrow{\rho}\cdot\overrightarrow{x_0}}{\lambda}\right)}{1+R^2}}$
Gauss	$I_0\sqrt{\frac{4\ln\left(2\varnothing\right)}{\pi}} \times e^{-4\ln 2\frac{r^2}{\varnothing^2}}$	$e^{-\frac{\left(\pi\varnothing\rho\right)^2}{4\ln 2}}$
Uniform disk	$\begin{cases} \frac{4}{\pi\varnothing^2} & \text{if } r < \frac{\varnothing}{2} \\ 0 & \text{otherwise} \end{cases}$	$\frac{2\mathrm{J}_1\left(\pi\varnothing\rho\right)}{\pi\varnothing\rho}$
Ring	$\frac{1}{\pi\varnothing}\delta\left(r - \frac{\varnothing}{2}\right)$	$\mathrm{J}_0(\pi\varnothing\rho)$
Exponential	$e^{-k_0 r}$, $k_0 \geq 0$	$\frac{k_0^2}{1+k_0^2\rho^2}$
Any circular object	$I(r)$	$2\pi\int_0^\infty I(r)\mathrm{J}_0(2\pi r\rho) r dr$
Pixel (image brick)	$\begin{cases} \frac{1}{lL} & \text{if } x < l \text{ and } y < L \\ 0 & \text{otherwise} \end{cases}$	$\frac{\sin(\pi x l)\sin(\pi y L)}{\pi^2 x y l L}$
Limb-darkened disk (linear)	$\begin{cases} I_0[1 - u_\lambda(1-\mu)] & \text{if } r < \frac{\varnothing}{2} \\ \mu = cos(2r/\varnothing) \end{cases}$	$\begin{cases} \frac{\left[\alpha\frac{\mathrm{J}_1(x)}{x} + \beta\sqrt{\pi/2}\frac{\mathrm{J}_{3/2}(x)}{x^{3/2}}\right]^2}{\left(\frac{\alpha}{2}+\frac{\beta}{3}\right)^2} \\ \alpha = 1 - u_\lambda \\ \beta = u_l\lambda \\ x = \pi\theta_{\mathrm{LD}}\frac{B}{\lambda} \end{cases}$

$\varnothing$ = either the diameter for a ring or uniform disk, or FWHM. r or $\overrightarrow{s}$ represent the angles in the image plane. $\rho = \frac{B}{\lambda}$ or $\overrightarrow{\rho}$ are the spatial frequencies.